# SBO spectrometer & center-to-limb variation

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## 1 Planck function

The Planck function is given by

$$B_\nu(\nu,T)=\frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/k_\mathrm{B}T}-1} \tag{1}$$

where $k_\mathrm{B}\approx 1.381\times10^{-23}\,\mathrm{J\,K^{-1}}$ is the Boltzmann constant, $h\approx 6.626\times10^{-34}\,\mathrm{J\,s}$ is the Planck constant, and $c\approx 2.998\times10^{8}\,\mathrm{m\,s^{-1}}$ is the speed of light.

The Planck function is defined such that $\int_{4\pi}\int_0^\infty B_\nu\,\mathrm{d}\nu\,\mathrm{d}\Omega=\sigma_\mathrm{SB}T^4$, where

$$\sigma_\mathrm{SB}=\frac{2k_\mathrm{B}^4\pi^5}{15c^2h^3}\approx 5.67\times10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}} \tag{2}$$

is the Stefan-Boltzmann constant. Instead of $B_\nu(\nu,T)$, one can also define $B_\lambda(\lambda,T)$, which is defined analogously such that $\int B_\lambda\,\mathrm{d}\lambda=\sigma_\mathrm{SB}T^4$. Since $B_\lambda\,\mathrm{d}\lambda=-B_\nu\,\mathrm{d}\nu$, we have $B_\lambda=B_\nu\,\mathrm{d}\nu/\mathrm{d}\lambda$ (the minus sign arises because the integration boundaries have been interchanged. Using $\nu=c/\lambda$, we have $-\mathrm{d}\nu/\mathrm{d}\lambda=c/\lambda^2$.

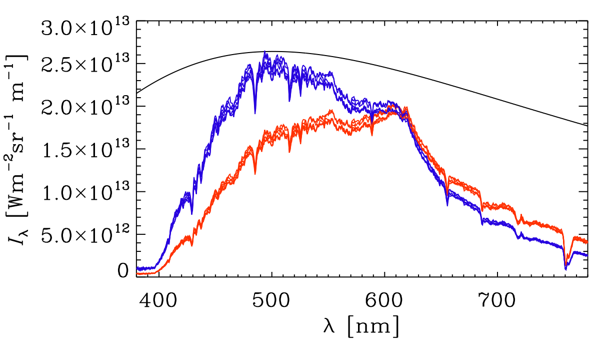


Figure 1: Planck spectrum $I_\lambda$ (black line), compared with the measured intensity at several positions near disk center (red lines) and the measured intensity assuming that it was $I_\nu$ which was therefore scaled by a $c/\lambda^2$ factor (blue lines).

In Fig. 1 we plot the resulting equilibrium intensity $I_\lambda=B_\lambda$ and compare with measurements of the Sun at disk center. Since we are not sure whether the SBO spectrometer really measures $I_\lambda$, we compare with the resulting plot assuming that it really measured $I_\nu$, which would then need to be converted to $I_\lambda$ with a $c/\lambda^2$ factor.

## 2 Center-to-limb variation

The intensity was measured from limb to limb as a function of linear position $x$ on the solar disk. This linear position was translated into a radius $r=x-x_0$, where $x_0$ is determined such that the curves for $I_\lambda(x=r)$ collapse onto those for $I_\lambda(x=-r)$. We then translate the $r$ dependence into a dependence on $\mu=\cos\theta$ through

$$\mu=\sqrt{1-r^2/R^2}, \tag{3}$$

where $R$ is the radius of the projected solar disk. There is uncertainty in the values of $R$, which introduces considerable variations in the final form of $I_\lambda(\mu)$. By default, we assume $R=R_0$, where $R_0=(x_{max}-x_\mathrm{min})/2$ is the range covered between the first and the last measurements. The actual value of $R$ is probably slightly larger than $R_0$.

Figure 3 shows the resulting function of $I_\lambda(\mu)$ for three values of $\lambda$. The decline with increasing distance from the disk center (smaller values of $\mu$) is slower for red colors ($\lambda=700\,\mathrm{nm}$) than for blue ones ($\lambda=420\,\mathrm{nm}$).

In Figs. 4–5 we show the resulting profiles of $I_\lambda(\mu)$ for $R=1.05\times R_0$ and $R=1.1\times R_0$, respectively. Nevertheless, in all cases the relative ordering of the lines for different wavelengths is the same.

Radius of the Sun: 136 windings.

## 3 Fit coefficients

Following early work of Pierce & Slaughter (1977), we first plot the intensity dependence as a function of $\xi=\cos\mu$ and then determine a quadric fit of the

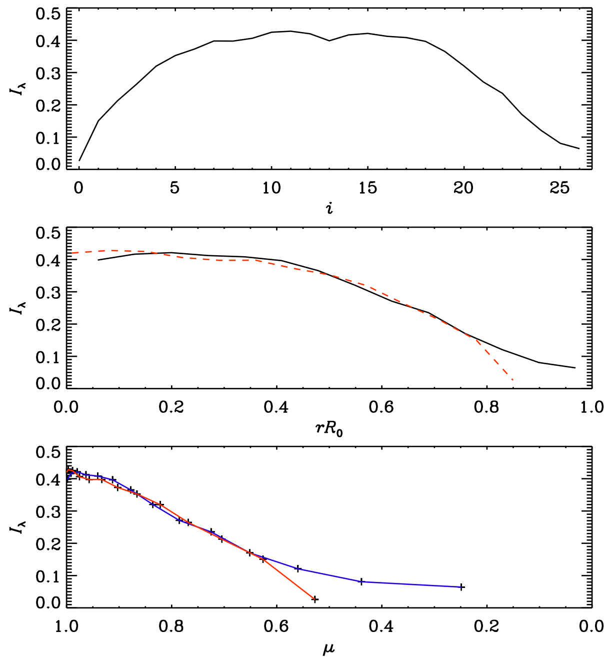


Figure 2: Raw data and figuring our the right values of the central point $x_0$ and the radius $R$.

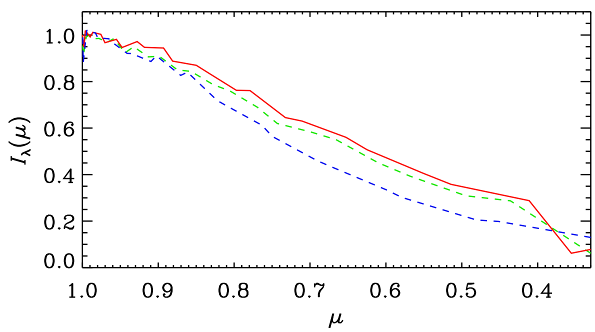


Figure 3: Center-to-limb variation for run `Rachel_Brandon_161310_trial_6_27parts at` $\lambda = 700\,\mathrm{nm}$ `(red line),` $\lambda = 500\,\mathrm{nm}$ `(green line), and` $\lambda = 420\,\mathrm{nm}$ `(blue line).`

form

$$I/I_0 = a + b\xi + c\xi^2. \tag{4}$$

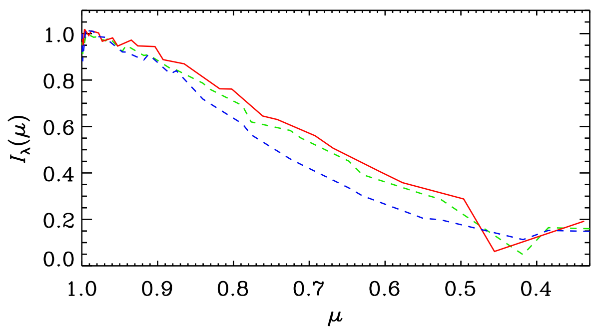


Figure 4: Same as Fig. 3, but assuming $R = 1.05 \times R_0$.

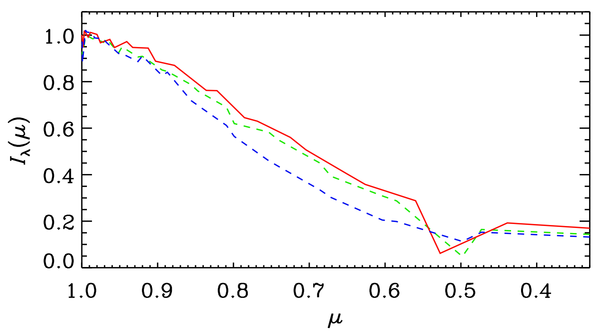


Figure 5: Same as Fig. 3, but assuming $R = 1.1 \times R_0$.

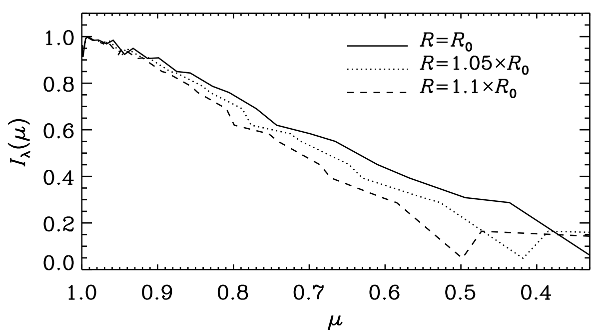


Figure 6: Comparison of the intensities for $\lambda = 500\,\mathrm{nm}$ for three values of $R/R_0$.

# References


Pierce, A. K., & Slaughter, C. D. 1977, Solar Phys., 51, 25 41Solar limb darkening. I - At wavelengths of 3033-7297

Table 1: Comparison of fit coefficients for Trial 3 of Max and those of Pierce & Slaughter (1977). These data are also plotted in Fig. 7.

| Max | | | | | | PS77 | | |
|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $a$ | $b$ | $c$ | $b/a$ | $c/a$ | $\lambda$ | $b/a$ | $c/a$ |
| 405.6 | 0.978 | 0.765 | 0.169 | 0.783 | 0.173 | 407 | 0.629 | 0.129 |
| 441.9 | 0.975 | 0.621 | 0.115 | 0.637 | 0.118 | 444 | 0.535 | 0.089 |
| 467.6 | 0.977 | 0.567 | 0.096 | 0.581 | 0.099 | 468 | 0.525 | 0.094 |
| 493.7 | 0.983 | 0.525 | 0.081 | 0.535 | 0.083 | 493 | 0.504 | 0.092 |
| 538.2 | 0.982 | 0.481 | 0.066 | 0.490 | 0.068 | 534 | 0.478 | 0.089 |
| 610.4 | 0.982 | 0.418 | 0.045 | 0.426 | 0.046 | 610 | 0.413 | 0.071 |
| 638.7 | 0.980 | 0.399 | 0.038 | 0.407 | 0.039 | 641 | 0.385 | 0.062 |
| 654.9 | 0.985 | 0.338 | 0.016 | 0.343 | 0.017 | 660 | 0.360 | 0.052 |
| 702.3 | 0.977 | 0.380 | 0.033 | 0.389 | 0.033 | 701 | 0.353 | 0.056 |
| 750.5 | 0.975 | 0.371 | 0.029 | 0.380 | 0.029 | 730 | 0.343 | 0.053 |

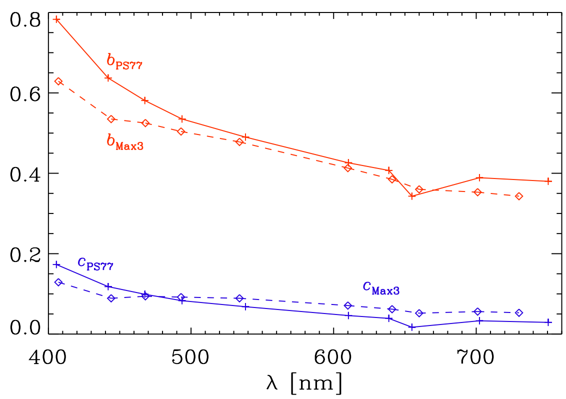


Figure 7: Comparison of the fit coefficients $b$ and $c$ as a function of $\lambda$ from Pierce & Slaughter (1977) (solid lines) and Trial 3 of Max (dashed).

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