Name: $\qquad$

1. Estimate the solar neutrino production given the Sun's luminosity of $4 \times 10^{26} \mathrm{~J} / \mathrm{s}$ and the fact that the production of each helium atom yields an energy of $E=m c^{2}$ where $m=0.03$ hydrogen masses of $\approx 2 \times 10^{-27} \mathrm{~kg}$. Remember that each time, two neutrinos are being produced (cf. Lecture 4, p. 5, pp. 8-11).
(a) Compute the amount of neutrinos produced per second
(b) Compute the neutrinos flux on Earth, assuming $4 \pi r_{E}^{2}=3 \times 10^{23} \mathrm{~m}^{2}$.
2. Describe qualitatively the solar butterfly diagram. Where do sunspots occur in the beginning and the end of the cycle, discuss the change in polarity of bipolar regions, and interpret this in terms of magnetically buoyant flux tubes (cf. Lecture 17, pp. 3-7).
3. Discuss the lower boundary of the cavity for sound waves in the Sun (cf. Lecture 12, p. 14, Stix, p. 203).
(a) Explain what happens to the trajectory of a sound wave in the Sun where the temperature (and hence the sound speed) increase with depth. Assume that the sound wave travels downward at an oblique angle. Sketch the trajectory together with the local wave front.
(b) Explain the lower boundary of the cavity mathematically by considering the dispersion relation for frequency $\omega$, horizontal wavenumber $k_{x}$, vertical wavenumber $k_{z}$, and sound speed $c_{\mathrm{s}}$ :

$$
\begin{equation*}
\omega^{2}=c_{\mathrm{s}}^{2}\left(k_{x}^{2}+k_{z}^{2}\right) . \tag{1}
\end{equation*}
$$

Solve for $k_{z}^{2}$, and now assume that both $c_{\mathrm{s}}$ and $k_{z}$ depend on the vertical coordinate $z$.
(c) Write down the condition for $k_{z}^{2}=0$ and compute $c_{\mathrm{s}}$ at that point. Assume $k_{x}=$ $0.1 \mathrm{Mm}^{-1}$ and $\omega=0.02 \mathrm{~s}^{-1}$. Give the result for $c_{\mathrm{s}}$ in $\mathrm{km} / \mathrm{s}$.
(d) Explain what happens at larger depths, where $c_{\mathrm{s}}$ is larger than the value you derived above.

