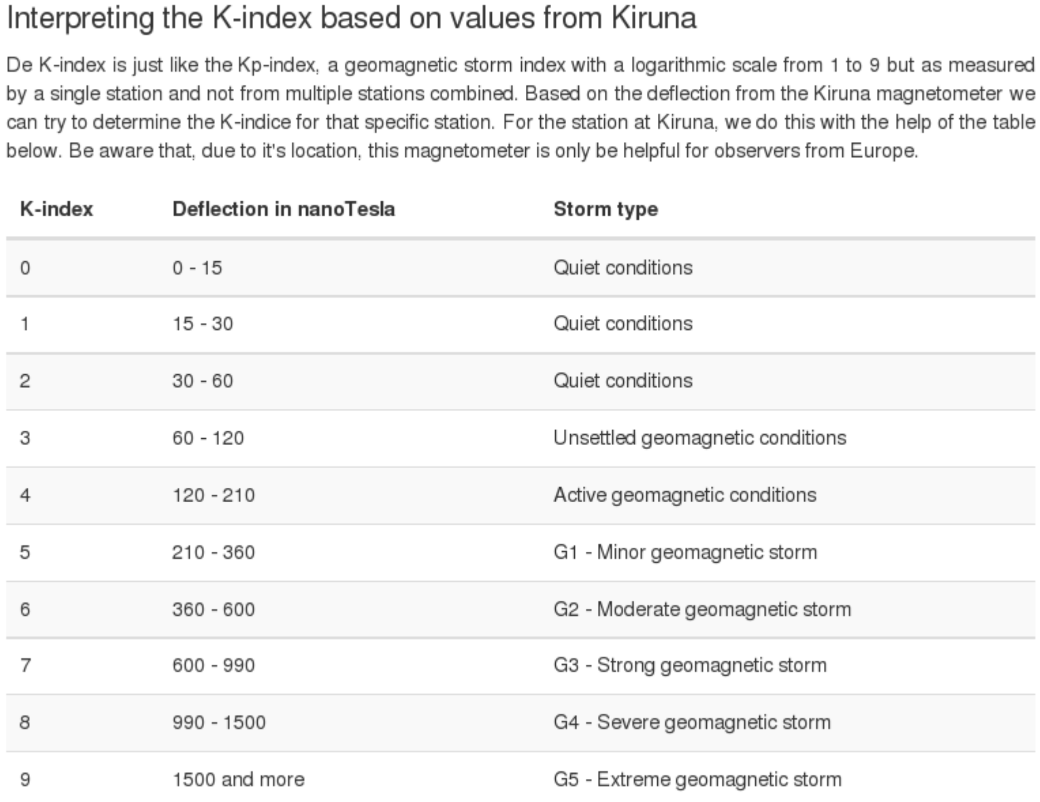
ASTR-3760: Solar and Space Physics ....Problem Set 1 (Due Mon., January 25, 2016)

Please try to be neat when writing up answers. In cases where calculations are called for, please show all of the intermediate steps, including any approximations you choose to make and any sketches you may need to illustrate what's what. Be careful to properly evaluate units and significant figures. Calculations given without 'showing the work' will receive zero credit, even if the final answer is correct.

---

1. K index and relation to B field. When googling for Kp index, I cam across the following site:
`http://www.spaceweatherlive.com/en/help/the-kiruna-magnetometer`
It relates the K index (from 0 to 9) to ranges of the magnetic field ($B_{\rm min}$ and $B_{\rm max}$) in nT; see below.



Interpreting the K-index based on values from Kiruna

De K-index is just like the Kp-index, a geomagnetic storm index with a logarithmic scale from 1 to 9 but as measured by a single station and not from multiple stations combined. Based on the deflection from the Kiruna magnetometer we can try to determine the K-indice for that specific station. For the station at Kiruna, we do this with the help of the table below. Be aware that, due to it's location, this magnetometer is only be helpful for observers from Europe.

K-index	Deflection in nanoTesla	Storm type
0	0 - 15	Quiet conditions
1	15 - 30	Quiet conditions
2	30 - 60	Quiet conditions
3	60 - 120	Unsettled geomagnetic conditions
4	120 - 210	Active geomagnetic conditions
5	210 - 360	G1 - Minor geomagnetic storm
6	360 - 600	G2 - Moderate geomagnetic storm
7	600 - 990	G3 - Strong geomagnetic storm
8	990 - 1500	G4 - Severe geomagnetic storm
9	1500 and more	G5 - Extreme geomagnetic storm

(a) Plot $B_{\rm min}$ and $B_{\rm max}$ versus Kp (either with the computer or by hand) and check that K does indeed grow logarithmically, and thus $B_{\rm min}$ and $B_{\rm max}$ grow exponentially with Kp.

(b) Try to describe the data with the formula

$$B = B_0 \exp(\mathrm{Kp}/\mathrm{Kp}_0)$$

and give the values B_0 and Kp_0. (Don't forget the give units, when appropriate and necessary.)

(c) To see the logarithmic behavior of Kp, it is more natural to plot Kp versus $B_{\rm min}$ and versus $B_{\rm max}$ (in the same graph). Check that

$$\mathrm{Kp} = \mathrm{Kp}_0 \ln(B/B_0)$$

---

2. Vector & scalar fields. Consider the following vector function in Cartesian space:

$$\mathbf{F} = (y+2xy)\hat{\mathbf{e}}_x + (x+x^2+3y^2z^2)\hat{\mathbf{e}}_y + (2y^3z)\hat{\mathbf{e}}_z$$

(a) Calculate the divergence and curl of **F**.

(b) Can **F** be an electric field? If so, under what circumstances?

(c) Can **F** be a magnetic field? If so, under what circumstances?

(d) Extra credit: Using a table of vector identities, it is possible for you to determine how the vector field **F** was "generated" from a simpler scalar function $\phi(x,y,z)$. Figure out how that was done, and calculate the functional form of $\phi(x,y,z)$.

---

3. A Not-So-Ordinary Differential Equation. Consider a one-dimensional "slab" of gas that starts at $x=0$ and ends at $x=D$, and is surrounded by empty space. A ray of light with intensity I_0 hits the slab at $x=0$ and shines through it parallel to the x axis. Inside the slab, the intensity obeys

$$\frac{dI}{dx} = \alpha\,(S-I)$$

where α and S are constants.

(a) Solve this equation for $I(x)$ at all points between $x=0$ and $x=D$.

(b) Define the quantity $\tau = \alpha D$. Give an approximate solution for the "emergent intensity" $I(D)$ under the three limiting cases:

- $\tau \ll 1$.
- $\tau \gg 1$ and $S \gg I_0$.
- $\tau \gg 1$ and $S \ll I_0$.

(c) Each of the three above cases matches with one of the following three physical analogies. Which do you think corresponds to which, and why?

- Shining a flashlight through a piece of dark smoky quartz.
- Shining a flashlight through the bright flame of a welder's torch.
- Shining a flashlight through a glass window pane.

Hint: The quantity τ can be thought of as the "optical depth" or opaqueness of the slab—i.e., how efficiently does the gas absorb (or otherwise eliminate) the incoming beam. The quantity S is a "source function" that describes how the gas in the slab generates its own light.

---

4. Electromagnetic Energy Conservation. Use Maxwell's equations, for a vacuum environment (i.e., $\mathbf{D} = \epsilon_0\mathbf{E}$ and $\mathbf{B} = \mu_0\mathbf{H}$), to show that

$$\frac{\partial}{\partial t}(U_{\mathrm{E}}+U_{\mathrm{B}})+\nabla\cdot\mathbf{S} = -\mathbf{E}\cdot\mathbf{J}$$

where

$$U_{\mathrm{E}} = \frac{\epsilon_0|\mathbf{E}|^2}{2}\ , \quad U_{\mathrm{B}} = \frac{|\mathbf{B}|^2}{2\mu_0}\ , \quad \mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B})\ .$$

Hint: The online version of the "useful formulae" document contains a new vector identity that didn't get included in time for the printed handouts on the first day. If vectors **A** & **B** depend on time t, then the chain rule for the dot product is given by

$$\frac{\partial}{\partial t}(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\cdot\frac{\partial\mathbf{B}}{\partial t} + \frac{\partial\mathbf{A}}{\partial t}\cdot\mathbf{B}\ .$$