

# Small-scale dynamos in simulations of stratified turbulent convection

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## Funding Information

George Ellery Hale Visiting Faculty. NSF Astronomy and Astrophysics Grants Program, 1615100. Academy of Finland ReSoLVE Centre of Excellence, 272157.

Small-scale dynamo action is often held responsible for the generation of quiet Sun magnetic fields. We aim to determine the excitation conditions and saturation level of small-scale dynamos in nonrotating turbulent convection at low magnetic Prandtl numbers. We use high-resolution direct numerical simulations of weakly stratified turbulent convection. We find that the critical magnetic Reynolds number for dynamo excitation increases as the magnetic Prandtl number is decreased, which might suggest that small-scale dynamo action is not automatically evident in bodies with small magnetic Prandtl numbers, such as the Sun. As a function of the magnetic Reynolds number ( $Rm$ ), the growth rate of the dynamo is consistent with an  $Rm^{1/2}$  scaling. No evidence for a logarithmic increase of the growth rate with  $Rm$  is found.

## KEYWORDS

convection – stars: magnetic fields – Sun: magnetic fields – turbulence

## 1 | INTRODUCTION

Magnetic fields are ubiquitous in astrophysical systems. These fields are, in most cases, thought to be generated by a dynamo process, involving either turbulent fluid motions or hydromagnetic instabilities. In dynamo theory (e.g., Brandenburg et al. 2012; Brandenburg & Subramanian 2005; Krause & Rädler 1980; Rüdiger & Hollerbach 2004), a distinction is made between large-scale (LSD) and small-scale dynamos (SSD), where the former produce fields whose length scale is greater than the scale of fluid motions, whereas in the latter, the two are comparable. An LSD can also produce small-scale magnetic fields through tangling, and the decay of active regions will similarly cause magnetic energy to cascade from larger to smaller scales, as is evidenced by the presence of a Kolmogorov-type energy spectrum in their proximity (Zhang et al. 2014, 2016).

Small-scale dynamos have been found in direct numerical simulations of various types of flows provided that the magnetic Reynolds number ( $Rm$ ) exceeds a critical value ( $Rm_c$ ). However, in many astrophysical conditions, molecular kinematic viscosity and magnetic diffusivity are vastly different, implying that their ratio, which is the magnetic Prandtl number ( $Pm$ ), is either very small or very large. For example, in the Sun,  $Pm = 10^{-3} \dots 10^{-6}$  (e.g., Ossendrijver 2003). Numerical simulations of forced turbulence and other idealized flows indicate that  $Rm_c$  increases as  $Pm$  decreases (Iskakov et al. 2007; Ponty et al. 2004; Schekochihin et al. 2004, 2005, 2007). Theoretical studies indicate a similar trend with an asymptotic value for  $Rm_c$  when  $Pm \rightarrow 0$  (Rogachevskii & Kleeorin 1997). The work of Iskakov et al. (2007) suggests that there is a value of  $Pm$  of around 0.1 where  $Rm_c$  is largest and that it somewhat decreases again, at even smaller values of  $Pm$ . In the nonlinear

regime, however, no significant drop in the magnetic energy is seen as  $\text{Pm}$  is decreased to and below 0.1 (Brandenburg 2011). More recently, Subramanian & Brandenburg (2014) found that the drop in the value of  $\text{Rm}_c$  may have been exaggerated by using a forcing wavenumber that was too close to the minimal wavenumber of the computational domain.

Simulations of turbulent convection have also been able to produce SSDs (e.g., Brandenburg et al. 1996; Cattaneo 1999; Favier & Bushby 2012; Hotta et al. 2015; Meneguzzi & Pouquet 1989; Nordlund et al. 1992; Pietarila Graham et al. 2010). Such small-scale magnetic fields may explain the network of magnetic fields in the Sun, which are independent of the solar cycle (Buehler et al. 2013; Rempel 2014; Stenflo 2014); see Brun & Browning (2017) and Borrero et al. (2017) for reviews. However, even the expected independence of the cycle does not go without controversy (Fau Robert & Ricort 2015; Utz et al. 2016). In fact, Jin et al. (2011) found evidence for an anticorrelation of small-scale fields with the solar cycle. This could potentially be explained by the interaction of the SSD with superequipartition large-scale fields from the global dynamo; see Karak & Brandenburg (2016). Small-scale magnetic fields may also play a role in heating the solar corona; see Amari et al. (2015) for recent work on that concept.

Small-scale dynamo-produced magnetic fields have been invoked (Bekki et al. 2017; Hotta et al. 2015, 2016) to explain the convective conundrum of the low levels of observed turbulent velocities compared to contemporary simulations (e.g., Gizon & Birch 2012; Miesch et al. 2012). However, subsequent work by Karak et al. (2018), who studied cases of large thermal Prandtl numbers conjectured to be due to the magnetic suppression of thermal diffusion, does cast some doubt on this idea.

Returning to the problem of magnetic Prandtl numbers, Thaler & Spruit (2015) studied the case  $\text{Pm} \geq 1$  from local solar surface convection simulations and found that the SSD ceases to exist for  $\text{Pm} = 1$ . However, this is mainly a shortcoming of low resolution. Global and semiglobal simulations of solar and stellar magnetism have also recently reached parameter regimes where SSDs are obtained (Hotta et al. 2016; Käpylä et al. 2017). These models suggest that the vigorous small-scale magnetism has profound repercussions for the LSD and differential rotation. However, due to resolution requirements, the global simulations are limited to magnetic Prandtl numbers of the order of unity or greater.

In the present paper, we therefore study high-resolution simulations of convection-driven SSDs in the case of small values of  $\text{Pm}$  by means of local models capturing more turbulent regimes. This regime was already addressed in an early paper by Cattaneo (2003), but no details regarding the dependence of the growth rates and saturation values of the magnetic field are available.

## 2 | THE MODEL

Our numerical model is the same as that of Käpylä et al. (2010) but without imposed shear or rotation. We use a Cartesian domain with dimensions  $L_x = L_y = 5d$  and  $L_z = d$  with  $0 < z < d$ , where  $d$  is the depth of the layer.

### 2.1 | Basic equations and boundary conditions

We solve the set of equations of magnetohydrodynamics:

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{U} \times \mathbf{B} - \eta \mu_0 \mathbf{J}, \quad (1)$$

$$\frac{D \ln \rho}{Dt} = -\nabla \cdot \mathbf{U}, \quad (2)$$

$$\frac{D\mathbf{U}}{Dt} = \mathbf{g} - \frac{1}{\rho} [\nabla p + \mathbf{J} \times \mathbf{B} + \nabla \cdot (2\nu \rho \mathbf{S})], \quad (3)$$

$$T \frac{Ds}{Dt} = \frac{1}{\rho} (\mu_0 \eta \mathbf{J}^2 - \nabla \cdot \mathbf{F}^{\text{rad}}) + 2\nu \mathbf{S}^2, \quad (4)$$

where  $D/Dt = \partial/\partial t + \mathbf{U} \cdot \nabla$  is the advective time derivative;  $\mathbf{A}$  is the magnetic vector potential;  $\mathbf{B} = \nabla \times \mathbf{A}$  is the magnetic field;  $\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}$  is the current density;  $\mu_0$  is the vacuum permeability;  $\eta$  and  $\nu$  are the magnetic diffusivity and kinematic viscosity, respectively;  $\mathbf{F}^{\text{rad}} = -K \nabla T$  is the radiative flux;  $K$  is the (constant) heat conductivity;  $\rho$  is the density;  $\mathbf{U}$  is the velocity;  $p$  is the pressure and  $s$  the specific entropy with  $Ds = c_p D \ln p - c_p D \ln \rho$ ; and  $\mathbf{g} = -g \hat{\mathbf{z}}$  is the gravitational acceleration. The fluid obeys an ideal gas law  $p = \rho e (\gamma - 1)$ , where  $p$  and  $e$  are the pressure and internal energy, respectively, and  $\gamma = c_p/c_v = 5/3$  is the ratio of specific heats at constant pressure and volume, respectively. The specific internal energy per unit mass is related to the temperature via  $e = c_v T$ . The rate of strain tensor  $\mathbf{S}$  is given by

$$\mathbf{S}_{ij} = \frac{1}{2} (U_{i,j} + U_{j,i}) - \frac{1}{3} \delta_{ij} \nabla \cdot \mathbf{U}. \quad (5)$$

In order to exclude complications due to overshooting and compressibility, we employ a weak stratification: the density difference between the top and bottom of the domain is 20%, and the average Mach number,  $\text{Ma} = u_{\text{rms}}/\sqrt{d}g$ , is always less than 0.1. The stratification in the associated hydrostatic initial state can be described by a polytrope with index  $m = 1$ . The stratification is controlled by the normalized pressure scale height at the surface

$$\xi_0 = \frac{(c_p - c_v) T_1}{gd}, \quad (6)$$

where  $T_1$  is the temperature at the surface ( $z = d$ ). In our current simulations, we use  $\xi_0 = 2.15$ .

The horizontal boundary conditions are periodic. We keep the temperature fixed at the top and bottom boundaries. For the velocity, we apply impenetrable, stress-free conditions according to

$$\partial_z U_x = \partial_z U_y = U_z = 0. \quad (7)$$

For the magnetic field, we use vertical field conditions

$$B_x = B_y = 0. \quad (8)$$

TABLE 1 Summary of weak field runs for  $\text{Pr} = 1$ 

Run	Pm	Ra ( $10^6$ )	Ma	Re	Rm	$\tilde{\lambda}$ ( $10^{-4}$ )	$\delta\tilde{\lambda}$ ( $10^{-4}$ )	Grid
A1	1	0.17	0.073	23	23	-52	47	$64^3$
A2	1	1.0	0.068	54	54	62	18	$128^3$
A3	1	4.2	0.064	101	101	162	20	$128^3$
A4	1	17	0.061	193	193	273	7	$256^3$
A5	1	67	0.056	354	354	453	2	$512^3$
B1	0.5	1.0	0.069	54	27	-128	24	$128^3$
B2	0.5	4.2	0.064	102	51	-27	12	$128^3$
B3	0.5	17	0.060	191	95	44	11	$256^3$
B4	0.5	67	0.056	360	180	155	18	$512^3$
B5	0.5	267	0.052	666	333	357	17	$1024^3$
C1	0.25	17	0.060	190	47	-144	22	$256^3$
C2	0.25	67	0.056	358	90	-35	28	$512^3$
D1	0.1	67	0.057	360	36	-237	56	$512^3$
D2	0.1	267	0.052	664	65	-139	77	$1024^3$
D3	0.1	740	0.050	1057	106	-10	94	$1024^3$

## 2.2 | Units, nondimensional quantities, and parameters

The units of length, time, velocity, density, specific entropy, and magnetic field are

$$[x] = d, \quad [t] = \sqrt{d/g}, \quad [U] = \sqrt{dg},$$

$$[\rho] = \rho_0, \quad [s] = c_p, \quad [B] = \sqrt{dg\rho_0\mu_0}, \quad (9)$$

where  $\rho_0$  is the density of the initial state at  $z_m = \frac{1}{2}d$ . The simulations are controlled by the following dimensionless parameters: thermal and magnetic diffusion in comparison to viscosity, which are measured by the Prandtl numbers

$$\text{Pr} = \frac{\nu}{\chi_0}, \quad \text{Pm} = \frac{\nu}{\eta}, \quad (10)$$

where  $\chi_0 = K/(c_p\rho_0)$  is the reference value of the thermal diffusion coefficient, measured in the middle of the layer,  $z_m$ , in the nonconvecting initial state. The efficiency of convection is measured by the Rayleigh number

$$\text{Ra} = \frac{gd^4}{\nu\chi_0} \left( -\frac{1}{c_p} \frac{ds}{dz} \right)_{z_m}, \quad (11)$$

again determined from the initial nonconvecting state at  $z_m$ . The entropy gradient can be presented as

$$\left( -\frac{1}{c_p} \frac{ds}{dz} \right)_{z_m} = \frac{\nabla - \nabla_{\text{ad}}}{H_p}, \quad (12)$$

where  $\nabla = (\partial \ln T / \partial \ln p)_{z_m}$  and  $\nabla_{\text{ad}} = 1 - 1/\gamma$  are the actual and adiabatic double-logarithmic temperature gradients, respectively, and  $H_p$  is the pressure scale height at  $z = z_m$ .

The effects of viscosity and magnetic diffusion are quantified respectively by the fluid and magnetic Rms

$$\text{Re} = \frac{u_{\text{rms}}}{\nu k_f}, \quad \text{Rm} = \frac{u_{\text{rms}}}{\eta k_f} = \text{PmRe}, \quad (13)$$

where  $u_{\text{rms}}$  is the root mean square (rms) value of the velocity, and  $k_f = 2\pi/d$  is the wavenumber corresponding to the depth

of the layer. Furthermore, we define the Péclet number as

$$\text{Pe} = \frac{u_{\text{rms}}}{\chi_0 k_f} = \text{PrRe}. \quad (14)$$

Except for the simulations of section 3.5, where  $\text{Pr} = \text{Pm}$  is varied, in all other simulations, we use  $\text{Pr} = 1$ , and thus,  $\text{Pe} = \text{Re}$ .

Error estimates are obtained by dividing the time series into three equally long parts. The largest deviation of the average for each of the three parts from that over the full time series is taken to represent the error.

The simulations were performed using the PENCIL CODE,<sup>1</sup> which uses sixth-order explicit finite differences in space and a third-order accurate time-stepping method. We use resolutions ranging from  $64^3$  to  $1024^3$ .

## 3 | RESULTS

### 3.1 | Description of the runs

We perform four sets of runs where we keep the magnetic Prandtl number fixed and vary Re and Rm; see Table 1. The lower-resolution ( $64^3$ ,  $128^3$ , and  $256^3$ ) runs were started from a nonconvecting state described in the previous section, whereas runs at  $512^3$  and  $1024^3$  were remeshed from saturated snapshots at lower resolutions; see Figure 1 for visualizations of specific entropy near the surface of the domain. After the convection has reached a statistically saturated state, we introduce a weak random magnetic field of the order of  $10^{-6}B_{\text{eq}}$ , where  $B_{\text{eq}}$  is the equipartition field strength with  $B_{\text{eq}}^2 = \langle \mu_0 \rho U^2 \rangle$ . We refer to these runs as weak field models and perform the data analysis in regimes where the magnetic fields remain dynamically unimportant. After an initial transient, the growth rate of the total rms magnetic field is

<sup>1</sup><https://pencil-code.github.com/>

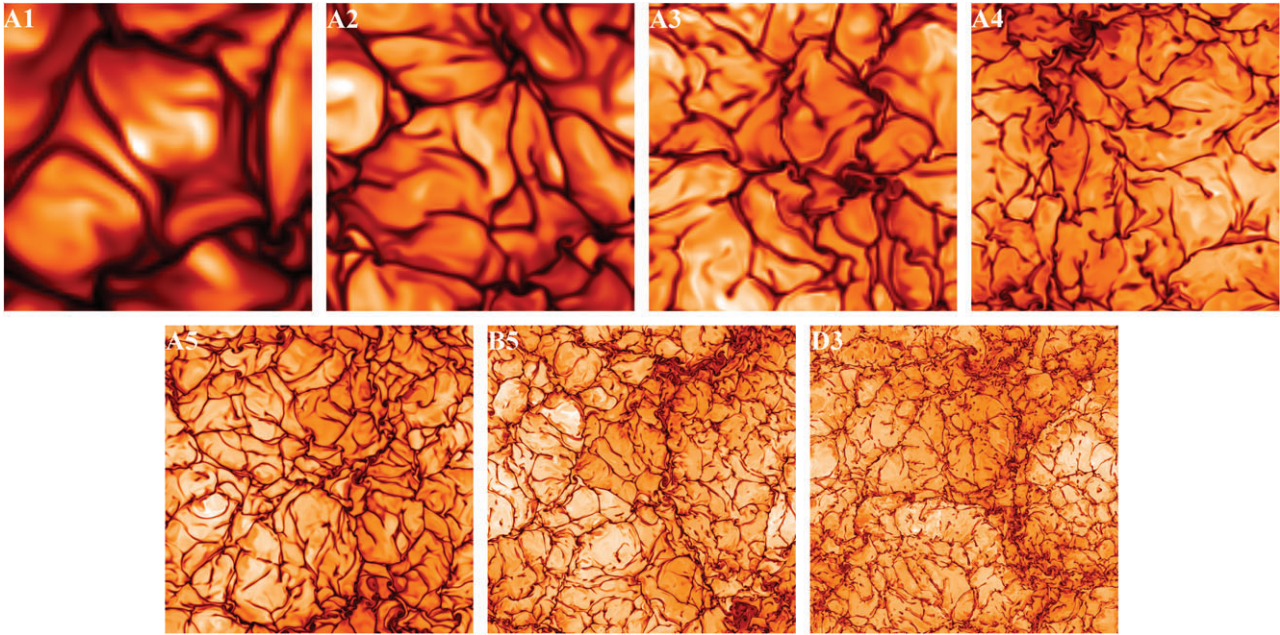


FIGURE 1 Upper row: specific entropy  $s/c_p$  near the surface  $z/d = 0.98$  for  $Re = Pe = 23, 54, 101,$  and  $193$ . Lower row:  $Re = Pe = 354, 666,$  and  $1057$

measured as

$$\lambda = \langle d \ln B_{\text{rms}} / dt \rangle_t, \quad (15)$$

where  $\langle \dots \rangle_t$  denotes time averaging. In the runs where the dynamo is clearly above or below critical, a short time series (few tens of turnover times) is sufficient to measure a statistically significant value of  $\lambda$ . The runs near the excitation threshold need to be run significantly longer (hundreds of turnover times). For the highest-resolution runs at low  $Pm$ , this is not feasible due to the computational cost, and thus, the error bars for these runs are typically significantly larger than in the low-resolution or  $Pm = 1$  runs.

### 3.2 | Growth rate in the kinematic regime

Figure 2 shows the growth rate of the magnetic field as a function of  $Rm$  for the four magnetic Prandtl numbers explored in the current study. For reference, we plot curves of the form

$$\gamma / \tau^{-1} \equiv \tilde{\gamma} = A + BRm^{1/2}, \quad (16)$$

where the value of the constant  $A$  changes as the magnetic Prandtl number is changed. Furthermore,  $B = 3.5 \times 10^{-3}$  for all values of  $Pm$ . The parameter  $A$  is negative, so the solutions will always decay for small values of  $Rm$ , but they increase with increasing values of  $Pm$ ; see Figure 2.

We find that the normalized growth rate for a given  $Rm$  decreases as  $Pm$  is decreased. Surprisingly,  $\tilde{\lambda}$  appears to follow an  $Rm^{1/2}$  trend for each value of  $Pm$ —even in the cases when an SSD is not excited. Such a dependence is predicted by theory for high  $Rm$ , that is, far away from excitation (Kleeorin & Rogachevskii 2012). However, given the relatively large error bars, the  $Rm^{1/2}$  scaling near the excitation threshold can, at this point, only be suggestive and far from definitive. Indeed, analytic theory yields a different scaling in this regime (Kleeorin & Rogachevskii 2012).

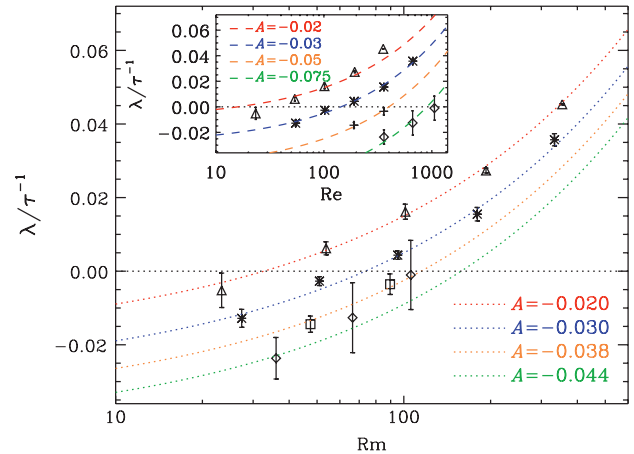
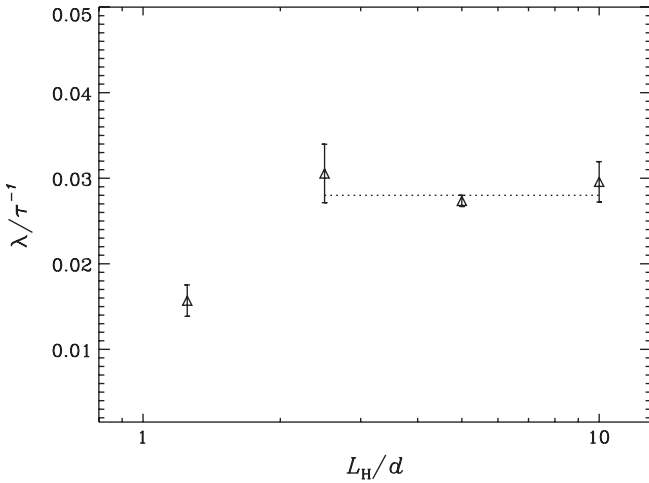


FIGURE 2 Growth rate  $\lambda$  of the rms magnetic field normalized by the inverse convective turnover time  $\tau^{-1} = u_{\text{rms}} k_f$  as a function of the magnetic Reynolds number  $Rm$ . The different symbols denote runs with  $Pm = 1$  (triangles),  $Pm = 0.5$  (stars),  $Pm = 0.25$  (squares), and  $Pm = 0.1$  (diamonds). The horizontal dotted line denotes marginal stability. The red, blue, orange, and green dotted lines are curves proportional to  $Rm^{1/2}$ ; see Equation 16 for different values of  $A$ , and  $B = 3.5 \times 10^{-3}$  is fixed. The inset shows the normalized growth rates for the same data as functions of  $Re$ . The dashed lines are proportional to  $Re^{1/2}$  according to a relation analogous to Equation 16 with  $B = 2.5 \times 10^{-3}$  and values of  $A$  indicated in the legend

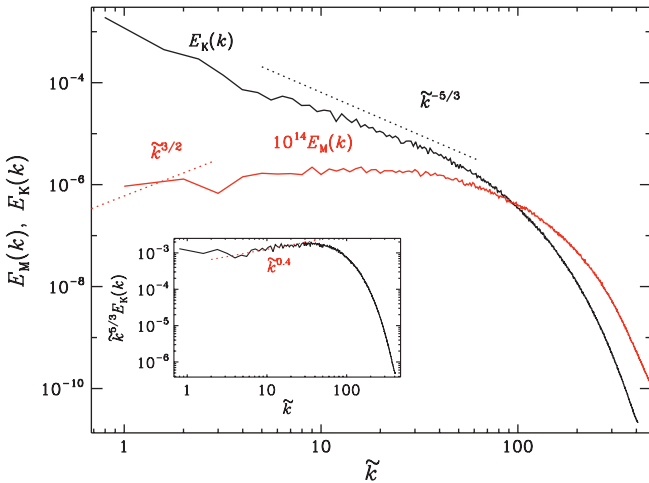
In the low- $Pm$  regime, the growth rate of the magnetic field due to the SSD is expected to scale with the  $1/2$  power of the fluid Reynolds number. We find that our simulation data is consistent with this for values of  $Pm$  of  $0.5$  and smaller; see the inset in Figure 2.

### 3.3 | Dependence on the box size

The dependence of the growth rate of the convection-driven SSD on the horizontal size of the domain and the presence of mesogranulation has been discussed in a recent paper by



**FIGURE 3** Growth rate  $\lambda$  of the rms magnetic field normalized by the inverse convective turnover time  $\tau^{-1} = u_{\text{rms}} k_f$  as a function of the horizontal box size for  $\text{Pm} = 1$



**FIGURE 4** Power spectra of velocity and magnetic field as functions of  $\tilde{k} = k/k_i$  near the top of the domain from Run B5.  $E_M$  has been multiplied by  $10^{14}$  for visualization purposes. The dotted line shows a  $k^{-5/3}$  scaling for reference. The inset shows the velocity power spectrum compensated by  $k^{5/3}$

Bushby et al. (2012). To address this issue, we show, in Figure 3, the growth rate of the magnetic field as a function of the horizontal box size for magnetic Prandtl number unity. Deviations from the constant trend are found for  $L_H/d = 1.25$ . For  $L_H/d = 0.5$ , no dynamo action is found. Our standard box size of  $L_H/d = 5$  is thus adequate and does not seem to suffer from the issues raised by Bushby et al. (2012). This is in spite of the fact that the flow is dominated by a single convection cell filling the whole domain, which is clear even on visual inspection of Figure 1.

### 3.4 | Energy spectra

In Figure 4 we show the kinetic and magnetic energy spectra,  $E_K$  and  $E_M$ , respectively, for Run B5 during the kinematic phase of the dynamo for  $\text{Pm} = 0.25$  and  $\text{Rm} = 654$ . The kinetic energy spectrum shows a clear  $k^{-5/3}$  spectrum along with a slightly shallower slope near the dissipative cutoff. This is

**TABLE 2** Summary of strong field runs

Run	Pm	Pr	Ra ( $10^6$ )	Re	Rm	Ma	$\tilde{B}_{\text{rms}}$	Grid
S1	1	1	17	169	169	0.053	0.0130	$256^3$
S2	0.5	0.5	33	361	180	0.057	0.0126	$256^3$
S3	0.25	0.25	67	760	190	0.060	0.0110	$256^3$
S4	0.1	0.1	167	2118	212	0.067	—	$512^3$

the bottleneck effect (Falkovich 1994), which has been held responsible for causing the increase of  $\text{Rm}_c$  near  $\text{Pm} = 0.1$  because the peak of the magnetic energy then lies fully within the inertial range of the kinetic energy spectrum (Boldyrev & Cattaneo 2004). The magnetic energy spectrum, on the other hand, is significantly shallower than the  $k^{3/2}$  spectrum expected from the work of Kazantsev (1968), which has been confirmed in several numerical simulations of kinematic dynamo action in forced turbulence (Haugen et al. 2004; Schekochihin et al. 2004) and supernova-driven turbulence (Balsara et al. 2004). The spectra shown in Figure 4 were taken from a run where the magnetic field has grown only by a factor of a few, and it is possible that the  $3/2$  scaling has not had enough time to develop yet. However, the flow exhibits a long-lived large-scale component, manifested by the peak at  $\tilde{k} = 1$ , which is not present in simpler forced turbulence simulations. Such flows may contribute to the relatively high magnetic power at large scales. In that case, the lack of a  $k^{3/2}$  spectrum in the kinematic regime would have a physical origin. These aspects will be explored further elsewhere.

### 3.5 | Saturation level

Another set of simulations was created to study the saturation level of the magnetic fields produced by the SSD; see Table 2. We refer to these runs as strong field models. These runs were either run to full saturation from the initial conditions described in section 2 (Run S1) or continued from a saturated snapshot of an earlier run (S2, S3, or S4). At each step, the kinematic viscosity is lowered to decrease  $\text{Pm}$  with the aim of avoiding the long kinematic stage of the dynamo. Another possible advantage of this procedure is that the SSD has been shown to operate in the nonlinear regime at an  $\text{Rm}$  value that would be subcritical in the kinematic case (Brandenburg 2011). While this procedure works for Runs S2 and S3, in Run S4 with  $\text{Pm} = 0.1$ , where  $\text{Rm} = 212$  and  $\text{Re} = 2118$ , the magnetic field is not sustained.

Figure 5 shows the saturation field strength for Runs S1–S3 with  $\text{Pm} = 0.25 \dots 1$ . Here, the magnetic  $\text{Rm}$  varies from 169 to 190 due to increasing  $u_{\text{rms}}$  when  $\text{Pm}$  decreases. Contrary to the results for forced turbulence, where the rms magnetic field was found to decrease by no more than a factor of two as  $\text{Pm}$  decreased from unity to 0.01 (Brandenburg 2011), here, we seem to find a somewhat stronger dependence of the

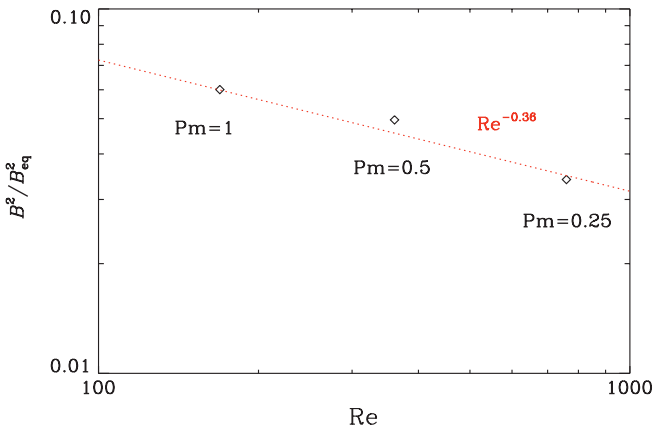


FIGURE 5 Saturation field strength for Runs S1–S3 with  $Pm = 1 \dots 0.25$  ( $Rm = 169 \dots 190$ ) as a function of the fluid Reynolds number

saturation field strength on the value of  $Pm$  and thus on  $Re$ . The current results suggest a scaling with  $Re$  with a power that is close to  $-1/3$ . However, one has to realize that for the run with the largest value of  $Re$  and  $Pm = 0.25$ , we used a resolution of  $256^3$ , which may be too low to resolve the flow at  $Re = 760$ .

## 4 | CONCLUSIONS

Our work has confirmed that in turbulent convection at low values of  $Pm$ , the value of  $Rm_c$  increases with decreasing  $Pm$ . This effect may well be connected with the bottleneck effect seen in the kinetic energy spectrum. The saturated field strength, however, is found to show a somewhat stronger dependence on  $Pm$  than in the case of forced turbulence.

For both small values of  $Pm$  and for  $Pm$  of unity, we find that the kinematic growth rate increases proportional to  $Rm^{1/2}$ . In particular, there is no evidence for a logarithmic dependence. A similar dependence on  $Rm$  has previously been seen in forced turbulence; see Haugen et al. (2004) for example.

Interestingly, however, in the kinematic regime, the magnetic energy spectrum is significantly shallower than the  $k^{3/2}$  spectrum expected for an SSD (Kazantsev 1968). This is also quite different from the case of forced turbulence, where a clear  $k^{3/2}$  spectrum is found during the kinematic growth phase. In other words, the current kinematic convection-driven dynamo shows a tendency of producing larger-scale magnetic fields than in forced turbulence. This is possibly caused by a persistent large-scale velocity pattern, which is a robust feature in the current simulations.

## ACKNOWLEDGMENTS

The computations were performed on the facilities hosted by CSC—IT Center for Science Ltd. in Espoo, Finland,

who are administered by the Finnish Ministry of Education. We also acknowledge the allocation of computing resources provided by the Swedish National Allocations Committee at the Center for Parallel Computers at the Royal Institute of Technology in Stockholm. This work was supported in part by the Academy of Finland ReSoLVE Centre of Excellence (Grant No. 272157; M.J.K. and P.J.K.), the NSF Astronomy and Astrophysics Grants Program (Grant No. 1615100), and the University of Colorado through its support of the George Ellery Hale Visiting Faculty appointment.

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**How to cite this article:** Käpylä PJ, Käpylä MJ, Brandenburg A. Small-scale dynamos in simulations of stratified turbulent convection. *Astron. Nachr./AN*. 2018;339:127–133. <https://doi.org/10.1002/asna.201813477>