ASTR-3760: Solar and Space Physics .... **(Wednesday, March 9, 2016)**

Third sample material for midterm exam (which is this week: Wednesday, March 16, 2016)

Name: ....

1. Be able to estimate the brightness temperature from a given plot of flux density versus radio wavelength (cf. Lecture 2, p. 20). Convert between $I_\nu(\nu,T)$ and $I_\lambda(\lambda,T)$ (cf. Lecture 3, p. 4).

   According to Wien's law, the intensity at long wavelengths is given by

$$I_\lambda = 2\pi c k_\mathrm{B} T (R_\odot / r_{\odot\oplus})^2$$

   where $r_{\odot\oplus}$ is the distance between the Sun and the Earth.

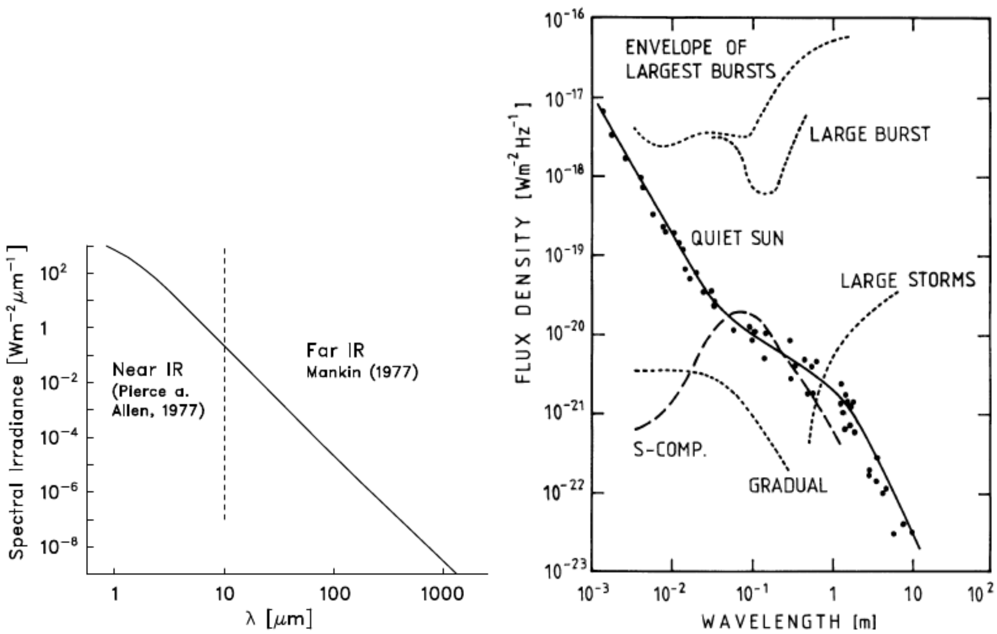

Figure 1: Left: intensity I_λ of the Sun at infrared wavelengths. Right: intensity I_ν of the Sun at radio wavelengths.

   (a) The two plots in Fig. 1 overlap at wavelength $\lambda = 1\,\mathrm{mm}$, but the two quantities are not quite the same. Remember that $I_\lambda\,\mathrm{d}\lambda = I_\nu\,\mathrm{d}\nu$ and that the frequency is given by $\nu = c/\lambda$. Verify that the values in the two plots are consistent. For this, you need to derive the conversion factor between the two. [Hint: this involves differentiation to compute $\mathrm{d}\nu/\mathrm{d}\lambda$.]

   (b) Looking at Fig. 1, you notice a kink in the function I_λ. Interpret this in terms of a change in the brightness temperature. What is the reason for this change?

2. Compute magnetic pressure gradient and tension force for a simple vector field (cf. Lecture 10, p. 15 and 16; Stix, Chap. 8.1.4).

   (a) Consider a two-dimensional magnetic field given by

$$\boldsymbol{B} = \begin{pmatrix} 1 \\ 2x \\ 0 \end{pmatrix} \tag{1}$$

   and calculate the current density (assuming $\mu_0 = 1$),

$$\boldsymbol{J} = \boldsymbol{\nabla} \times \boldsymbol{B} = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2x \\ 0 \end{pmatrix} = \tag{2}$$

   Give a rough sketch of the magnetic field lines.

   (b) Calculate the magnetic pressure (again assuming $\mu_0 = 1$)

$$\tfrac{1}{2}\boldsymbol{B}^2 = \tag{3}$$

   (c) Calculate the magnetic pressure gradient

$$\boldsymbol{\nabla}(\tfrac{1}{2}\boldsymbol{B}^2) = \tag{4}$$

   (d) Give the expression for

$$\boldsymbol{B} \cdot \boldsymbol{\nabla} = \begin{pmatrix} 1 \\ 2x \\ 0 \end{pmatrix} \cdot \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ 0 \end{pmatrix} = \tag{5}$$

   Hint: Remember that this is a dot product, so the resulting operator is a scalar operator.

   (e) Calculate the magnetic tension force

$$\boldsymbol{B} \cdot \boldsymbol{\nabla}\boldsymbol{B} = (.........) \begin{pmatrix} 1 \\ 2x \\ 0 \end{pmatrix} = \tag{6}$$

   Here, (.........) is the result from the previous question. In which direction does the tension force point? Indicate this in the sketch of magnetic field vectors.

   (f) Verify that

$$\boldsymbol{J} \times \boldsymbol{B} = \boldsymbol{B} \cdot \boldsymbol{\nabla}\boldsymbol{B} - \boldsymbol{\nabla}(\tfrac{1}{2}\boldsymbol{B}^2) \tag{7}$$

   (g) Optional: can you think of field lines that are similarly shaped, but where the magnetic pressure does not depend on x?