ASTR-3760: Solar and Space Physics . . . . (Wednesday, March 9, 2016)

Sample material for midterm exam (which is next week: Wednesday, March 16, 2016)

Name: . . . .

1. Relation between electric field vector and Stokes parameters (cf. Lecture 8, p. 17).

   Given an electric field of the form $E_x = \xi_x \cos\phi$, $E_y = \xi_y \cos(\phi + \epsilon)$, with $\xi_x = 2$, $\xi_y = 1$, arbitrary phase ϕ, and $\epsilon = 0$.

   (a) Compute the 4 Stokes parameters

   $$I = \xi_x^2 + \xi_y^2 = \tag{1}$$

   $$Q = \xi_x^2 - \xi_y^2 = \tag{2}$$

   $$U = 2\xi_x\xi_y \cos\epsilon = \tag{3}$$

   $$V = 2\xi_x\xi_y \sin\epsilon = \tag{4}$$

   (b) Compute

   $$I^2 = \tag{5}$$

   (c) Compute

   $$Q^2 + U^2 + V^2 = \tag{6}$$

   (d) ... and compare.

2. Given a certain profile of specific entropy, explain in words when the stratification is convectively stable (cf. Lecture 14, p. 9).

3. What causes changes in specific entropy (cf. Lecture 15, p. 12).

4. Compute magnetic pressure gradient and tension force for a simple vector field (cf. Lecture 10, p. 15 and 16; Stix, Chap. 8.1.4).

   (a) Consider a two-dimensional magnetic field given by

$$\boldsymbol{B} = \begin{pmatrix} y \\ -x \\ 0 \end{pmatrix} \tag{7}$$

   and calculate the current density (assuming $\mu_0 = 1$),

$$\boldsymbol{J} = \boldsymbol{\nabla} \times \boldsymbol{B} = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ 0 \end{pmatrix} \times \begin{pmatrix} y \\ -x \\ 0 \end{pmatrix} = \tag{8}$$

   (b) Calculate the magnetic pressure (again assuming $\mu_0 = 1$)

$$\tfrac{1}{2}\boldsymbol{B}^2 = \tag{9}$$

   (c) Calculate the magnetic pressure gradient

$$\boldsymbol{\nabla}(\tfrac{1}{2}\boldsymbol{B}^2) = \tag{10}$$

   (d) Give the expression for

$$\boldsymbol{B} \cdot \boldsymbol{\nabla} = \begin{pmatrix} y \\ -x \\ 0 \end{pmatrix} \cdot \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ 0 \end{pmatrix} = \tag{11}$$

   Hint: Remember that this is a dot product, so the resulting operator is a scalar operator.

   (e) Calculate the magnetic tension force

$$\boldsymbol{B} \cdot \boldsymbol{\nabla}\boldsymbol{B} = (.........) \begin{pmatrix} y \\ -x \\ 0 \end{pmatrix} = \tag{12}$$

   Here, (.........) is the result from the previous question.

   (f) Verify that

$$\boldsymbol{J} \times \boldsymbol{B} = \boldsymbol{B} \cdot \boldsymbol{\nabla}\boldsymbol{B} - \boldsymbol{\nabla}(\tfrac{1}{2}\boldsymbol{B}^2) \tag{13}$$