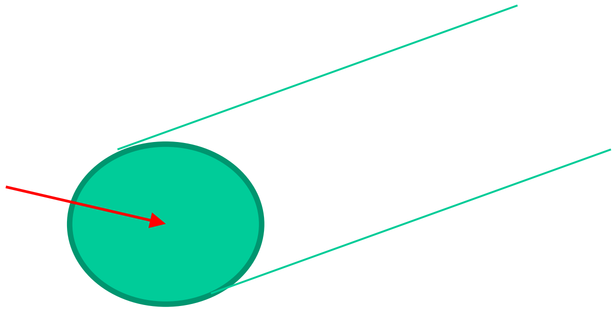
Lecture 27

- Details on midterm exam
  - Magnetic buoyancy
  - Solar cycle polarity reversals
  - Refraction in the Sun
- More on convection

Flux tubes: what happens when total interior pressure exceeds exterior pressure

interior pressure

$$P_{\text{gas}}^{\text{int}} + \underbrace{\mathbf{B}^2 / 2\mu_0}_{P_{\text{mag}}}$$



exterior pressure

$$P_{\text{gas}}^{\text{ext}}$$

A. Tube rises

B. Tube sinks

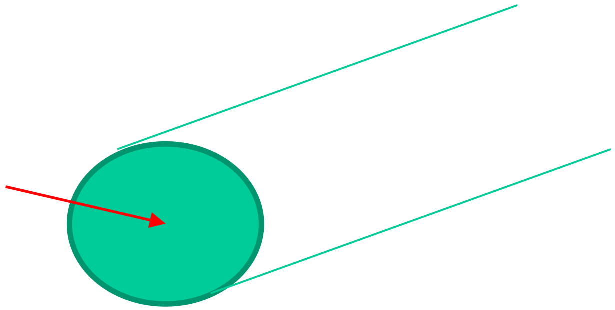
C. Tube expands

D. Tube shrinks

Flux tubes: what happens when total interior pressure *exceeds* exterior pressure

interior pressure

$$P_{\text{gas}}^{\text{int}} + \underbrace{\mathbf{B}^2 / 2\mu_0}_{P_{\text{mag}}}$$



exterior pressure

$$P_{\text{gas}}^{\text{ext}}$$

A. Tube rises

B. Tube sinks

C. Tube expands

D. Tube shrinks

…by a small amount until again in equilibrium

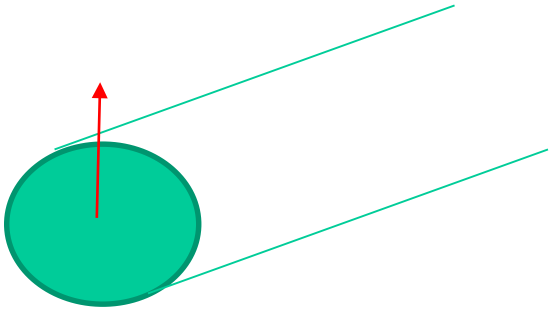
What happens next?

equilibrium $$P_{\text{gas}}^{\text{int}} + \underbrace{\mathbf{B}^2 / 2\mu_0}_{P_{\text{mag}}} = P_{\text{gas}}^{\text{ext}}$$

so $$P_{\text{gas}}^{\text{int}} < P_{\text{gas}}^{\text{ext}}$$ because $$\underbrace{\mathbf{B}^2 / 2\mu_0}_{P_{\text{mag}}} > 0$$

thus $$\rho_{\text{gas}}^{\text{int}} < \rho_{\text{gas}}^{\text{ext}}$$ if T=constant

→ tube is lighter



A. *Tube rises*

B. Tube sinks

C. Tube expands

D. Tube shrinks

Homework 2, problem 3

(a) The total pressure is given by

$$P_{\rm tot} = P_{\rm gas} + P_{\rm mag} = \frac{\rho k_{\rm B} T}{\mu m_{\rm H}} + \frac{B^2}{2\mu_0} \ .$$

The interior of the tube has both gas and magnetic pressure. The surroundings have weak or negligible magnetic field, so that allows us to assume that the surroundings have *only* gas pressure. Thus, if the total pressure is equal at a given height z in the two regions, then

$$\begin{aligned} P_{\rm tot,T} &= P_{\rm tot,S} \\ \frac{\rho_{\rm T} k_{\rm B} T}{\mu m_{\rm H}} + \frac{B^2}{2\mu_0} &= \frac{\rho_{\rm S} k_{\rm B} T}{\mu m_{\rm H}} \end{aligned}$$

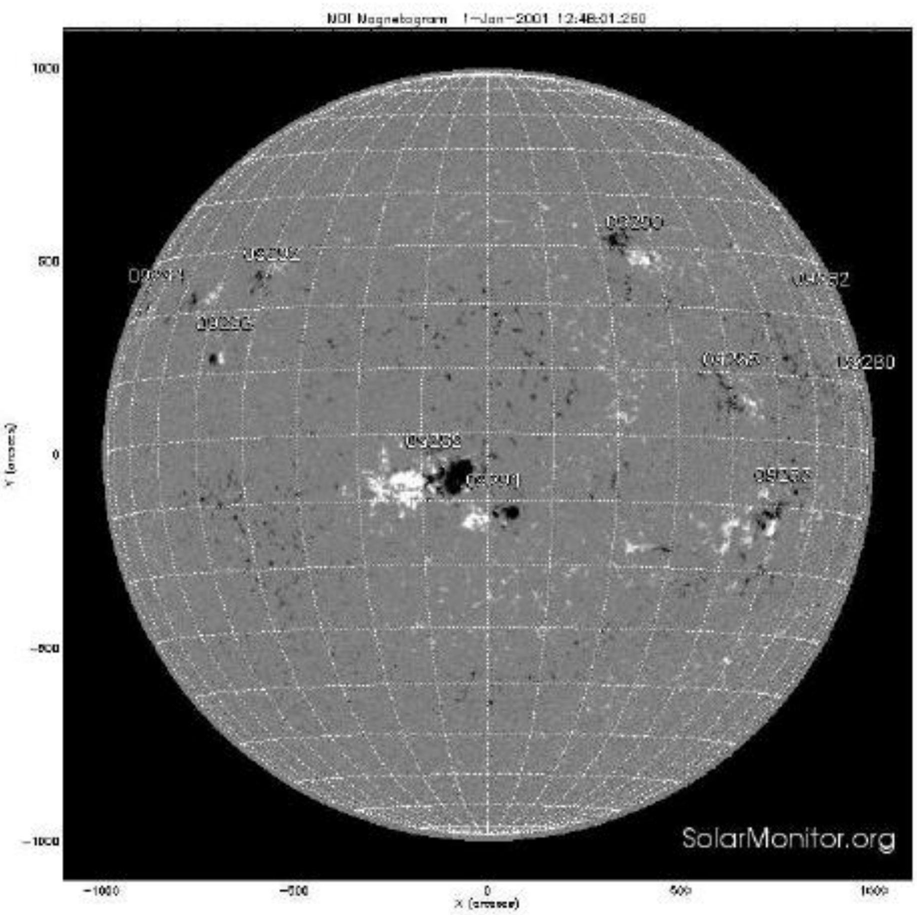
and thus, we can solve for

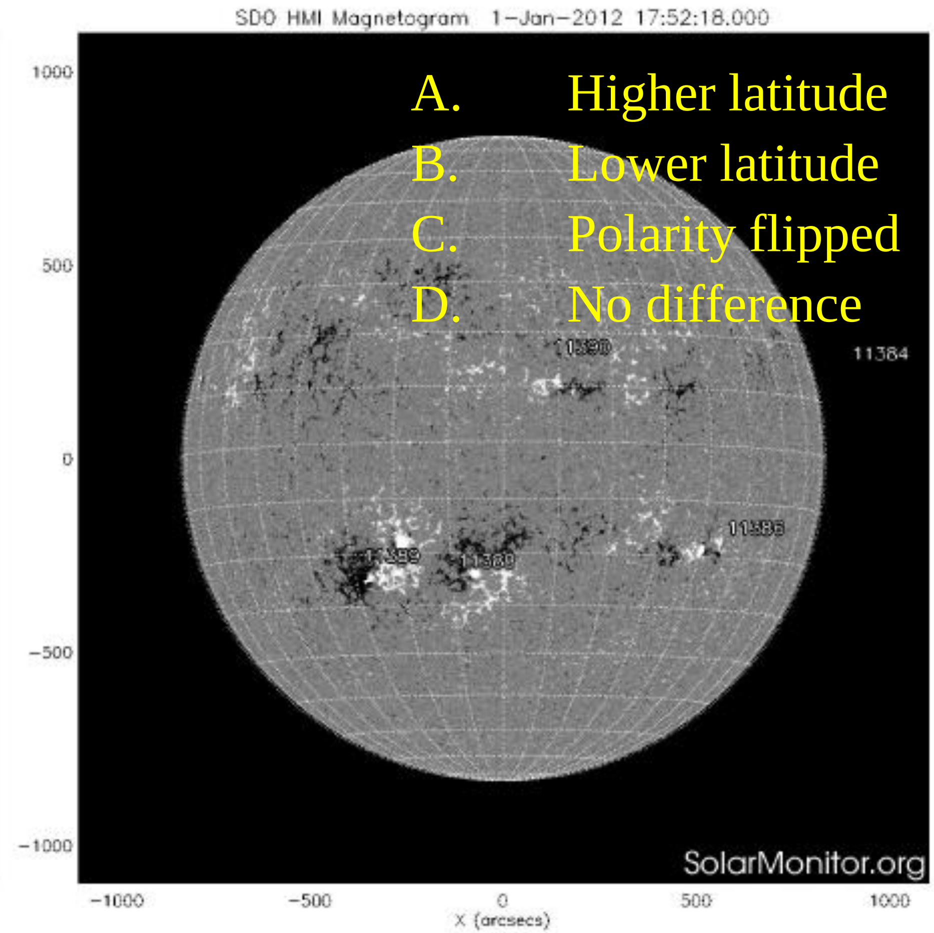
$$\frac{B^2}{2\mu_0} = \frac{(\rho_{\rm S} - \rho_{\rm T}) k_{\rm B} T}{\mu m_{\rm H}} = \left[\frac{(\rho_{0,\rm S} - \rho_{0,\rm T}) k_{\rm B} T}{\mu m_{\rm H}}\right] \exp\left(-\frac{z}{H}\right) \ .$$

Exterior=surroundings=S

Interior=tube=T (was less dense; lighter)

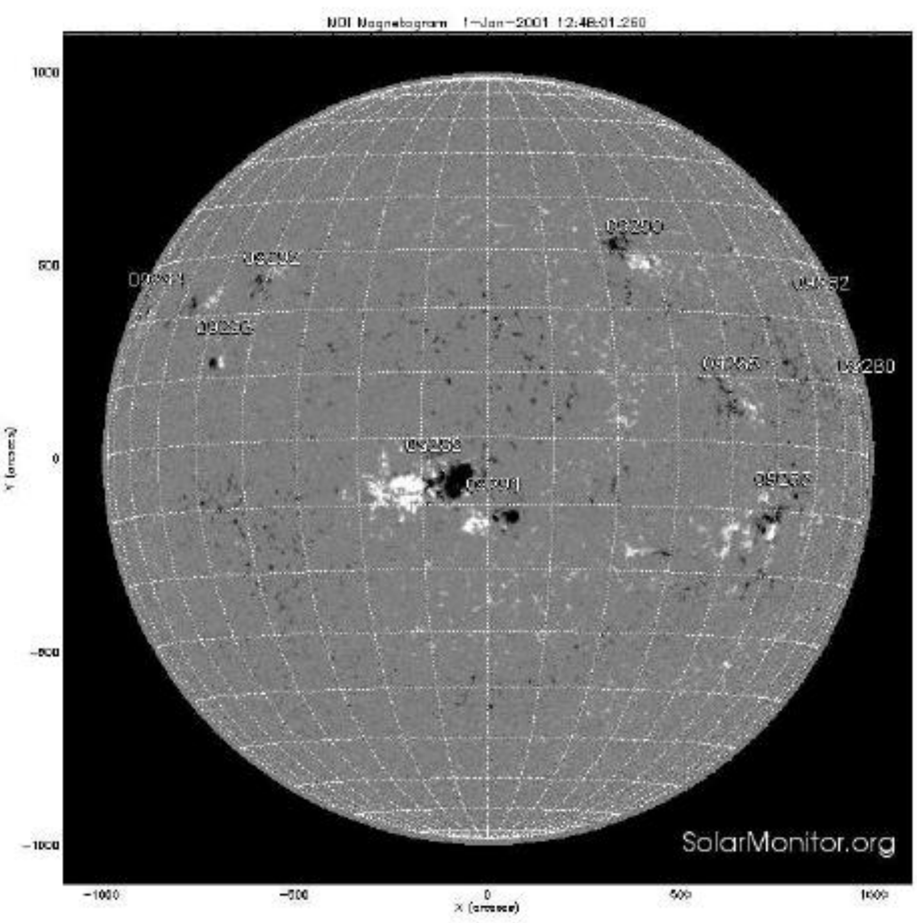
What is different: (i) 2001 and 2012

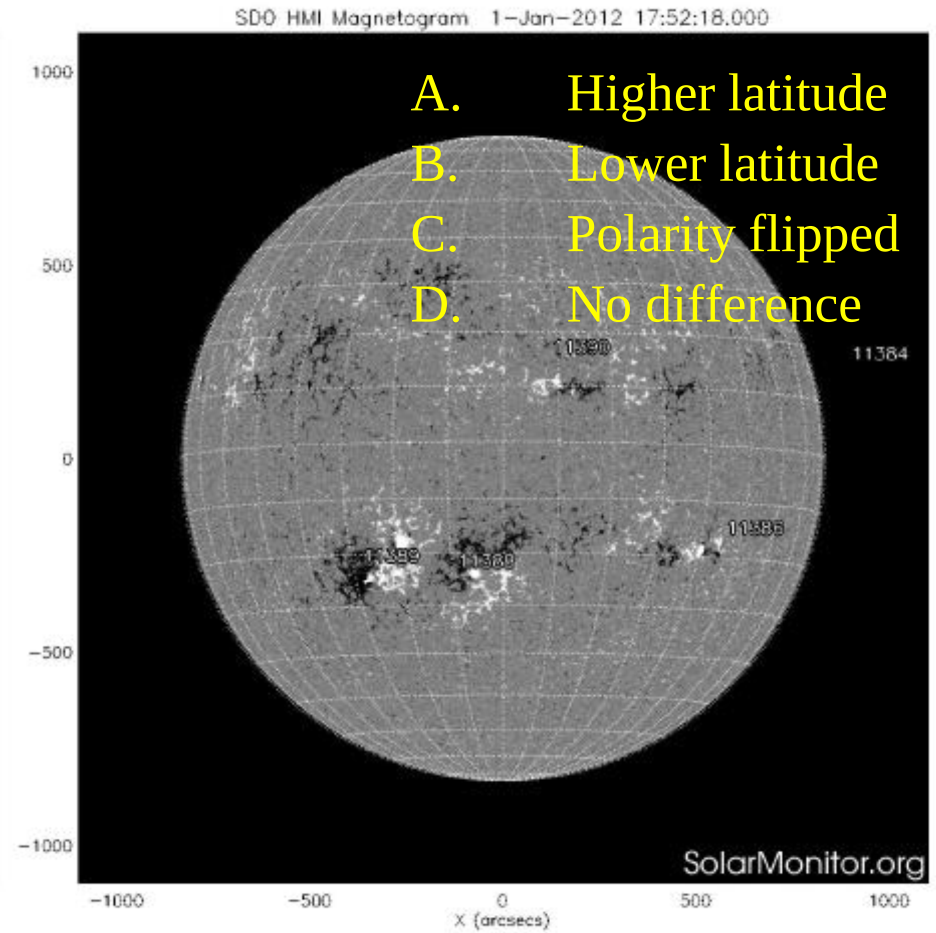




A. Higher latitude
B. Lower latitude
C. Polarity flipped
D. No difference

What is different: (ii) north & south





A. Higher latitude

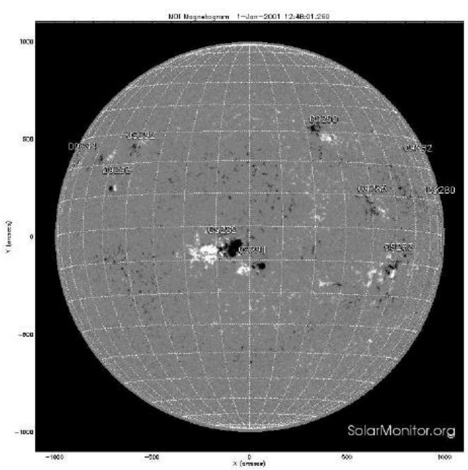
B. Lower latitude

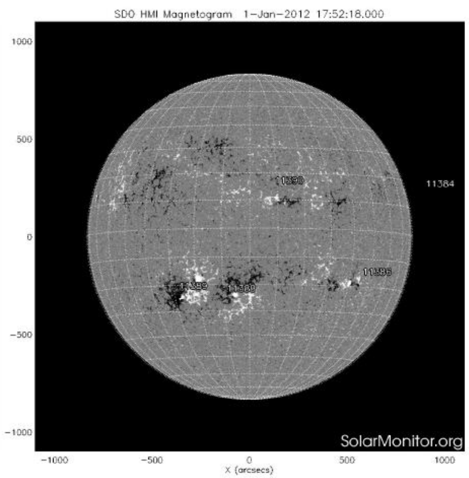
C. Polarity flipped

D. No difference

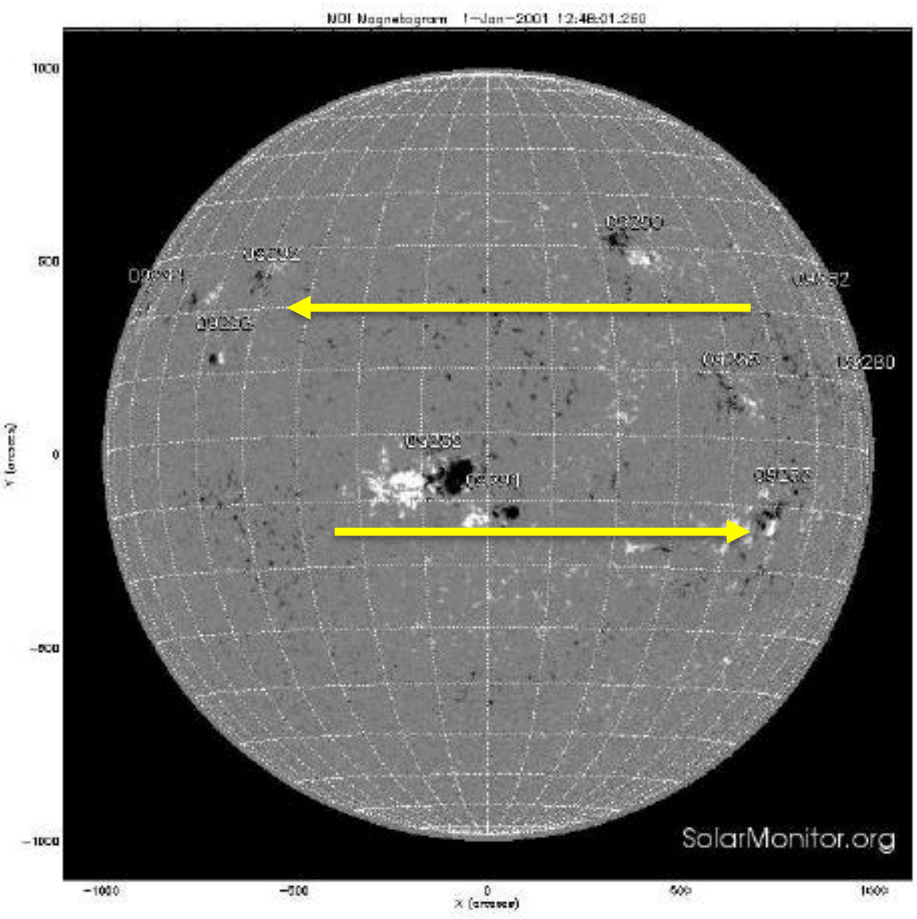
(i) 2001 and 2012, (ii) north and south

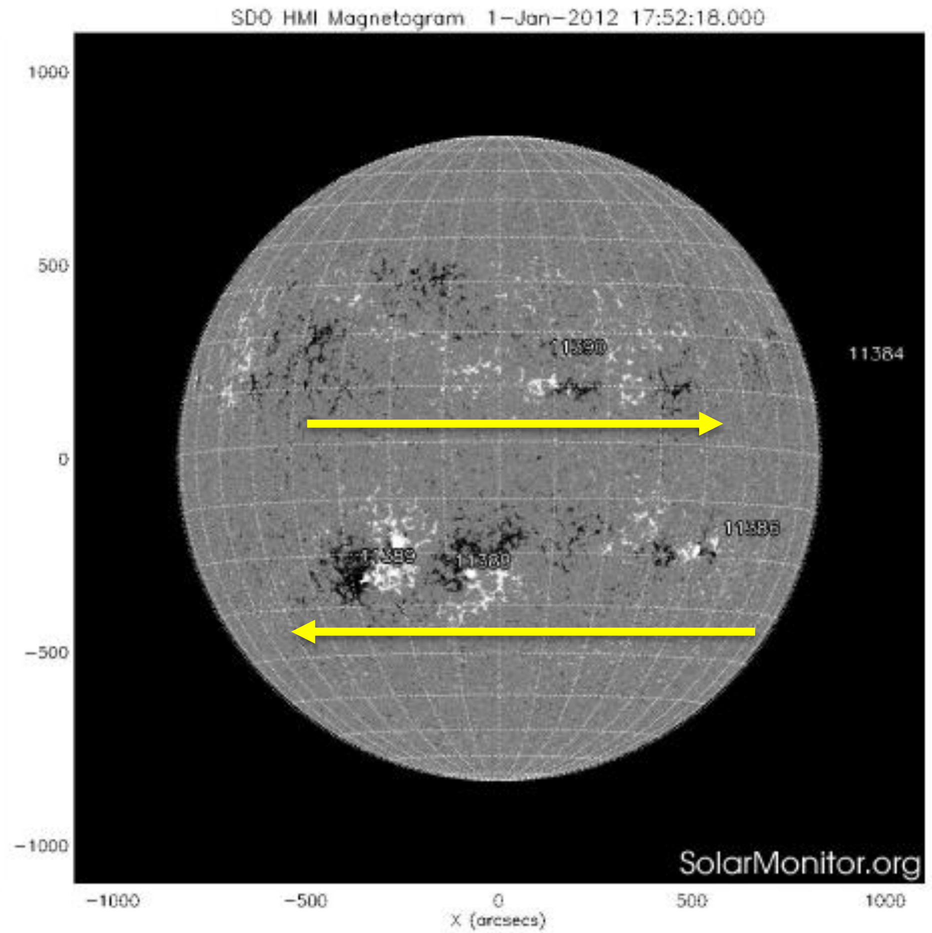
- Cycle period ~11 yr
  - latitude shound be same
- Polarity flipped
  - In time (2001/2012), and
  - in space (north/south)





Flips in space & time





Lecture 12: vertical wavenumber

Dispersion relation of Lecture 11
$$\omega^2 = \frac{\Re T}{\mu} k_z^2$$

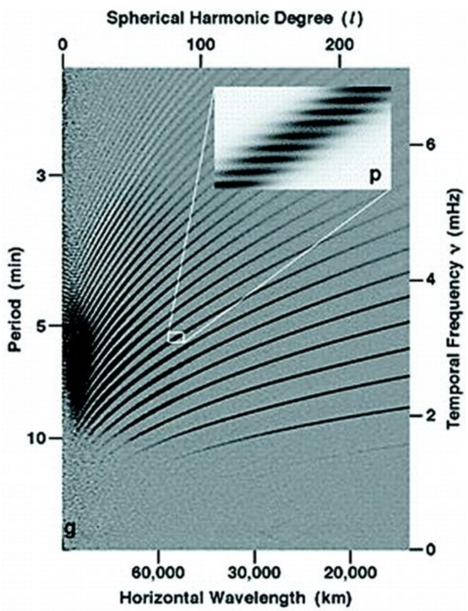
In 3-D
$$\omega^2 = c_s^2 \, (\underbrace{k_x^2 + k_y^2}_{k_\mathrm{hor}^2} + \underbrace{k_z^2}_{k_\mathrm{vert}^2})$$

"Solve" for k_z
$$k_z^2 = \frac{\omega^2}{c_s^2} - k_\mathrm{hor}^2$$

Consider $c_s = c_s(\mathrm{r})$ [oops?]

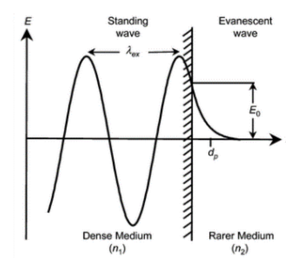
In quantum mechanics:
WKB approximation

→ Jeffreys-Wrentzel-Kramers-Brilloin
→ Tunnel effect → Gamow!!

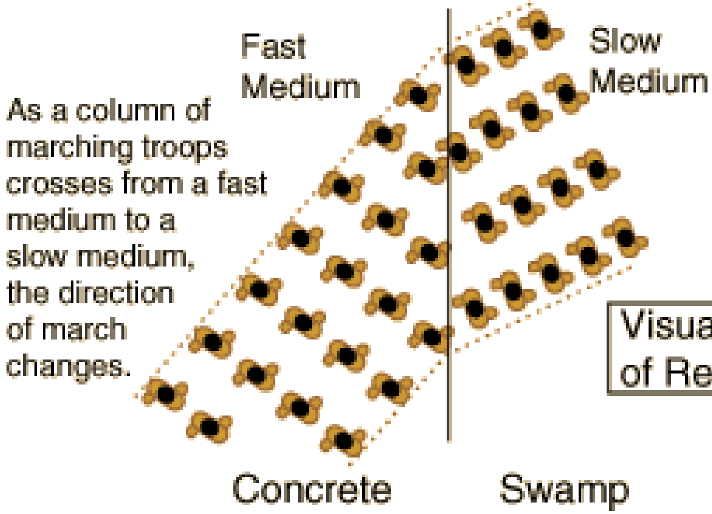


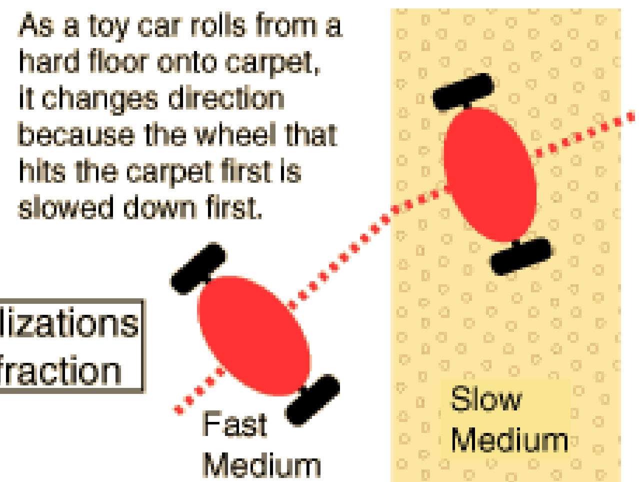
Example
$$\omega = \frac{2\pi}{300\mathrm{s}} = 0.02\mathrm{s}^{-1}$$
$$k_\mathrm{hor} = \frac{\ell}{R} = \frac{100}{700\,\mathrm{Mm}}$$
$$c_s = \frac{\omega}{k_{hor}} = 0.02 \times 7 \,\tfrac{\mathrm{Mm}}{\mathrm{s}} = 140\,\tfrac{\mathrm{km}}{\mathrm{s}}$$

Deeper down: k_z imaginary!?
$$e^{ik_z z} = e^{\mp |k_z| z}$$



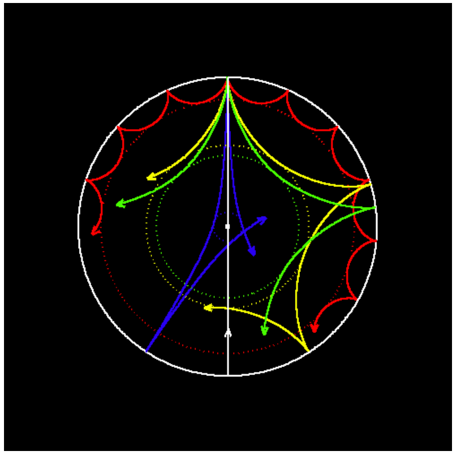
Refraction analogy





Application to the Sun:
Upward bending

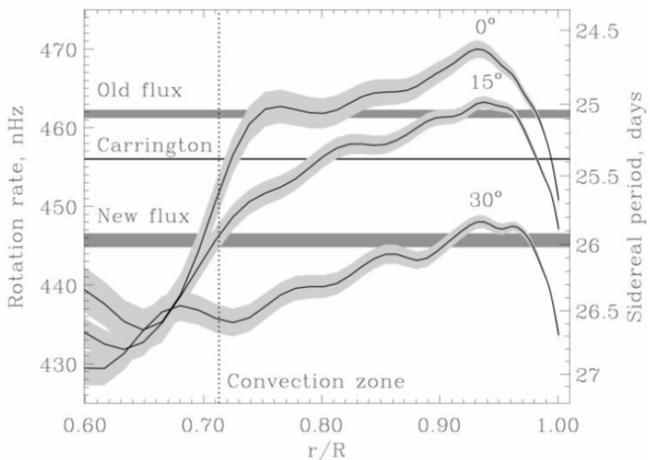
at the top: reflection when wavenlength ~ density scale height

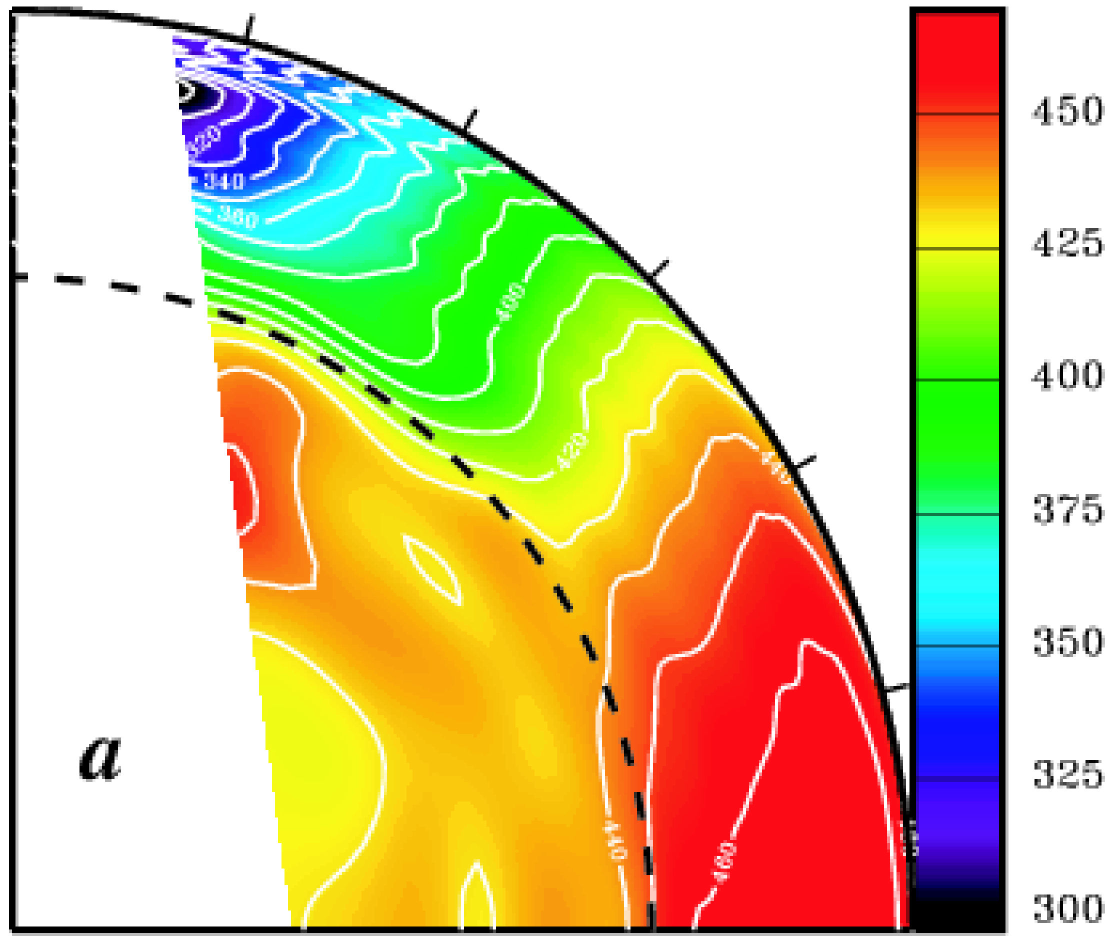


Deeper down:
Sound speed large

$$c_s^2 = \frac{RT}{\mu}$$

Internal angular velocity from helioseismology

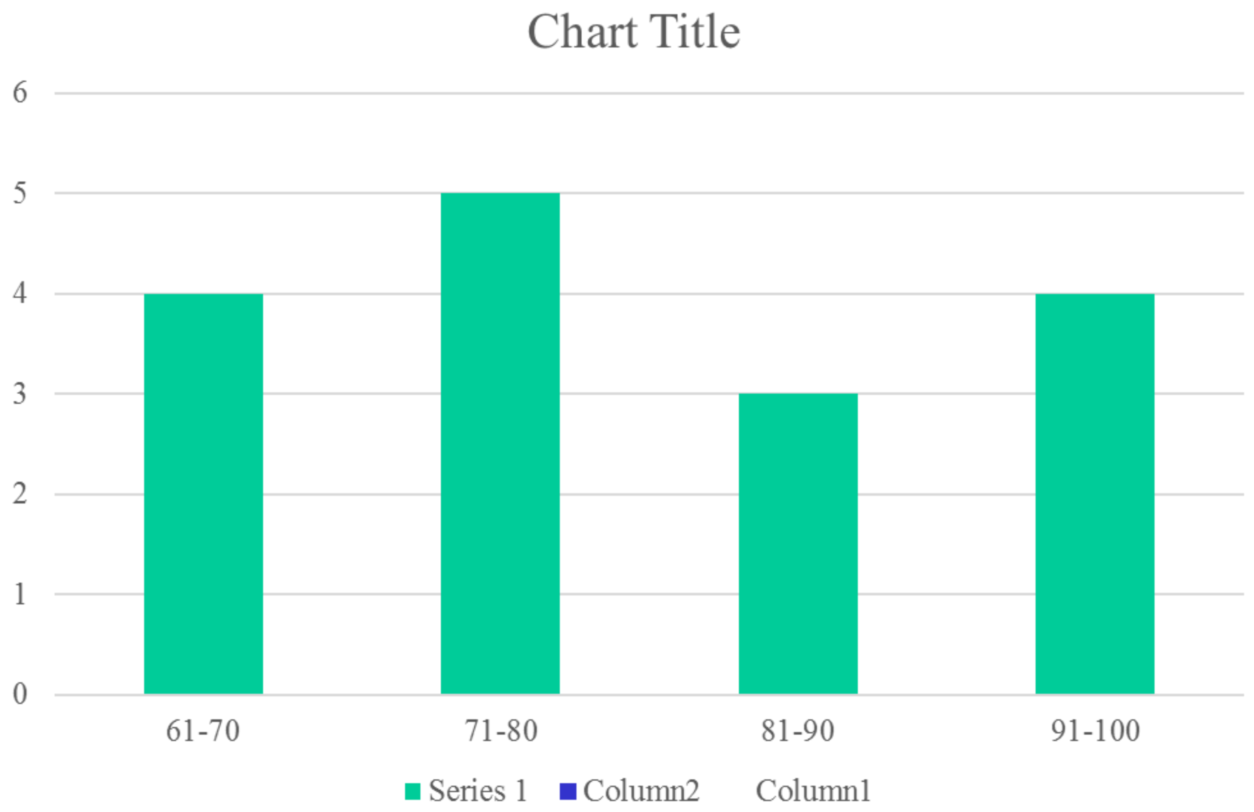




spoke-like at equ.
dΩ/dr>0 at bottom

? dΩ/dr<0 at top

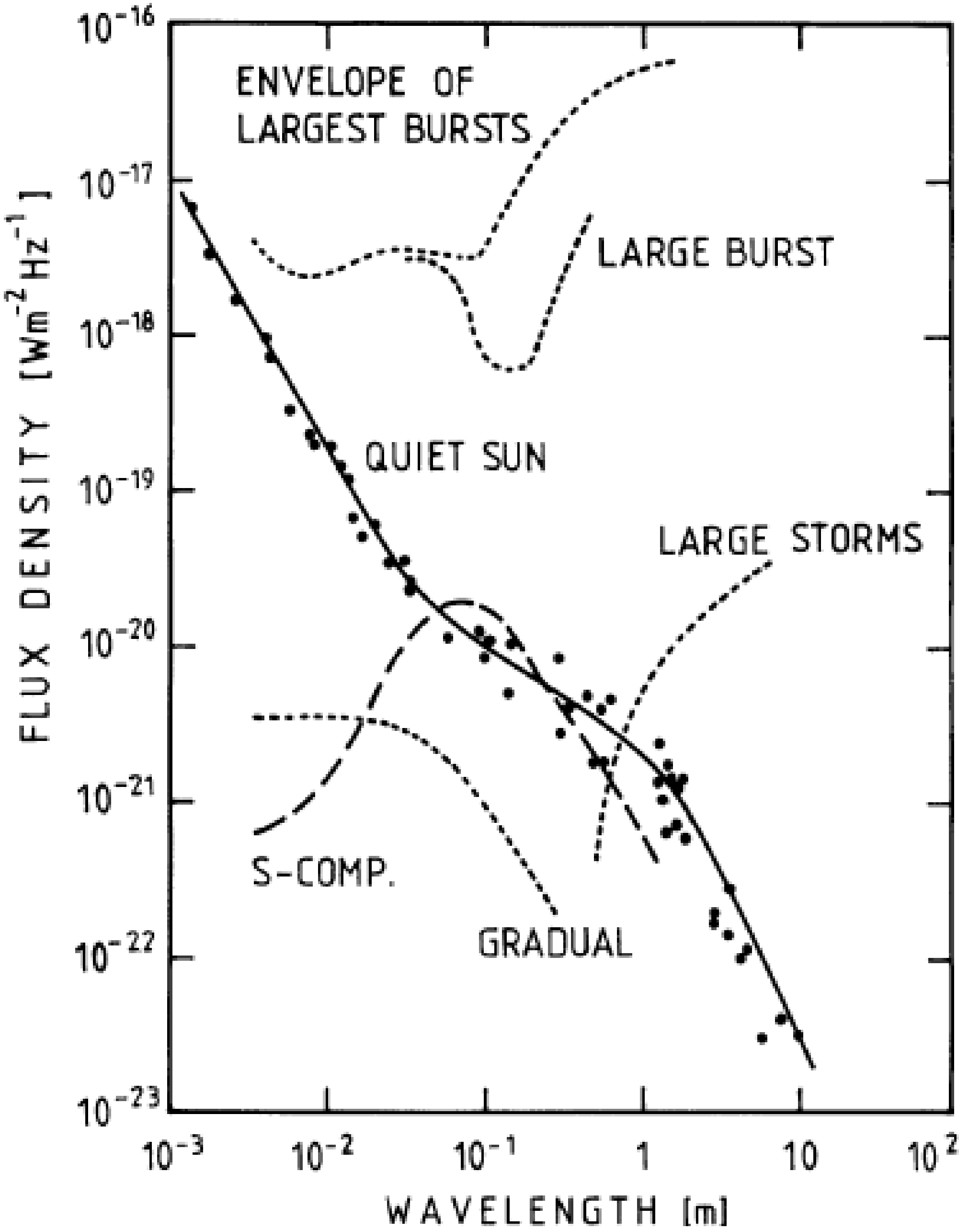
Midterm exam outcome



Lecture 2: Radio: interesting break

- Clue about hot corona
- Brightness temperature

$$S(\lambda) \simeq 2\pi ckT\lambda^{-4}(r_\odot/A)^2$$



Conversion & numbers

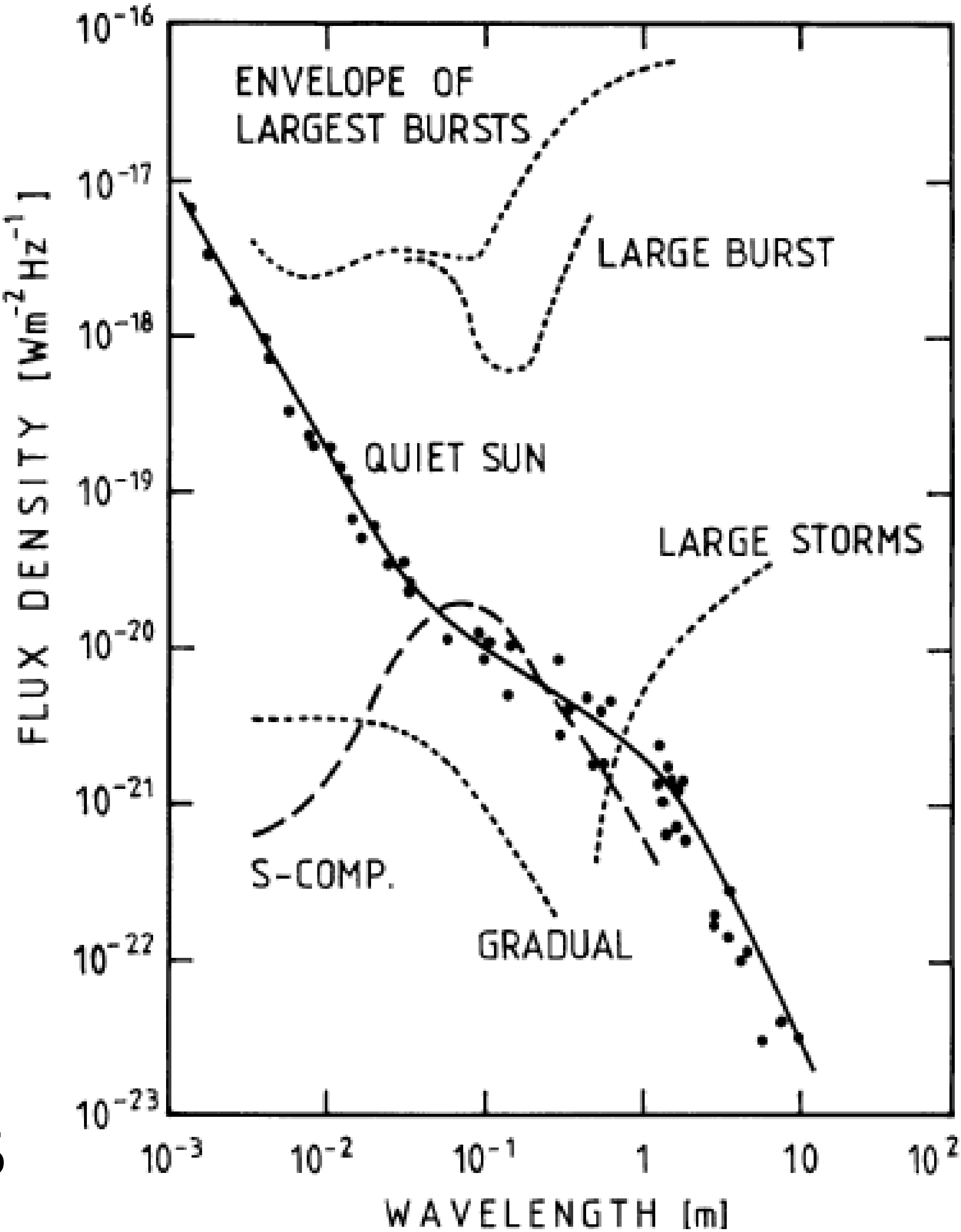
$$S(\lambda) \simeq 2\pi ckT\lambda^{-4}(r_\odot/A)^2$$

$$I_\lambda = I_\nu \frac{c}{\lambda^2}$$

$$T = \frac{I_\nu \lambda^2}{2\pi k_B}\frac{r^2}{R^2}$$

$$T = \frac{10^{-17}(10^{-3})^2}{6\times 10^{-23}}\frac{(10^{11})^2}{(10^9)^2}$$

$$= \tfrac{1}{6}10^4 \quad \text{but} \quad \frac{(1.5)^2}{(0.7)^2} = 5$$



and at 1m

$$T = \frac{I_\nu \lambda^2}{2\pi k_B} \frac{r^2}{R^2}$$

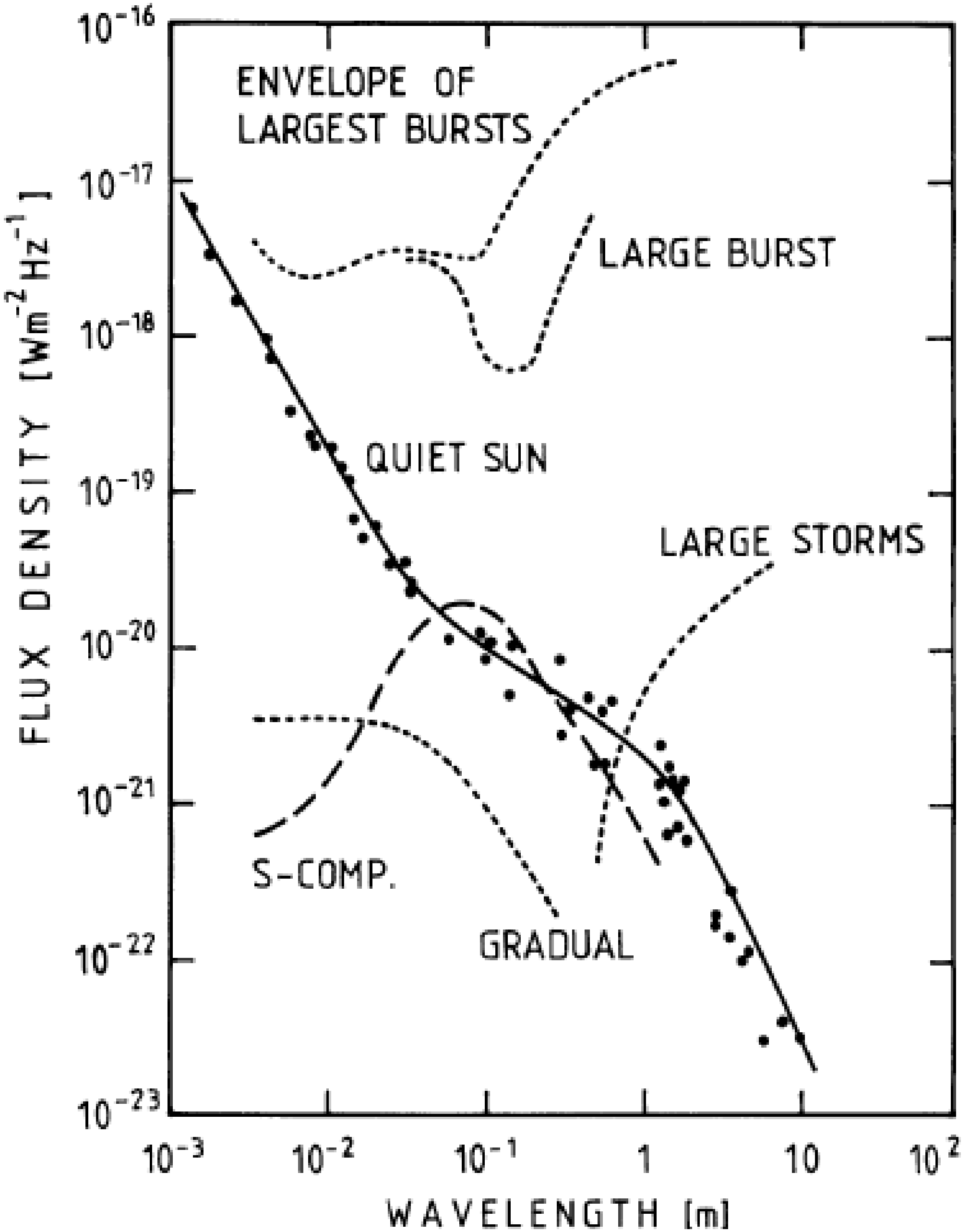
$$T = \frac{10^{-21}}{6\times10^{-23}} \frac{(10^{11})^2}{(10^9)^2}$$

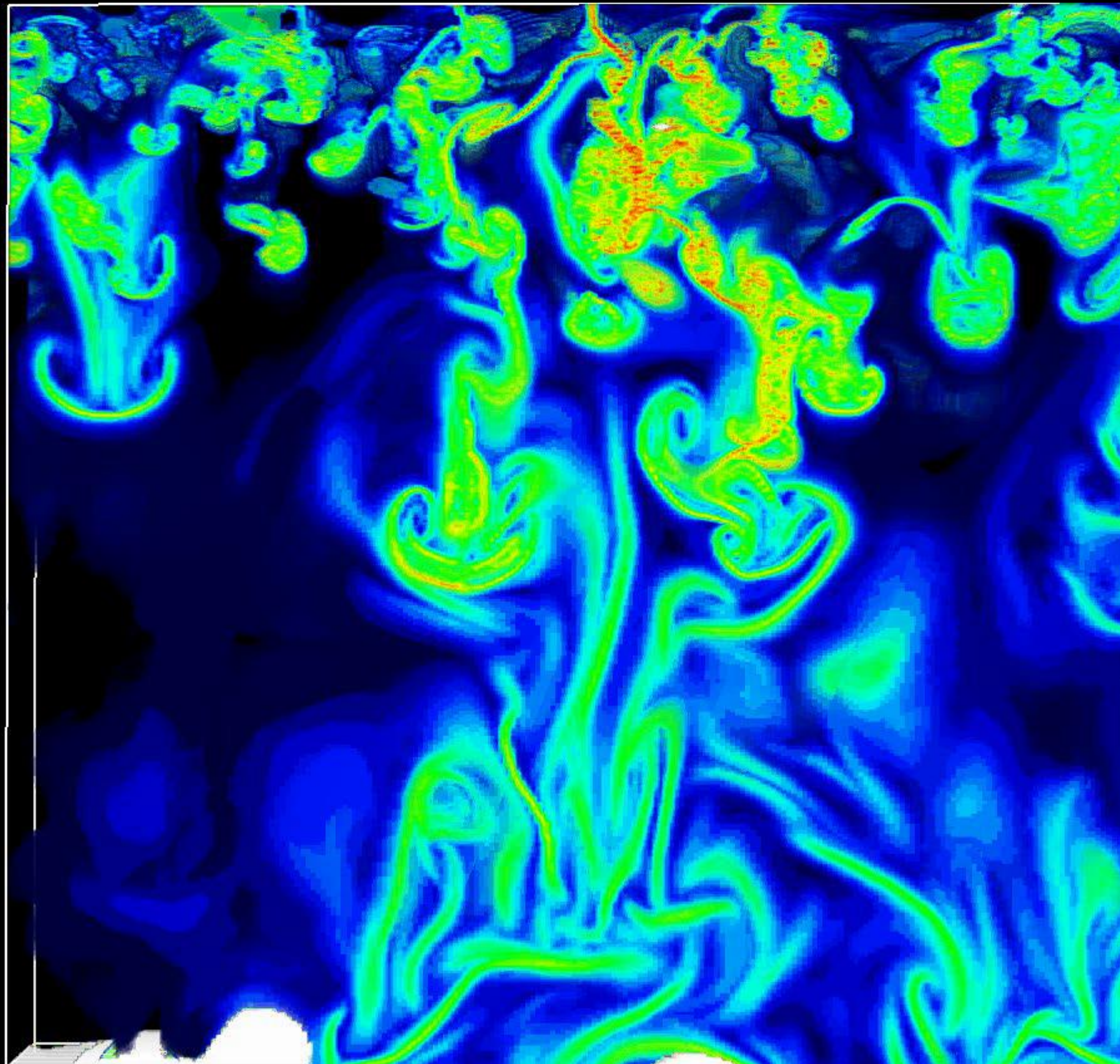
100 times larger

A. Larger λ more energetic?

B. More potential for solar storms?

C. Hotter corona





Courtesy: Bob Stein (MSU)

Convective velocity

Enthalpy flux

$$F_{\mathrm{conv}} = \overline{\rho u c_p \delta T}$$

Mixing length approximation

$$u^2 / \ell \sim g \delta T / T$$

Scaling behavior

$$F_{\mathrm{conv}} = \overline{\rho}\, u_{\mathrm{rms}}^3$$

→ Slower with depth

...ospheric structure through which it is moving. The ... depth the downflows come to dominate the energy tran...

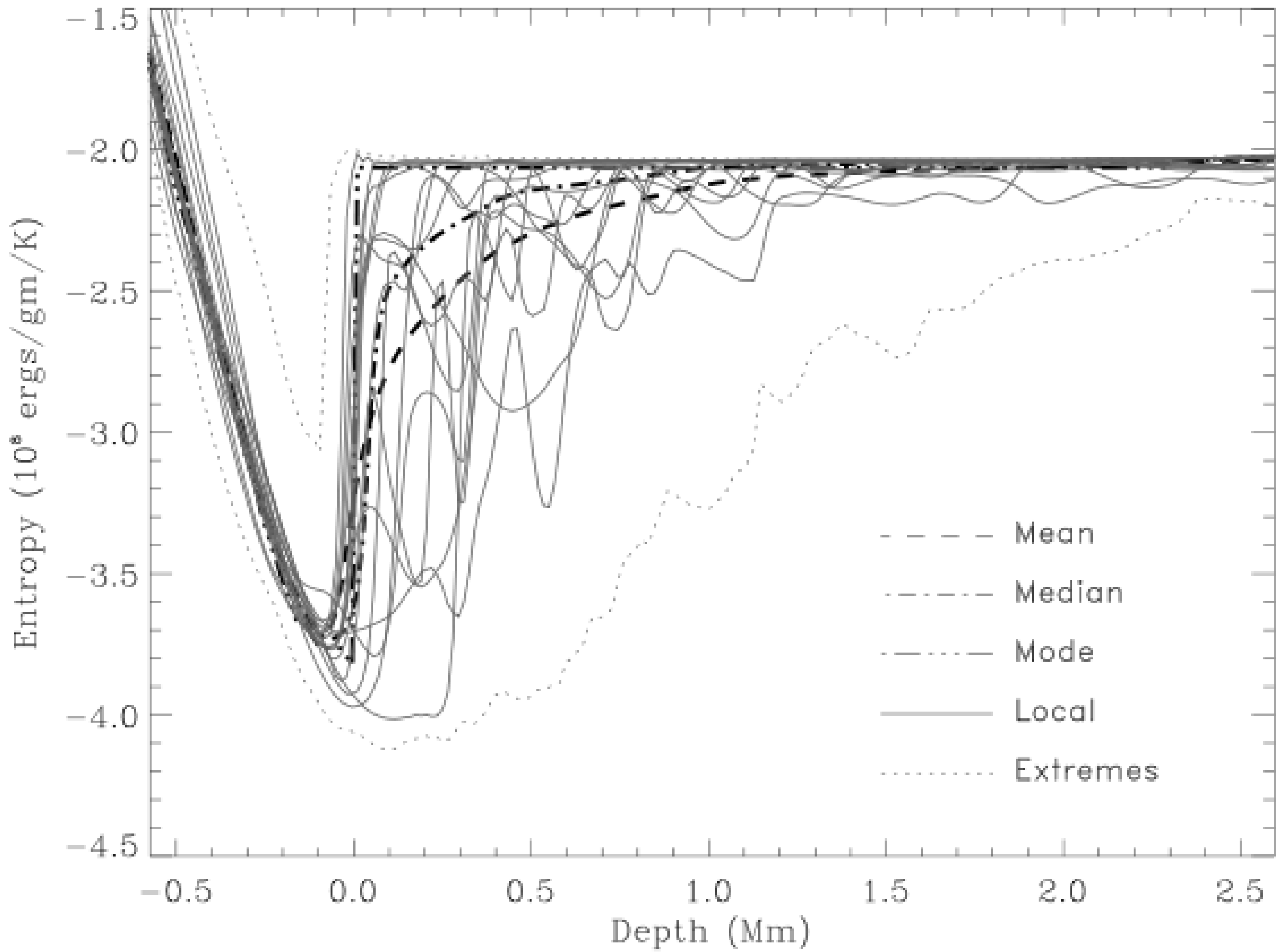


FIG. 13.—Entropy as a function of depth at several horizontal locations plus the average, median, modal and extreme values. Except very near the surfa... ...st of the entropy fluctuations occur in the downdrafts. The range decreases with depth due to entrainment, mixing, and thermal diffusion.

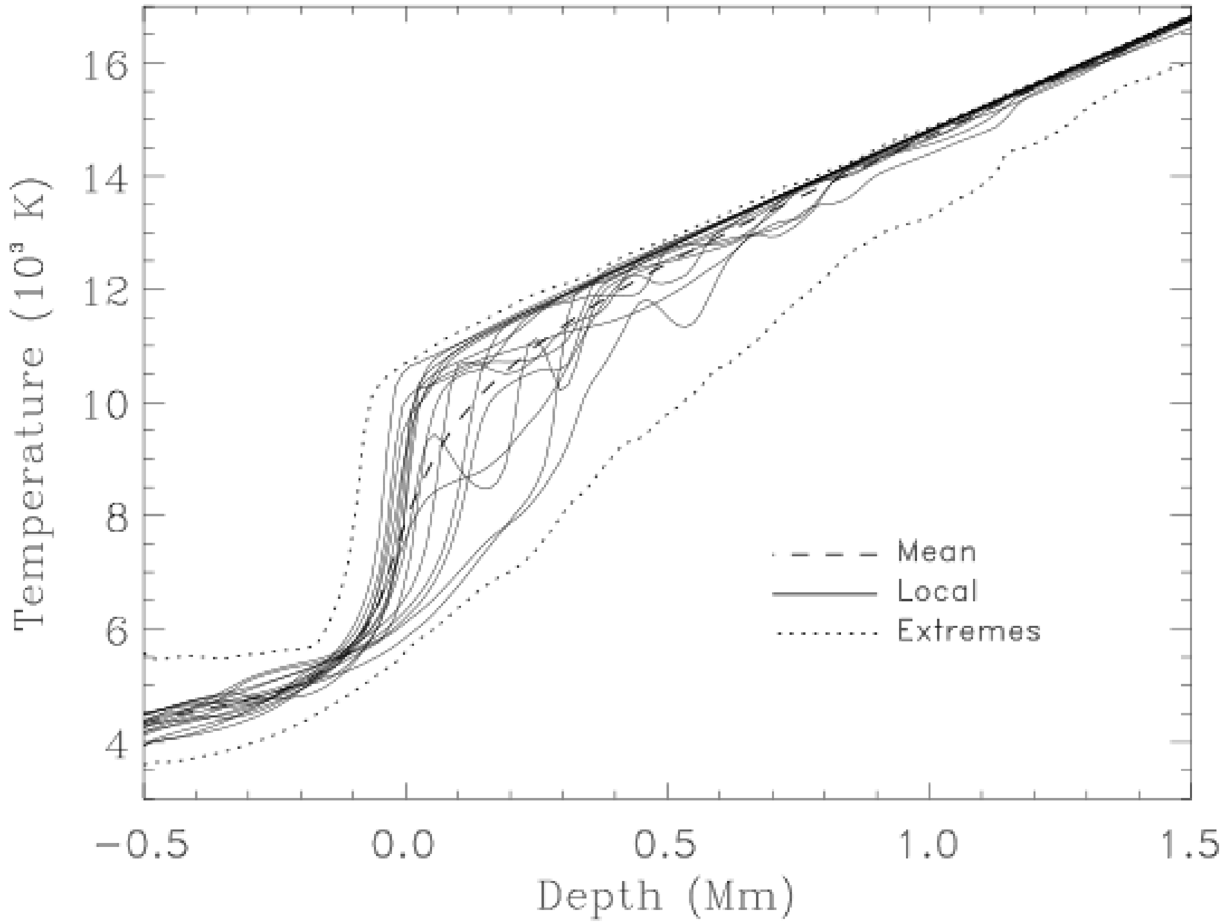


FIG. 14.—Temperature as a function of geometric depth at several horizontal locations plus the average temperature profile. Locally the temperature file is much steeper than the average profile.

rt. In the near surface layers, energy is transported pri-

$\langle \rho u_z \rangle$), $(\sim \frac{1}{3})$ and thermal energy, $F_{\rm thermal} =$ $P/\rho)(\rho u_z - \langle \rho u_z \rangle)$, $(\sim \frac{1}{3})$. The kinetic energy flux, $F_{\rm KE} =$ $^2(\rho u_z - \langle \rho u_z \rangle)$, is about 10% of the enthalpy flux and nsports energy downward in the faster downdrafts g. 16).

Driving of the convective flows comes primarily fro intergranular lanes and downdrafts. The buoyancy driving the convective motions are significantly larger downflows than in the upflows, below the surface, b the entropy fluctuations are much larger in the dow

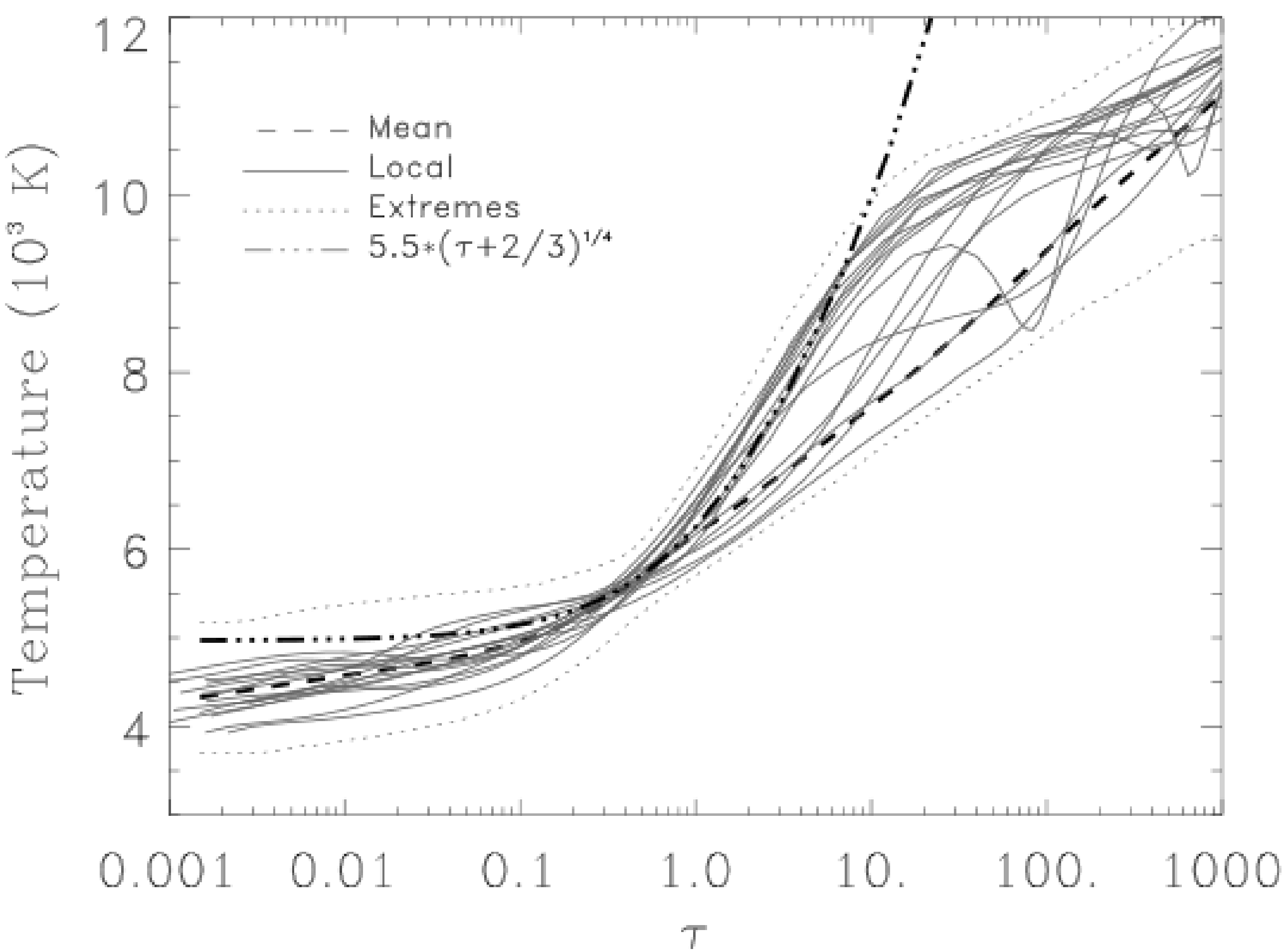
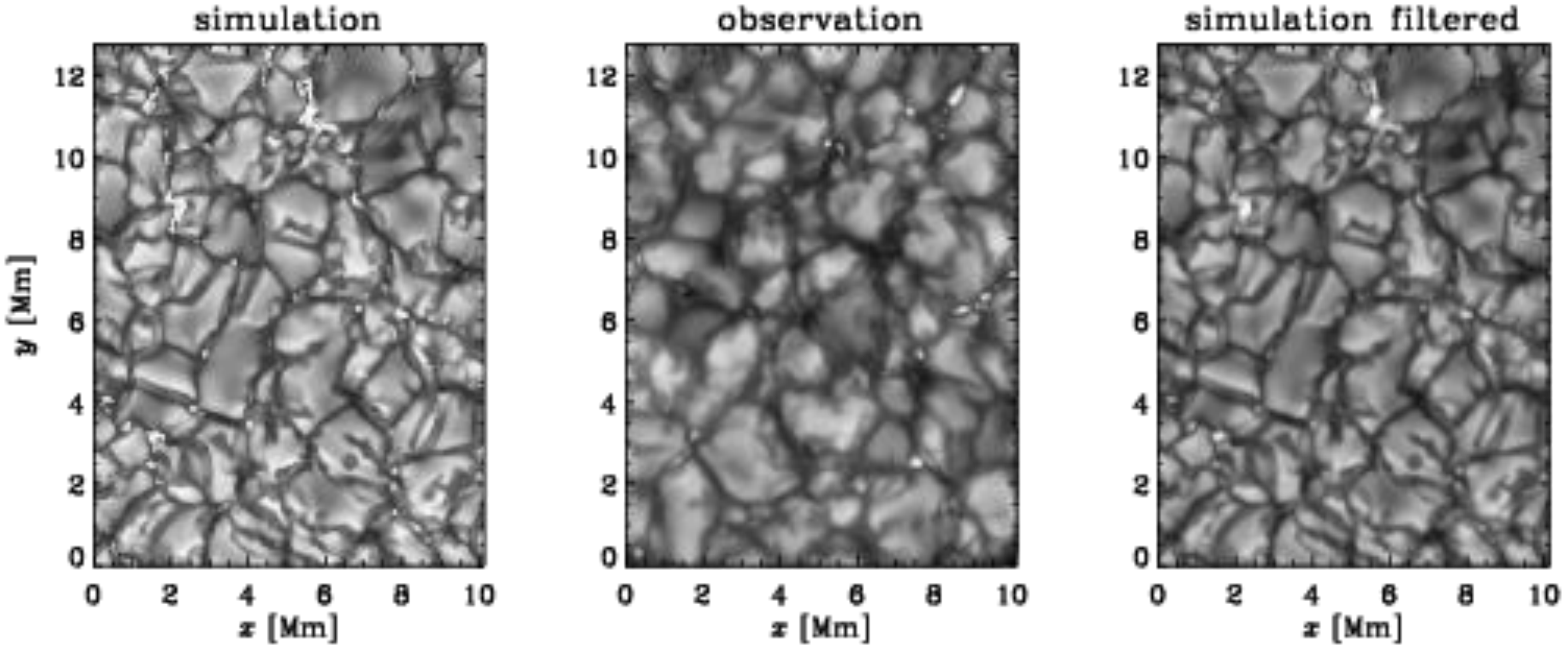


FIG. 15.—Temperature as a function of optical depth at several horizontal locations plus the average temperature profile. On an optical depth s perature profile is nearly the same at all places in the simulation domain, whether in warm upflows or cool downflows. Thus, the temperature stru rly in radiative-convective equilibrium everywhere on the solar surface.

Agreement with observations



Why does convective velocity decrease with depth?

A. Because of cooling only from the top
B. Because density increases downward
C. Because the gas spreads over large scales

What we learned

- Magnetic buoyancy
- Solar cycle polarity reversals
- Refraction in the Sun
- Convection
- Eddington approximation