STOCHASTIC EXCITATION OF GRAVITY WAVES BY OVERSHOOTING CONVECTION IN SOLAR-TYPE STARS

BORIS DINTRANS[1], AXEL BRANDENBURG[2], ÅKE NORDLUND[3] and
ROBERT F. STEIN[4]

[1]*Laboratoire d'Astrophysique, Observatoire Midi-Pyrénées, 14 avenue Edouard Belin,
31400 Toulouse, France*
[2]*Nordic Institute for Theoretical Physics, Blegdamsvej 17, 2100 Copenhagen Ø, Denmark*
[3]*Theoretical Astrophysics Center, and Astronomical Observatory, Juliane Maries Vej 30,
2100 Copenhagen Ø, Denmark*
[4]*Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA*

Abstract. The excitation of gravity waves by penetrative convective plumes is investigated using 2D direct simulations of compressible convection. The oscillation field is measured by a new technique based on the projection of our simulation data onto the theoretical g-modes solutions of the associated linear eigenvalue problem. This allows us to determine both the excited modes and their corresponding amplitudes accurately.

Keywords: Stars: oscillations – Convection

1. Introduction

Two-dimensional simulations of compressible convection have shown that it is possible to excite internal gravity waves (IGW) in radiative zones of solar-type stars from the downward penetrating plumes (Hurlburt et al., 1986, hereafter HTM86; Hurlburt et al., 1994; Kiraga and Jahn, 1995). However, detecting IGW with confidence is challenging given the stochastic nature of the excitation mechanism.

We propose here a new detection method which allows us to measure rigorously both the spectrum and amplitude of excited g-modes. This method is first applied to the g-mode oscillations of an isothermal atmosphere and then, to IGW generated in a 2D-simulation of a convective zone embedded between two stable ones.

2. Detecting g-Modes using the Anelastic Subspace

In hydrosimulations, wave fields are commonly measured using two main methods: *(i)* the simplest one consists in recording the vertical velocity at a fixed point and then performing the Fourier transform of the sequence (see, e.g., HTM86); *(ii)* a more complicated method consists in taking two Fourier transforms, in space and time, of the vertical mass flux (Stein and Nordlund, 1990). However, these two methods are not well adapted to detect IGW in our problem because the Fourier
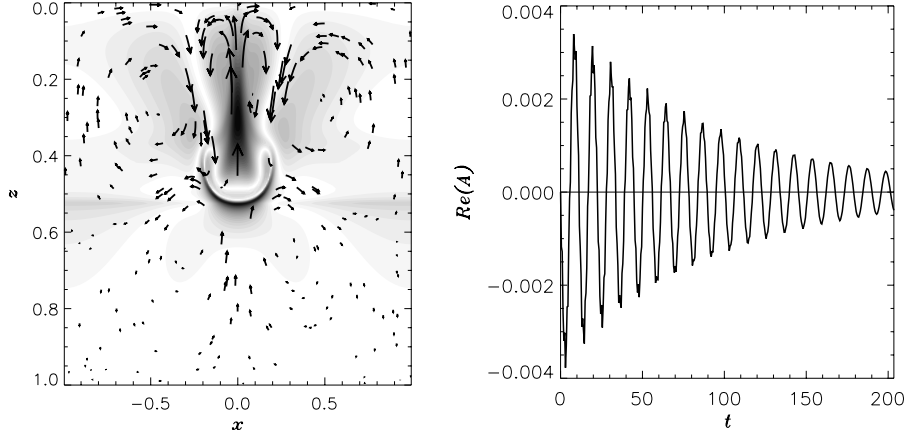
Figure 1. Left: slide of the velocity field superimposed on the contours of entropy perturbations for a 2D-simulation of an oscillating entropy bubble embedded in an isothermal atmosphere. Right: time-evolution of the real part of its projection coefficient A_{10} (here t is in units of d/c_s).

transforms are calculated over the *whole* simulation while IGW are *stochastically* excited by penetrating plumes.

Our new detection method takes into account the random nature of this excitation. Indeed, it is based first, on projections of the simulated velocity field $\vec{v}(k, z, t)$ onto the anelastic eigenvectors $\vec{\psi}_{kn}(z)$ as

$$\vec{v}(k, z, t) = \sum_{n=0}^{\infty} < \vec{\psi}_{kn}, \vec{v} > \vec{\psi}_{kn}(z) = \sum_{n=0}^{\infty} A_{kn}(t)\vec{\psi}_{kn}(z), \tag{1}$$

and second, on time-frequency diagrams of the complex coefficients $A_{kn}(t)$. As a consequence, the immediate spectrum (the set of frequencies ω) and amplitudes (defined as $|A_{kn}|$) of stochastically excited g-modes are reached and not only their 'mean' values over the whole simulation. It is instructive to consider the simplest possible case, that is, the propagation of a single gravity mode with horizontal wavenumber k through the computation domain: applying Eq. (1) leads in this case to a projection coefficient $A_{kn}(t) \propto \exp(i\omega_{kn}t)$, where ω_{kn} denotes the frequency of the anelastic eigenmode of degree k and order n.

3. Results

3.1. OSCILLATIONS OF AN ISOTHERMAL ATMOSPHERE

As a first test, we apply our method to detect IGW excited by an oscillating entropy bubble embedded in an isothermal atmosphere of depth d (see Fig. 1, left panel). In

this case, the building of the anelastic subspace is simplified since we found analytic solutions for the eigenfrequencies and their associated eigenvectors (Dintrans et al., 2002).

In Figure 1 (right panel), we show the real part of the projection coefficient $A_{10}(t)$, i.e. we projected the left panel velocity field onto the first anelastic eigenmode of the isothermal atmosphere at $k = 1$ and $n = 0$. As expected, we found that $A_{10}(t)$ behaves like $\exp(i\omega_{10}t)$ (with $\omega_{10} \simeq 0.569c_s/d$, where c_s is the constant adiabatic sound speed) whereas the mode amplitude $|A_{10}|$ decreases as $\exp(-t/t_\nu)$ with $t_\nu \propto \nu^{-1}$ (ν being the constant kinematic viscosity of the simulation).

3.2. 2D-SIMULATIONS OF PENETRATIVE CONVECTION

Once our method validated, we study the excitation of IGW by overshooting convection using high-resolution two-dimensional simulations of a three-layer polytropic model. That is, we solve the following equations:

$$
\begin{cases}
\dfrac{D\ln\rho}{Dt} = -\operatorname{div}\vec{u}, \\[2ex]
\dfrac{D\vec{u}}{Dt} = -(\gamma - 1)\left(\vec{\nabla}e + e\vec{\nabla}\ln\rho\right) + \vec{g} + \dfrac{1}{\rho}\vec{\nabla}\cdot(\rho\nu\vec{\mathcal{S}}), \\[2ex]
\dfrac{De}{Dt} = -(\gamma - 1)e\operatorname{div}\vec{u} + \dfrac{1}{\rho}\vec{\nabla}\cdot(\mathcal{K}\vec{\nabla}e) + \nu\vec{\mathcal{S}}^2 - \dfrac{e - e_0}{\tau(z)},
\end{cases}
\tag{2}
$$

where $\vec{u}$ denotes the velocity, e the internal energy, ρ the density, $\mathcal{K} = K/c_v$ where K is the radiative conductivity and c_v the specific heat at constant volume, $\vec{\mathcal{S}}$ the stress tensor and, finally, $\tau(z)$ is a cooling time (see Brandenburg et al., 1996 for more details).

Figure 2 (left panel) shows an example of such a simulation of a convective zone of depth $d = 1$ ($0 < z < 1$) embedded between two stable ones ($-0.15 < z < 0$ and $1 < z < 3$). IGW are excited in the bottom radiative zone by penetrating downward plumes and the evolution of the projection coefficient $A_{10}(t)$ is now more chaotic (see right panel). However, by applying a time-frequency diagram on this sequence, we extracted three IGW events (with $\omega_{10} \simeq 0.251\sqrt{g/d}$), emphasized as thick lines in the figure.
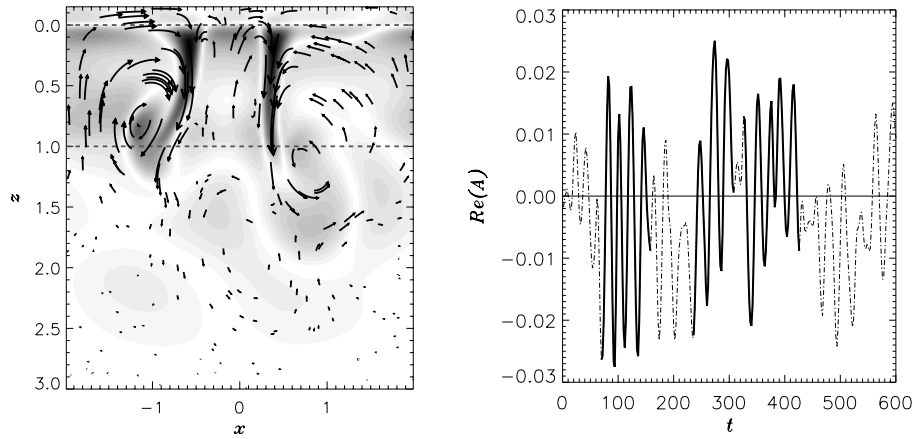
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Figure 2. Left: slide of the velocity field superimposed on the contours of entropy perturbations for a two-dimensional convection simulation. Two strong downwards plumes penetrate into the bottom radiative zone and excite IGW. Right: time-evolution of the real part of the projection coefficient A_{10} (here t is in units of $\sqrt{d/g}$). Thick lines emphasize IGW events detected using a time-frequency diagram.

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