ASTR-3760: Solar and Space Physics .....Resit for Problem Set 2
Maximum number of points: 75 ..... (Due Mon., March 30, 2016)

Please try to be neat when writing up answers. In cases where calculations are called for, please show all of the intermediate steps, including any approximations you choose to make and any sketches you may need to illustrate what's what. Be careful to properly evaluate units and significant figures. Calculations given without 'showing the work' will receive zero credit, even if the final answer is correct.

1. In Lecture 5, it was shown that to leading order the center-to-limb variation is given by

$$I_\nu(\mu) = B_\nu - \frac{\cos\theta}{\rho\kappa_\nu}\frac{\mathrm{d}B_\nu}{\mathrm{d}r} \tag{1}$$

   Compare this solution with that found for problem 1 of Homework 2 assume that the source function is $S = B_\nu$ with $S\tau = 0.1 + 1.35\tau - 0.45\tau^2$. Sketch this solution in the key to Homework 2 for problem 1.

2. In Lecture 4, we looked at the periodic table and computed the energy mc^2 gain in the reaction $4\mathrm{H} \rightarrow \mathrm{He}$. Compare with the corresponding energy gain (or loss) for the hypothetical reactions $7\mathrm{H} \rightarrow \mathrm{Li}$, $9\mathrm{H} \rightarrow \mathrm{Be}$, $10\mathrm{H} \rightarrow \mathrm{Bo}$, $12\mathrm{H} \rightarrow \mathrm{C}$, $14\mathrm{H} \rightarrow \mathrm{N}$, $16\mathrm{H} \rightarrow \mathrm{O}$, $19\mathrm{H} \rightarrow \mathrm{F}$, and $10\mathrm{H} \rightarrow \mathrm{B}$.

3. Using the Planck law,

$$I(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/k_\mathrm{B}T} - 1}, \tag{2}$$

   to derive the limits for large and small frequencies ν.

4. Ignoring effects of the atmosphere, what is the theoretical effective temperature on Titan, which has a distance of 1.427×10^{12} m from the Sun. You may use the material of Lecture 4 for this exercise.

5. Using index notation, show that

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{\nabla}\boldsymbol{\nabla} \cdot \boldsymbol{A} - \nabla^2 \boldsymbol{A} \tag{3}$$

6. (a) Using the material of Lecture 8, show that $Q^2 + U^2 + V^2 = I^2$. (b) Discuss why this relation is not obeyed for "wave packets" of finite length (see Sect. 3.5.2 of Stix 2002).

7. What is the mean molecular weight μ for an ionized mixture with hydrogen abundance of (i) $X = 0.735$ or (ii) $X = 0.739$, assuming $Y = 1 - X$ for the helium abundance.