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# The evolution of primordial magnetic fields since their generation

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## Abstract

We study the evolution of primordial magnetic fields in an expanding cosmic plasma. For this purpose we present a comprehensive theoretical model to consider the evolution of MHD turbulence that can be used over a wide range of physical conditions, including cosmological and astrophysical applications. We model different types of decaying cosmic MHD turbulence in the expanding Universe and characterize the large-scale magnetic fields in such a medium. Direct numerical simulations of freely decaying MHD turbulence are performed for different magnetogenesis scenarios: magnetic fields generated during cosmic inflation as well as electroweak and QCD phase transitions in the early Universe. Magnetic fields and fluid motions are strongly coupled due to the high Reynolds number in the early Universe. Hence, we abandon the simple adiabatic dilution model to estimate magnetic field amplitudes in the expanding Universe and include turbulent mixing effects on the large-scale magnetic field evolution. Numerical simulations have been carried out for non-helical and helical magnetic field configurations. The numerical results show the possibility of inverse transfer of energy in magnetically dominated non-helical MHD turbulence. On the other hand, decay properties of helical turbulence depend on whether the turbulent magnetic field is in a weakly or a fully helical state. Our results show that primordial magnetic fields can be considered as a seed for the observed large-scale magnetic fields in galaxies and clusters. Bounds on the magnetic field strength are obtained and are consistent with the upper and lower limits set by observations of extragalactic magnetic fields.

Keywords: cosmic magnetic fields, primordial magnetogenesis, turbulence

(Some figures may appear in colour only in the online journal)

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## 1. Introduction

Understanding the origin and evolution of cosmic magnetism is one of the challenging questions of modern astrophysics. The major questions include theoretical as well as observational aspects of the problem: when and how was the cosmic magnetic field generated? How did it evolve during the expansion of the Universe? What are modern observational constraints on the magnetic fields at large scales? Are magnetic fields observed at galactic and extragalactic scales of cosmological or astrophysical origin? The types of turbulence considered here are characterized by a strong random initial magnetic field. The interaction with the velocity field leads to inverse spectral transfer towards large scales that is unknown in non-magnetic turbulence<sup>10</sup>.

The goal is to identify important properties of cosmic magnetic turbulence in the expanding Universe. Properties of decaying MHD turbulence in primordial plasma link magnetogenesis scenarios operating in the early Universe with the constraints on the large-scale magnetic fields set by present observations. Hence, studying the magnetic field evolution, we can identify likely magnetogenesis scenarios responsible for exciting seed fields in the early Universe and exclude unlikely ones using constraints set by modern or future observations.

The problem of cosmological magnetogenesis is guided by recent observations of large-scale magnetic fields. Indeed, galaxies are known to have magnetic fields that are partly coherent on the scale of the Galaxy with field strengths reaching  $10^{-6}$  Gauss (G) (see [1–6] and references therein). These magnetic fields are the result of amplification of initial weak seed fields of unknown nature. Moreover, it is now clear that  $\mu$ G-strength magnetic fields were already present in normal galaxies (like our Milky Way) when the Universe was less than half of its present age [7–9]. This poses strong limits on the seed magnetic field strength and its amplification timescale.

From a theoretical point of view there are two scenarios that can lead to the generation of magnetic fields at extragalactic scales [10]: a bottom-up (astrophysical) scenario, where the seed field is typically very weak and the observed large-scale magnetic field is transported from local sources within galaxies to larger scales [11], and a top-down (cosmological) scenario where a significant seed field is generated prior to Galaxy formation in the early Universe on scales that are large at the present time [12]. The major theme of this review is to discuss the evolution, structure, and effects of cosmic magnetic fields with the goal to better understand its origin and observational signatures.

We will briefly discuss cosmic magnetohydrodynamic (MHD) turbulence in order to understand the magnetic field evolution. MHD turbulence in the context of astrophysical plasma processes has been studied for a long time. On the

other hand, the effects of MHD turbulence in cosmological contexts has received attention only in recent years [13]. Simulations show that the kinetic energy of turbulent motions in Galaxy clusters can be as large as 5–10% of the thermal energy density [14]. This can influence the physics of clusters [15], and at least should be modeled correctly when performing large-scale simulations [16–21]. Turbulent motions can also affect cosmological phase transitions; see [22–24] and references therein). Turbulence can be generated by a small initial cosmological magnetic fields. Understanding mechanisms for exciting primordial turbulence is an important goal. We argue that even if the total energy density present in turbulence is small, its effects might be substantial because of the strongly nonlinear nature of the relevant physical processes.

Recent important observations [25, 26, 28–33] (also [34] for recent study, and [35] for discussions on possible uncertainties in the measurements of blazar spectra), suggest the existence of magnetic fields in the Universe at scales large enough to suggest a primordial origin [10]. This result is robust to potential plasma instabilities of the two-stream family [36–38]. Prior to these observations, there existed only *upper* limits of the order of a few nG for the intergalactic magnetic field. These were obtained through Faraday rotation of the cosmic microwave background (CMB) polarization plane [39–49] and Faraday rotation of polarized emission of distant quasars [50–53]. Other tests to derive *upper* limits on large-scale correlated magnetic fields are based on their effect on the CMB (see [54] and references therein), [55–82], CMB distortions [83–92], the broken isotropy limits, [93–102], big bang nucleosynthesis (BBN) data [103–105], or large scale structure (LSS) formation [106–128]. The *lower limit* on the intergalactic magnetic field in voids of order  $10^{-18}$  G on 1 Mpc scales is a puzzle of modern astrophysics (see [129]), and could very well be the result of the amplification of a primordial cosmological field [31].

In what follows we review recent efforts which include the pioneering studies of primordial magnetic field evolution through cosmological phase transitions; see [130–140]. The decay of cosmic magnetic field in the Universe has been analyzed through numerical simulations of decaying MHD turbulence. Major findings include (i) the possibility of the inverse transfer of *non-helical* causally generated magnetic fields [135]; (ii) fast growth of vorticity in the magnetized Universe [132]; (iii) growth of helical structures at large scales for partially helical magnetic fields generated at cosmological phase-transitions [133, 134, 136, 137], and more interestingly the absence of the inverse cascade for inflation-generated fully or partially helical magnetic fields [139, 140].

## 2. Modeling MHD turbulence in the universe

The origin of the cosmic magnetic field has been discussed for decades, starting with Enrico Fermi's paper of 1949 [141]. The approach presented below is novel in several ways. (i) Primordial magnetic fields are generally analyzed in the 'frozen-in' approximation due to a high conductivity of

<sup>10</sup> The paper is based on the presentation by Tina Kahniashvili *Cosmic Magnetic Fields: Origin, Evolution, and Signatures* at the Turbulent Mixing and Beyond Workshop 2014 'Mixing in Rapidly Changing Environments—Probing Matter at the Extremes'.

cosmic plasma, when the magnetic field evolves only due to the dilution of field lines as the Universe expands. In contrast, we account for the actual coupling between the magnetic field and the cosmic plasma, which leads to major differences with the frozen-in approximation at some epochs. (ii) Much work on MHD turbulence is focused on specific astrophysical objects (such as galaxies, clusters, interstellar medium, or stellar magnetosphere). Instead we have developed a comprehensive theoretical framework to consider the evolution of MHD turbulence over a wide range of physical conditions, beyond any specific application. (iii) Cosmic MHD turbulence is usually studied within one of two limiting cases, the viscous (optically thick) or free-streaming (optically thin) regimes. These two regimes differ in the form of viscous or drag forces. Realistic turbulent behavior is somewhere in between these two limits, and the numerical simulations have the capability to describe adequately a smooth transition between these two regimes.

As noted above, several astrophysical observations show the presence of a large-scale correlated magnetic fields in the Universe. The recent study by Dolag *et al* [31] concludes that these magnetic fields are most likely seeded by a field of primordial origin. In fact, many different mechanisms of cosmological seed magnetic field generation have been proposed. Some of these employ symmetry breaking during phase transitions (e.g. electroweak or QCD) [142–162]. On the other hand, if the magnetic field originated during a cosmological phase transition, its configuration is strongly limited by causality [163]: the correlation length of the magnetic field cannot exceed the Hubble horizon at the moment of field generation. The causality condition combined with the divergence-free field condition implies a magnetic energy spectrum at large scales  $E_M(k) \propto k^4$  [164] (the so-called Batchelor spectrum [165]). Recent numerical simulations [131–135] confirm that cosmological turbulence produces a Batchelor spectrum completely independently of initial conditions present in the cosmic plasma. Combining this *causal* spectrum with the requirement that the total energy density of the magnetic field be less than 10% of the radiation energy density (to be consistent with standard BBN) leads to a strong limit on the smoothed amplitude of the magnetic field at large scales of the order of  $10^{-26}$  to  $10^{-19}$  Gauss at 1Mpc [166], although the effective value of the magnetic field derived through its total energy density is high enough, of the order of  $10^{-6}$  Gauss [130]. (Note that this argument does not account for further evolution of the magnetic field in MHD turbulence). Taking into account that the magnetic field effects are mostly determined by effective values (i.e. the total energy density), and noticing that the extremely low limits at large scales of causal fields are consequences of normalization (smoothing procedure), the upper bounds have been re-determined in terms of the effective strength of the magnetic field; see [44, 46, 122]. The BBN limits have also been re-analyzed by accounting for the MHD evolution of *toy* magnetic fields throughout expansion of the Universe [134].

Our particular interest lies in helical magnetic fields that can be generated in the early Universe; see [167–179] and

references therein. There are two main motivations for considering helical seed magnetic fields: (i) the presence of helical magnetic fields in the early Universe can be related to the lept- and baryogenesis problems [180]; (ii) it sheds light on the evolution of helical magnetic fields in stellar magnetospheres, AGNs, and voids [181, 182].

An exception to the Batchelor spectrum (spectral index  $n=4$ ) is the possibility of inflationary magnetogenesis, in which the spectral index of the magnetic field could be less than  $+1$ , and the simplest option is a scale-invariant spectrum with  $n \rightarrow -1$  [183–207]. Inflation-generated magnetic field scenarios should be considered with some caution due to the possibility of significant backreaction [208–211], which is not an issue for the phenomenological, effective classical model; see [212] and references therein. The first simulations describing the inflation-generated magnetic field coupled to the primordial plasma suggested that the presence of an initial magnetic field leads to large-scale turbulent motions in the rest plasma [132]. Ongoing research consists in the study of inflation-generated helical magnetic field (with a scale-invariant  $k^{-1}$  spectrum) evolution during the expansion of the Universe [138–140]. Simulations show that inflation-generated magnetic fields retain information about initial conditions. In other words, they decay very slowly when compared with phase transition-generated fields. Magnetic fields are almost ‘frozen-in’ the primordial plasma at large scales, where causality allows interaction only at scales smaller than the Hubble horizon and they correspondingly retain their initial spectral shape. On the other hand, within the causal horizon, the magnetic seed field interacts with cosmic plasma leading to the excitation of kinetic motions (turbulent velocities); see below. We also discuss an alternative approach where cosmic magnetic fields originate during the late stages of the evolution of the Universe [106, 213]. In this case the correlation length is strongly limited by the causality requirement. Due to the sharp spectral shape at large scales, the magnetic field amplitude might be low. The simplest astrophysical magnetogenesis mechanism invokes the ejection of magnetic flux from compact systems such as AGNs or supernovae [214, 215]. In this scenario the generation of a strong magnetic field is ensured by its extremely fast generation due to rapid rotation of the object [11]. Other mechanisms are based on the generation of a small seed by plasma processes [129, 216, 217], which are then amplified by MHD dynamo mechanisms [218, 219].

### 3. The model

As mentioned above, we focus on the magnetic field evolution during the expansion of the Universe from the moment of magnetic field generation until today. Over this lengthy period, the magnetic field is affected by different physical processes that result in amplification as well as damping: the complexities of the problem are due to the strong coupling between magnetic field and turbulent motions. First, to account for the cosmological expansion we must reformulate the MHD equations in terms of comoving quantities [220].

Specific epochs most relevant to the final configuration of the primordial magnetic field are related to cosmological phase transitions, neutrino decoupling, nucleosynthesis, recombination, and reionization; see [10, 221] for reviews and [110, 222–227].

In the following, we discuss numerical simulations performed with the PENCIL CODE. This public MHD code (<https://github.com/pencil-code>) (see also [228]) is particularly well suited for simulating turbulence owing to its high spatial (sixth order) and temporal (third order) accuracy, while still taking advantage of the finite difference in terms of speed and straightforward parallelization. Recent results from the PENCIL CODE include MHD turbulence simulations at the electroweak or QCD phase transitions [131–135].

### 3.1. Numerical technique

By default, the PENCIL CODE solves the MHD equations for the logarithmic density  $\ln \rho$ , flow velocity  $\mathbf{v}$ , and the magnetic vector potential  $\mathbf{A}$  as follows:

$$\frac{D}{D\eta} \ln \rho = -\nabla \cdot \mathbf{v}, \quad (1)$$

$$\frac{D}{D\eta} \mathbf{v} = \mathbf{J} \times \mathbf{B} - c_s^2 \nabla \ln \rho + \mathbf{f}_{\text{visc}}, \quad (2)$$

$$\frac{\partial}{\partial \eta} \mathbf{A} = \mathbf{v} \times \mathbf{B} + \mathbf{f}_M + \lambda \nabla^2 \mathbf{A}. \quad (3)$$

Here,  $\eta$  is the conformal time and  $D/D\eta \equiv \partial/\partial\eta + \mathbf{v} \cdot \nabla$  is the advective time derivative,  $\mathbf{f}_{\text{visc}} = \nu(\nabla^2 \mathbf{v} + \frac{1}{3} \nabla \nabla \cdot \mathbf{v} + \mathbf{G})$  is the compressible viscous force for constant  $\nu$ ,  $G_i = 2S_{ij} \nabla_j \ln \rho$ , and  $S_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}) - \frac{1}{3} \delta_{ij} v_{k,k}$  is the traceless rate-of-strain tensor. The pressure is given by  $p = \rho c_s^2$ , where  $c_s = 1/\sqrt{3}$  is the speed of sound in the case of an ultra-relativistic gas, and  $\mathbf{J} = \nabla \times \mathbf{B}/4\pi$  is the current density.

In our simulations we use a vanishing magnetic forcing term  $\mathbf{f}_M = \mathbf{0}$  everywhere, except for the purpose of producing initial conditions, as explained below.

### 3.2. Initial conditions

To produce initial conditions, we run the simulation for a short time ( $\Delta t \approx 0.5 \lambda_l / c_s$ ) with a random (in time)  $\delta$ -correlated magnetic force  $\mathbf{f}_M$  in equation (3). The forcing term is composed of plane monochromatic waves pointing randomly in all possible directions with an average wavenumber  $k_0$  and fractional helicity  $\langle \mathbf{f}_M \cdot \nabla \times \mathbf{f}_M \rangle / \langle k_0 \mathbf{f}_M^2 \rangle = 2\sigma / (1 + \sigma^2)$ . Here  $\sigma$  is the parameter characterizing the initial forcing. Initial conditions for the magnetic and velocity fields produced from such a procedure have the advantage of being turbulent, still self-consistent solutions of the MHD equations.

### 3.3. Effective magnetic field characteristics

A magnetic field generated during phase transitions through any magnetogenesis scenario should satisfy the causality

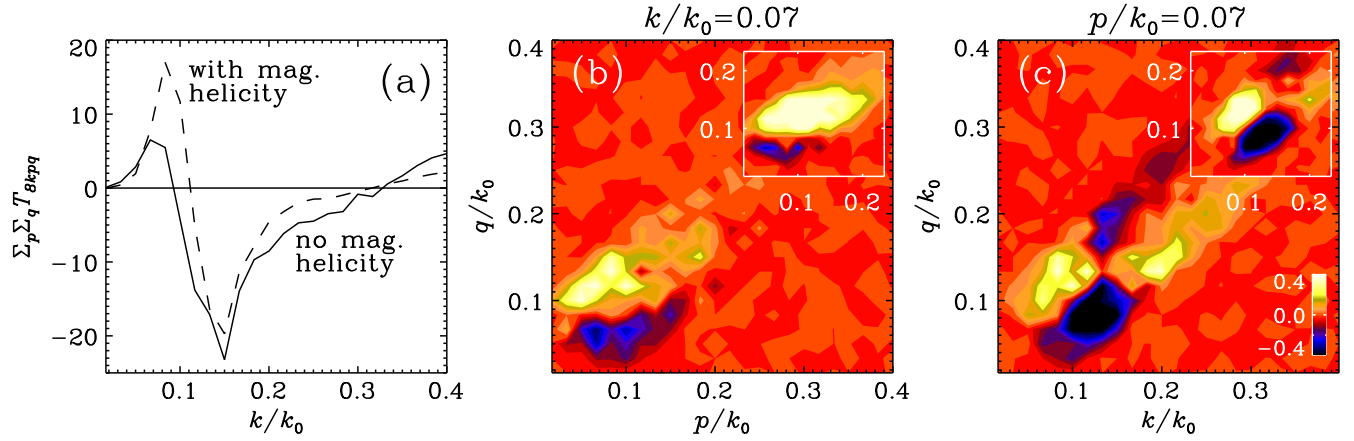
condition [163, 164, 220]. The maximal correlation length  $\xi_{\text{max}}$  of a causally generated primordial magnetic field should be shorter than the Hubble radius at the time of generation,  $H_*^{-1}$ . We define the parameter  $\gamma = \xi_{\text{max}}/H_*^{-1} \leq 1$ , which can describe the number of primordial magnetic field bubbles inside the Hubble radius, and thus  $N \propto \gamma^{-3}$ . To account for the Universe expansion we use comoving length, which is measured today and corresponds to the Hubble radius at the moment of magnetic field generation. Comoving length should be inversely proportional to the phase transition temperature ( $T_*$ ):

$$\lambda_{H_*} = 5.8 \times 10^{-10} \text{ Mpc} \left( \frac{100 \text{ GeV}}{T_*} \right) \left( \frac{100}{g_*} \right)^{1/6}. \quad (4)$$

For the QCD phase transition ( $g_* = 15$  and  $T_* = 0.15$  GeV), the comoving length equals 0.5 pc, while for the electroweak phase transition ( $g_* = 100$  and  $T_* = 100$  GeV) it should be equal to  $6 \times 10^{-4}$  pc. In all cases, the correlation length of the primordial magnetic field should not exceed the comoving value of the Hubble radius:  $\xi_{\text{max}} \leq \lambda_H$ . Obviously, the latter condition accounts *only* for the increase of the correlation length due the expansion of the Universe, and does not account for the effects of cosmic MHD turbulence (free decay or an inverse cascade in the case if primordial magnetic fields have nonzero helicity). Note, that the number of bubbles inside the Hubble radius is around 6 ( $\gamma \simeq 0.15$ ) for the QCD phase transition and around 100 ( $\gamma \simeq 0.01$ ) for the electroweak phase transition. Thus, the maximal correlation length for the QCD and electroweak phase transitions should be 0.08 pc and  $6 \times 10^{-6}$  pc, respectively. On the other hand, the correlation length is unlimited in the case of inflation-generated magnetic fields.

As mentioned above, the primordial magnetic field contributes to the relativistic component and thus the total energy density of the primordial magnetic field  $\rho_B(a_N)$ , where  $a_N$  is the scale factor during nucleosynthesis, is limited by the BBN bound: it cannot exceed 10% of the radiation energy density  $\rho_{\text{rad}}(a_N)$ . It is straightforward to see that the maximal value of the effective magnetic field defined through the total magnetic energy does not depends on the temperature at the moment of generation ( $T_*$ ), and depends weakly on the relativistic degrees of freedom ( $\gamma$ ) at the moment of the magnetic field generation.

The dominant contribution to the magnetic field energy density comes from the given length-scale, the so-called *integral scale*, where the magnetic field strength reaches its maximum. Thus, when dealing with phase transition-generated magnetic fields, we adopt the following idealizing approximation: we generate initial conditions for freely decaying turbulence simulations by running a numerical simulation of forced MHD equations for a short time interval. The external electromagnetic force, intended to generate a turbulent state, is introduced in the form of  $\delta$  functions that peak at a characteristic wavenumber,  $k_0 = 2\pi/\xi_0^{-1}$ . This yields random magnetic fields with correlation length  $\xi_0$ . Thus, the magnetic field strength at the characteristic length scale is  $B^{(\text{eff})} = \sqrt{8\pi\rho_B}$ . The characteristic length scale of the



**Figure 1.** Spectral transfer function  $T_{kpq}$ , (a) as a function of  $k$  and summed over all  $p$  and  $q$ , (b) as a function of  $p$  and  $q$  for  $k/k_1 = 4$ , and (c) as a function of  $k$  and  $q$  for  $p/k_1 = 4$ . The dashed line in (a) and the insets in (b) and (c) show the corresponding case for a direct numerical simulations with helicity; both for  $\text{Pr}_M = 1$ . See figure 3 of [135].

initial turbulent state  $\xi_0$  is set by the size of the largest magnetic eddies, because the primordial magnetic field evolves with the primordial MHD turbulence. In this approach, the characteristic length scale of the magnetic field is set by the bubble size of the phase transition when magnetogenesis occurs.

### 3.4. Magnetic field spectrum

The interaction between magnetic field and plasma gives rise to kinetic motions, and the turbulent backreaction results in spreading of the spectral energy density of magnetic field over a range of wavenumbers. At scales longer than the integral scale of the turbulence (small wavenumbers), the spectral energy density develops into the form of a power law  $E_M = Ak^n$ , where  $A$  is a normalization constant, and  $n$  is the spectral index. The spectrum of the turbulent magnetic field can be determined by the spectral expansion of the two-point correlation function of the magnetic field  $\langle B_i(x)B_j(\mathbf{x} + \mathbf{r}) \rangle$ , whose Fourier transform with respect to  $\mathbf{r}$  gives the spectral function

$$F_{ij}^M(\mathbf{k}) = P_{ij}(\mathbf{k}) \frac{E^M(k)}{4\pi k^2} + i\varepsilon_{ijl} k_l \frac{H^M(k)}{8\pi k^2}. \quad (5)$$

Here,  $P_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j / k^2$ ,  $\varepsilon_{ijl}$  is the antisymmetric tensor, and  $H^M(k)$  is the magnetic helicity spectrum. In this case, a white noise spectrum corresponds to the spectral index  $n = 2$  [163], while the Batchelor spectrum corresponds to the spectral index  $n = 4$  [164]. The power law of large-scale MHD turbulence spectrum extends down to the integral scale  $\xi_M$ , which is itself a time-dependent quantity throughout the turbulence decay process. At scales shorter than the integral scale the spectral energy density of the magnetic field decreases rapidly due to the combined action of turbulent decay and viscous damping.

### 3.5. Decay laws

The correlation length of the turbulent magnetic field evolves in time during the free decay of turbulence. We may describe

the decay laws of the magnetic correlation length  $\xi_M(\eta)$  and the spectral energy density  $\mathcal{E}_M(\eta)$  using two power law indices  $n_\xi$  and  $n_E$ :

$$\xi_M(\eta) = \xi_M(\eta_0) \left( \frac{\eta}{\eta_0} \right)^{n_\xi}, \quad (6)$$

$$\mathcal{E}_M(\eta) = \mathcal{E}_M(\eta_0) \left( \frac{\eta}{\eta_0} \right)^{n_E}. \quad (7)$$

The spectral energy density of the primordial MHD turbulence spectrum can be split into its large-scale and short-scale components, above and below the time dependent integral scale:

$$E_M(k, \eta) = E_0(\eta) \begin{cases} \bar{k}^4 & \text{when } k < k_I(\eta) \\ \bar{k}^{-5/3} & \text{when } k > k_I(\eta) \end{cases}, \quad (8)$$

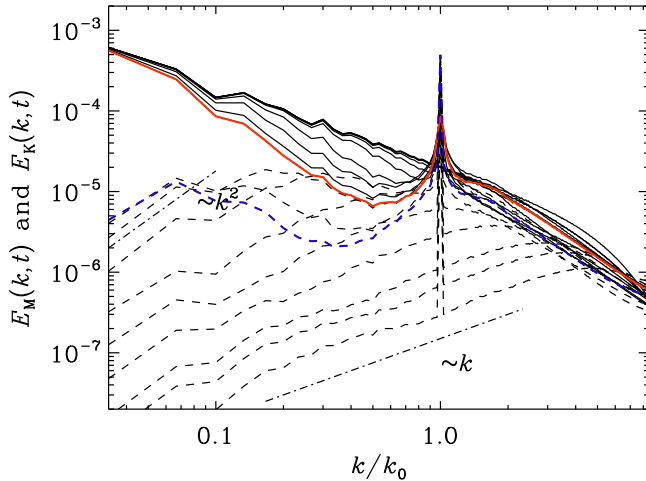
where  $\bar{k} = k/k_I$  and  $k_I(\eta) = 2\pi/\xi_M(\eta)$ . Hence, equations (6) and (7) can be used to describe the time evolution of spectral amplitude of magnetic field  $E_0(\eta)$  for a given turbulent spectrum:

$$E_0(\eta) = \frac{5}{17\pi} \xi_M(\eta_0) \mathcal{E}_M(\eta_0) \left( \frac{\eta}{\eta_0} \right)^{n_\xi + n_E}. \quad (9)$$

## 4. Results

### 4.1. The inverse transfer for non-helical fields

The inverse cascade is by now a well-known effect in helical magnetic turbulence [229]. One of the remarkable results is the presence of *non-helical* inverse transfer for magnetically dominated (causally generated) MHD turbulence; see figure 1, where we show spectral energy transfer rates, which demonstrate that the inverse transfer is about half as strong as with helicity. However, in both cases the magnetic gain at large scales results from velocity at similar scales interacting with smaller-scale magnetic fields [135]. This result has not



**Figure 2.** Magnetic (solid lines) and kinetic (dashed lines) energy spectra in regular time intervals.  $Re \approx 170$ . The magnetic and kinetic spectra at the last time are additionally marked in red and blue, respectively.

been emphasized in previous studies, see [230, 231] and references therein, and has now been confirmed by independent research groups [232–234].

Recent high resolution simulations with different magnetic Prandtl numbers  $Pr_M = \nu/\lambda$  [135] have shown a clear  $k^{-2}$  spectrum in the inertial range. This is the first example of fully isotropic magnetically dominated MHD turbulence (governed by the phase transition-generated magnetic fields) exhibiting what we argue to be weak turbulence scaling [235]. On the other hand, the Kolmogorov scaling  $k^{-5/3}$  has been recovered for the case of the inflation-generated helical magnetic fields [139].

#### 4.2. Inflation generated magnetic fields

Magnetic fields generated during the inflation should be affected by cosmological phase transitions occurring at later times during the expansion of the Universe. In this case, a separate study of the imprint of phase transitions on cosmic magnetic fields is needed. For this purpose we adopt a general approach that can be applied to both, QCD and electroweak phase transitions. In each case, turbulence forcing is determined by the phase transition bubble size. Rapid phase transitions generate turbulence, which then decays slowly at large scales. In contrast to previous studies, the inflation-generated magnetic field is not frozen into the cosmic plasma. Turbulence is generated during a short forcing period, which then is followed by slow decay (see [131, 132] for details). Recent simulations showed an increasing characteristic length scale of the velocity field and the establishment of a  $k^2$  (white noise) spectrum at large scales. This increase of vorticity perturbations occurs until it reaches equipartition with the magnetic field [134]. Figure 2 shows the evolution of kinetic and magnetic field spectra from those simulations. Numerical results show that inflation-generated magnetic fields are not significantly modified at large scales by their coupling to the plasma during a cosmological phase transition. The coupling

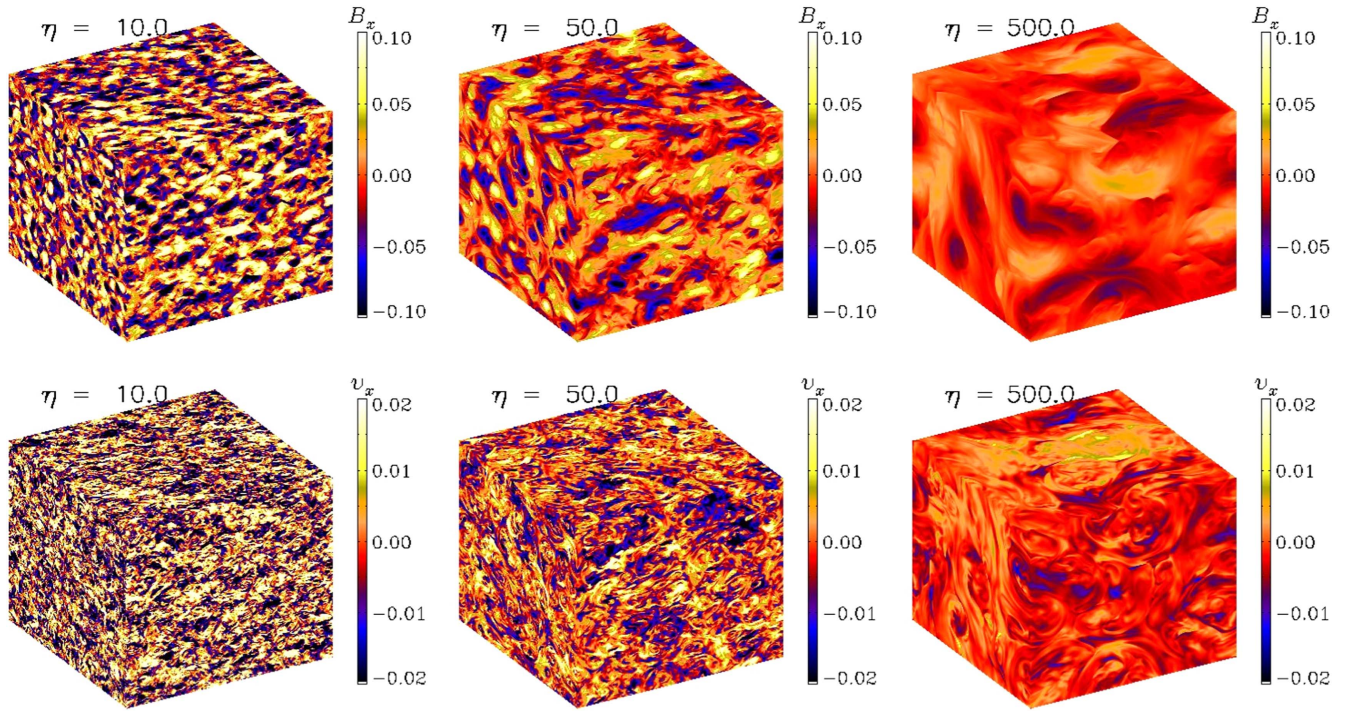
of cosmic magnetic field with the phase transition-generated fluid turbulence leads to deviations of the magnetic field spectrum from the initial scale-invariant shape only at intermediate scales. Figure 3 shows different snapshots of the decaying helical MHD turbulence. Figure 4 shows magnetic field and density fields developed using different values of initial forcing.

Ongoing research consists in pursuing high resolution numerical simulations of helical inflation-generated magnetic field evolution. Such a field, being subject to inflationary expansion, is characterized by a scale-invariant spectrum  $n \rightarrow -1$ , and its correlation length can be as large as Hubble horizon today or even larger (i.e., even when the total energy density  $\mathcal{E}_M$  is finite, the correlation length  $\xi_M \propto \int dk E_M(k)/k$  diverges for  $k \rightarrow 0$ ). In contrast to well known helical magnetic field decay laws [13, 165, 226, 236–238, 240–244], an absence of the inverse cascade has been found for inflation-generated magnetic fields. Furthermore, an unusually slow growth of the correlation length and conservation of helicity has been recovered even for the case of partially helical magnetic fields. These unexpected and unknown features of magnetic helicity are the result of a substantial turbulent power at large scales and the impossibility of the redistribution of helical fields at small wavenumbers (only the forward cascade is possible). A more thorough investigation of this phenomenon will be performed through varying initial conditions and basic parameters of primordial plasma.

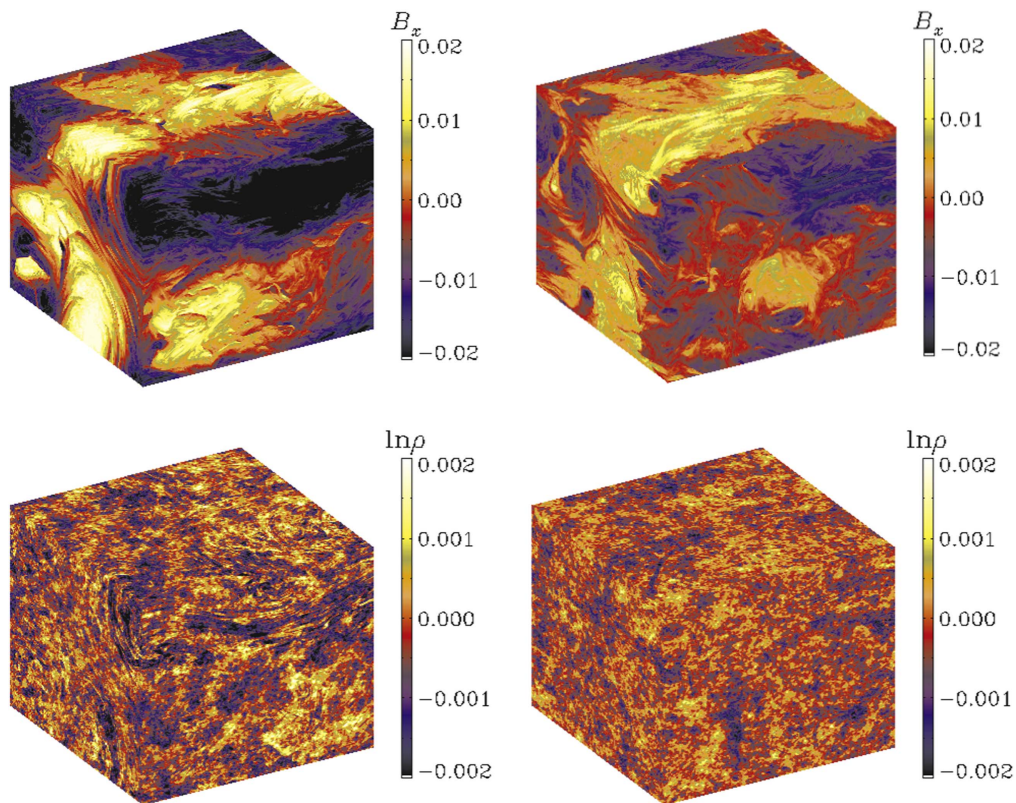
#### 4.3. Growth of helical structures

It is long known that the magnetic helicity plays a crucial role in determining the evolution pattern of MHD turbulence. Distinct evolution characteristics are known for helical and non-helical fields. In recent simulations, a partially helical initial magnetic field was used [133], assuming a tiny initial magnetic helicity during the QCD phase transition. It was shown that at late times the resulting field attains the maximally allowed magnetic helicity. This result is important since helicity crucially affects the MHD dynamics, and has very interesting consequences in astrophysical objects (e.g. galactic magnetic fields [245–247], for example). The resulting magnetic field has an amplitude of around 0.04 nG and a correlation length of order 20 kpc, which (assuming realistic scenarios of amplification [31]) serves as a seed for galactic magnetic fields. At this point the electroweak phase transition-generated magnetic fields are less promising due to a smaller initial correlation length, but are not completely excluded [248], in particular for fully helical magnetic fields [134].

Magnetic helicity is a crucial factor that affects the evolution of primordial magnetic fields. The evolution of the primordial magnetic field that has been produced with weak initial magnetic helicity that undergoes two consecutive stages. During the first stage, the evolution of a partially helical magnetic field spectrum is very similar to that of non-helical magnetic fields, and is sometimes described as a direct cascade. At this stage the spectral energy density cascades from large to small scales, where it undergoes de-correlation

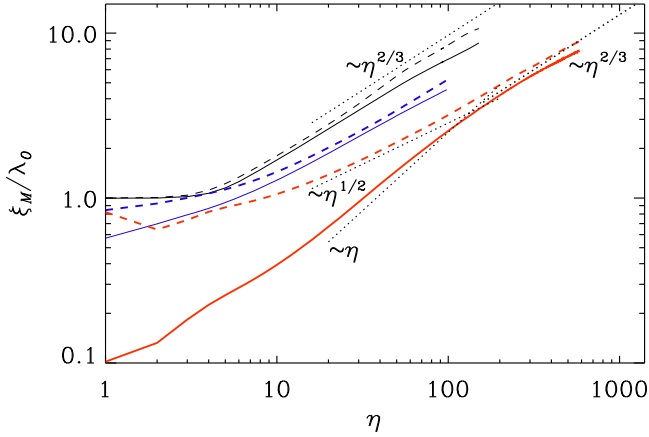


**Figure 3.** Visualizations of  $B_x$  (upper row) and  $v_x$  (lower row) at three times during the magnetic decay of a weakly helical field with  $\sigma = 0.03$  generated during QCD phase transitions. See figure 2 of [133].



**Figure 4.** Comparison of  $B_x$  (upper row) and  $\ln \rho$  (lower row) for an inflation-generated magnetic field with  $\sigma = 1$  (left) and  $\sigma = 0.03$  (right).





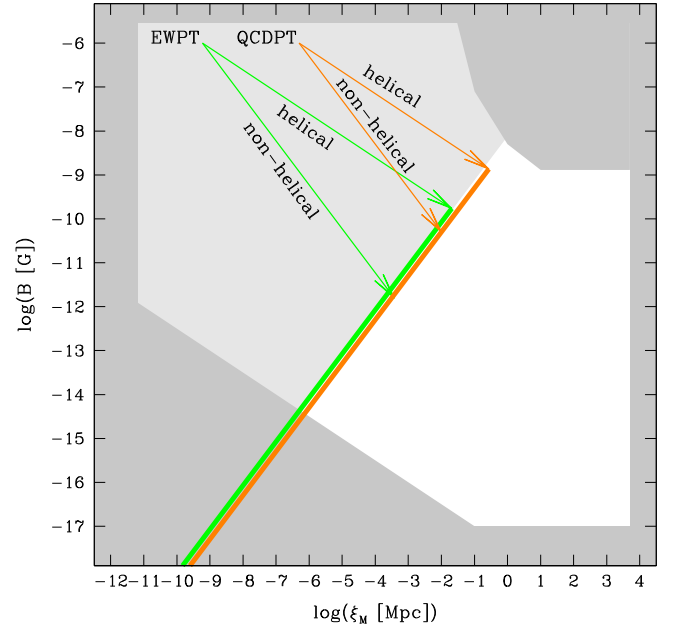
**Figure 5.** Evolution of turbulent correlation length  $\xi_M(\eta)$  (solid) and minimal correlation length  $\xi_M^{\min}(\eta)$  (dashed) of a helical state for  $\sigma = 1$  (black), 0.1 (blue) and 0.03 (red). (Figure 3 of [133]).

and viscous damping. Magnetic helicity is conserved and hence its fractional value increases during the turbulent decay process. The second stage in the primordial magnetic field evolution sets in when the turbulent state with maximal helicity is reached. The maximal value of the magnetic helicity that can be reached is limited by the realizability condition. Indeed, the conservation of magnetic helicity leads to a decay of the magnetic energy density inversely proportional to the correlation length of the turbulence:

$$\xi_M(\eta) \geq \xi_M^{\min}(\eta) \equiv |\mathcal{H}_M(\eta)|/2\mathcal{E}_M(\eta), \quad (10)$$

where  $\xi_M^{\min}(\eta)$  is the minimal correlation length of the turbulent state. Hence, an inverse cascade develops during the second stage of the primordial magnetic field evolution. In [133], we have studied the  $\xi_M(\eta)$  and  $\xi_M^{\min}(\eta)$  for  $\sigma = 1, 0.1$ , and 0.03 in the case of the QCD phase transition. It seems that, especially at lower  $\sigma$ , the increase of  $\xi_M$  is slow ( $\sim \eta^{1/2}$ ) while  $\xi_M(\eta) \gg \xi_M^{\min}(\eta)$ . However, since magnetic helicity conservation implies that  $\mathcal{E}_M$  decreases as  $\eta^{-1}$ , the minimal correlation length  $\xi_M^{\min}(\eta)$  soon reaches  $\xi_M(\eta)$ ; see figure 5. When the correlation length of the turbulent magnetic field reaches a minimal correlation length, the turbulence reaches its fully helical state. Then the turbulence decays according to the helical turbulent decay laws:  $\xi_M \sim \eta^{2/3}$  and  $\mathcal{E}_M \sim \eta^{-2/3}$ . Hence, we identify two distinct phases in the MHD turbulence evolution: the phase of *weakly helical* turbulence decay with  $n_\xi = 1/2$  and  $n_E = -1$ , and the phase of *fully helical* turbulent decay with  $n_\xi = 2/3$  and  $n_E = -2/3$ . The fully helical case is characterized by an inverse cascade where  $E_0(\eta) \propto \xi_M(\eta)\mathcal{E}_M(\eta) = \text{const}$  (see equation (9)). These results are in full agreement with earlier works [13, 165, 236–238, 240, 242]. The effective coupling of the primordial magnetic fields and cosmic plasma ends at the time of recombination. At later stages, primordial magnetic fields exhibit much slower developments [220].

Knowing the initial values of the turbulent magnetic correlation length  $\xi_M(\eta_0)$ , the minimal correlation length  $\xi_M^{\min}(\eta_0)$  set by the realizability condition, we can calculate the time interval  $\eta_{\text{fully}}$  needed for the turbulence to reach its



**Figure 6.** Evolution of the effective magnetic field  $B^{(\text{eff})}$  and correlation length  $\xi_M$  in the case of magnetogenesis at electroweak phase transition (green) and QCD phase transition (orange). Arrows indicate the evolutionary path of the strength and integral scale of helical and non-helical turbulent magnetic fields during the radiation-dominated era up to their final values. Thick solid line(s) show possible present day field strengths and integral scales of the magnetic field generated during phase transitions (see figure 7 of [134]).

fully helical state during the decay process. Since these two scales approach each other as  $\eta^{1/2}$ , the result is  $\eta_{\text{fully}} = \eta_0 [\xi_M(\eta_0)/\xi_M^{\min}(\eta_0)]^2$ . Hence, the time interval needed for the development of a fully helical state can be calculated using the initial values of turbulent energy and helicity:

$$\eta_{\text{fully}} = 4\eta_0 \xi_M^2 \mathcal{E}_M^2 / \mathcal{H}_M^2. \quad (11)$$

Note that this time increases inversely proportional to the square of the initial helicity of the turbulent state  $\mathcal{H}_M$ . Assuming that the initial magnetic helicity is  $\xi_M/\lambda_{H^*}$ , times lower than the maximal helicity in the case of strong CP violation during phase transition, we can calculate the time needed for the cosmic turbulence to reach its maximally helical state:  $\eta_{\text{fully}} = \eta_0/\gamma^2$ .

## 5. Discussion

Let us now discuss our results in the broader context of the workshop ‘Mixing in Rapidly Changing Environments—Probing Matter at the Extremes’. Our work has demonstrated a rather generic trend of decaying MHD turbulence to display an increase of energy at large length scales. This process is well-known in helical MHD turbulence [229], but to a lesser extent it also occurs in non-helical MHD turbulence [135]. Moreover, if there is small fractional helicity initially, this fraction will increase with time proportional to the square root

of time. At a certain time, the fractional helicity will be 100%, after which the decay of energy slows down and the increase of the correlation length speeds up.

An increase of energy at large length scales is not a common phenomenon in hydrodynamic turbulence and has never been found for passive scalars. This special behavior in MHD is likely to lead to unconventional mixing properties, although this has not yet been well quantified for decaying MHD turbulence. However, for statistically stationary turbulence, an increase of energy at small wavenumbers is usually described as non-diffusive turbulent transport, which is particularly well known in mean-field dynamo theory [250]. In a sense, this is more reminiscent of what might look like ‘anti-mixing’. This is to some extent due to the fact that vector fields behave differently from scalar fields. This can in part be due to the presence of additional conservation laws. In particular, magnetic helicity provides an extremely powerful constraint.

The significance of studying decaying MHD turbulence is manifold. On the one hand, our results will help to better understand the nature of cosmic magnetism and will gain insight into the decay laws of cosmic MHD turbulence. On the other hand, our analysis can be applied not only to cosmological scales, but to molecular clouds or even protoplanetary disks where decaying magnetic turbulence can crucially affect the global state or the formation of local structures. The results of our research therefore have important implications in many areas, including fluid dynamics, early Universe physics, high energy astrophysics, MHD modeling, and large-scale structure formation in the Universe.

## 6. Conclusions

We have discussed the evolution of primordial magnetic fields during the expansion of the Universe and have addressed some of the observational signatures. The coupling between the magnetic fields and cosmic turbulence leads to novel results as compared to previously adopted frozen-in approximations when magnetic field evolution was considered solely due to the field line dilution in the expanding Universe. For this purpose, we have developed a comprehensive theoretical model to consider the evolution of MHD turbulence over a wide range of physical conditions, beyond any specific astrophysical application.

Numerical simulations have shown novel effects in the evolution of magnetic fields in cosmic turbulence. It seems that inverse transfer that normally occurs in helical MHD turbulence, can also take place for non-helical magnetic fields if the MHD turbulence is magnetically dominated. In this case, large-scale velocity perturbations power up the magnetic field, leading to substantial increase of magnetic power spectra at large scales and corresponding inverse transfer.

An analysis of the inflation-generated magnetic field is carried out to find out how they are affected by the cosmic phase transitions (QCD and electroweak phase transition). Numerical results show that large-scale magnetic fields survive phase transitions, and thus, phase transitions cannot rule

out inflationary magnetogenesis as the source of seed magnetic field in the Universe.

On the other hand, magnetic fields generated during phase transitions can have tiny helicity. We have shown that during the evolution in MHD turbulence magnetic field helicity grows until it reaches maximal helical state. It seems that helicity growth rate is fast enough to reach maximal helicity well before the epoch of recombination, when the primordial magnetic field decouples from cosmic turbulence.

We have used results of high resolution simulations to set limits on the large-scale magnetic field generated during early stages of the evolution of the Universe (inflation or cosmological phase transitions). Indeed, it seems that magnetic fields produced during this epochs and subsequently modified by cosmic MHD turbulence can reach amplitudes similar to the lower bounds of the observational magnetic fields even accounting for the effects of large-scale turbulent decay as well as additional Alfvén wave damping. The extremely low values derived for smoothed magnetic field [166] do not imply that the effective magnetic field is also small in the 1 pc–1 kpc range and cannot lead to the observational signatures in blazar emission spectra. Using the effective magnetic field approach we obtain results that do not depend on the specific spectral shape of the magnetized turbulence. Observational signatures of the magnetogenesis during electroweak phase transition can come from future observations, if weak magnetic fields with  $10^{-14}$ – $10^{-15}$  G amplitude and few pc correlation length are detected. While a somewhat stronger magnetic field with a correlation length of the order of kpc might indicate the presence of QCD phase transition magnetogenesis. In turn, magnetic fields with extremely large correlation lengths (1Mpc or higher) will indicate inflationary magnetogenesis.

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