ASTR/GEOL-2040-001: Search for Life in the Universe

Homework #4 (Friday Oct 6, 2017) <u>model solutions</u>

Using rounded values is acceptable as long as the steps of the calculation are correct.

1. **Temperatures on planets**. *In the following, I use rounded values, which is good enough to get the idea of what is going on. That way you don't need a calculator. Also, you'll see that, instead of writing 6000, I sometimes prefer to write* 0.6×10^4*, because now the prefactor* 0.6 *is closer to 1 than 6 would be. To raise this to the fourth power, I square it twice, so* $(0.6)^2 = 0.36$ *and* $(0.36)^2 \approx 0.1$*, so* $(0.6)^4 \approx 0.1$*. Finally, raising* 10^4 *to the fourth power gives* $10^{4\times 4} = 10^{16}$*.*

(i) At the solar surface we have thus

$$F_{\rm Sun} = \sigma_{\rm SB} T_{\rm Sun}^4 = 5.67\times 10^{-8}\times 6000^4\,{\rm W\,m^{-2}} \approx 6\times 10^{-8}\times(0.6\times 10^4)^4\,{\rm W\,m^{-2}} \approx 6\times 10^{-8}\times 0.1\times 10^{16}\,{\rm W\,m^{-2}}.$$

Thus, we have

$$F_{\rm Sun} \approx 6\times 10^7\,{\rm W\,m^{-2}}.$$

By the way, σ is the Greek letter sigma, and the indix SB refers just to the surnames Stefan and Boltzmann. [2pts]

(ii) The $6\times 10^7\,{\rm W\,m^{-2}}$ tells you the power per unit area, so to get the total power emitted by the Sun, we have to multiply this by the total surface area of the Sun, which is $4\pi R_{\rm Sun}^2$ and $R_{\rm Sun} = 7\times 10^8\,{\rm m} = 0.7\times 10^9\,{\rm m}$ was given. Thus,

$$\text{Power} = 6\times 10^7\,{\rm W\,m^{-2}}\times 4\pi R_{\rm Sun}^2 \approx 6\times 10^7\,{\rm W\,m^{-2}}\times 12\times(0.7\times 10^9\,{\rm m})^2$$

so the ${\rm m}^2$ cancels out and with $0.7^2 = 0.49 \approx 0.5$, we get

$$\text{Power} \approx 6\times 10^7\,{\rm W}\times 12\times 0.5\times 10^{18} = 36\times 10^{25}\,{\rm W} = 3.6\times 10^{26}\,{\rm W}.$$

[2pts]

(iii) The light spreads out from the surface of the Sun (radius or distance from the center is $0.7\times 10^9\,{\rm m}$) to the distance of the Earth ($1.5\times 10^{11}\,{\rm m}$). This distance is approximately 200 times larger than the distance from the center of the Sun to the surface, because $(1.5/0.7)\times 10^{11-9} \approx 2\times 10^2$. The light gets diluted by the square of the distance ratio, so we have to divide the flux $6\times 10^7\,{\rm W\,m^{-2}}$ obtained under (i) by $200^2 = 4\times 10^4$, so we get

$$\text{Flux at Earth} = \frac{6\times 10^7\,{\rm W\,m^{-2}}}{4\times 10^4} = 1.5\times 10^3\,{\rm W\,m^{-2}} = 1500\,{\rm W\,m^{-2}}$$

This is the power we could potentially tap with a solar cell *in the day time.* [2pts]

(iv) The surface area of the Earth that is exposed to the Sun is

$$\pi R_{\rm Earth}^2 \approx 3\times(0.6\times 10^7\,{\rm m})^2 = 3\times 0.36\times 10^{14}\,{\rm m}^2 \approx 10^{14}\,{\rm m}^2.$$

Since it gets $1.5\times 10^3\,{\rm W\,m^{-2}}$ from the Sun, the total power

$$\text{received from the Sun} = 1.5\times 10^3\,{\rm W\,m^{-2}}\times 10^{14}\,{\rm m}^2 = 1.5\times 10^{17}\,{\rm W}.$$

[2pts]

(v) If 30% of the sunlight is reflected, we get only 70%, so we have to multiply our 1.5×10^{17} W by 0.7, which gives

$$\text{received from the Sun through clouds etc} = 0.7 \times 1.5 \times 10^{17}\,\text{W} \approx 10^{17}\,\text{W}.$$

[2pts]

(vi) The Earth itself radiates, just like the Sun, except that its temperature is only 300 K instead of 6000 K. Therefore, instead of our calculation in (i), we have

$$F_{\text{Earth}} = \sigma_{\text{SB}} T_{\text{Earth}}^4 = 5.67 \times 10^{-8} \times 300^4\,\text{W}\,\text{m}^{-2} \approx 6 \times 10^{-8} \times (0.3 \times 10^3)^4\,\text{W}\,\text{m}^{-2}.$$

Thus, we have

$$F_{\text{Earth}} \approx 6 \times 10^{-8} \times 0.01 \times 10^{12}\,\text{W}\,\text{m}^{-2} = 6 \times 10^2\,\text{W}\,\text{m}^{-2} = 600\,\text{W}\,\text{m}^{-2}.$$

This is being radiated in all directions, and also into the night sky, so the total area over which it is radiated is

$$4\pi R_{\text{Earth}}^2 \approx 12 \times (0.6 \times 10^7\,\text{m})^2 = 12 \times 0.36 \times 10^{14}\,\text{m}^2 \approx 4 \times 10^{15}\,\text{m}^2,$$

which is 4 times more than the surface exposed to the Sun. So the power

$$\text{lost into space} = 600\,\text{W}\,\text{m}^{-2} \times 4 \times 10^{14}\,\text{m}^2 = 2400\,\text{W}\,\text{m}^{-2} \times 10^{14}\,\text{m}^2 = 2.4 \times 10^{17}\,\text{W}.$$

[2pts]

(vii) The power lost into space is 2.4×10^{17} W, which is more than what it receives from the Sun through clouds, which is $\approx 10^{17}$ W. The Earth would therefore cool, unless it wraps itself into a thick blanket. This is what the Earth does, through the various greenhouse gases.

[2pts]

2. Name that "–troph". In the reaction

$$2H_2O + Fe_2O_3 \rightarrow 2Fe(OH)_2 + \tfrac{1}{2}O_2.$$

everything on the left-hand side is inorganic, and no photosynthesis is involved,

(i) This kind of -troph gets its energy from molecules, so it is *chemo* [2pts].
(ii) The electron donor (molecule with lots of H) is H_2O, which is inorganic, so it is *litho* [2pts].
(iii) And the carbon (here iron) source is inorganic, so it is an *auto* [2pts],

and thus it is a chemolithoautotroph.

You can read more about this on: `https://en.wikipedia.org/wiki/Iron-oxidizing_bacteria`