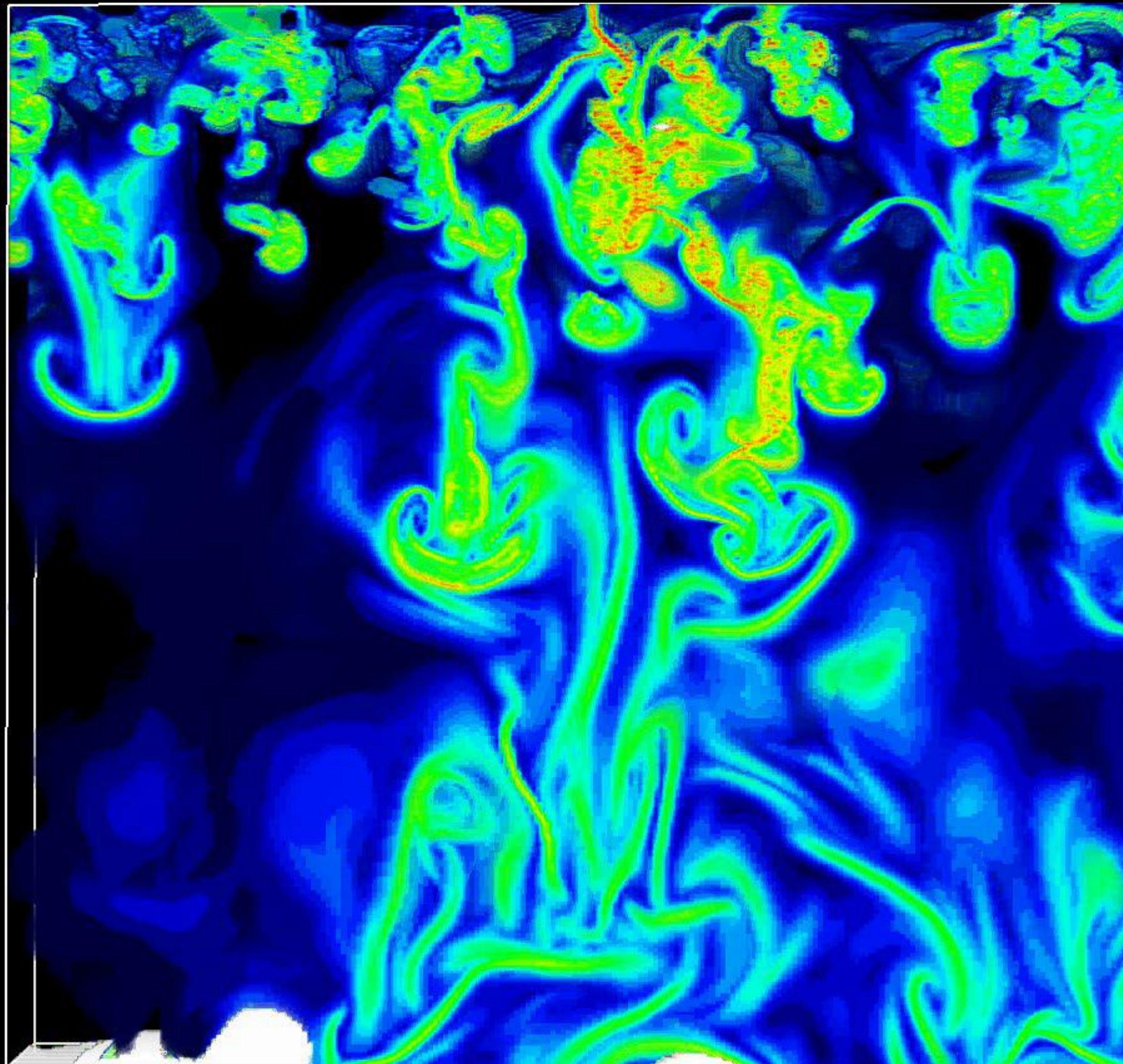
Lecture 28

- Eddington approximation (Stix pp 52-54)
- 2-stream approximation

Last time…

- Magnetic buoyancy
- Solar cycle polarity reversals
- Refraction in the Sun
- Convection
- Eddington approximation



Courtesy: Bob Stein (MSU)

Why does convective velocity decrease with depth?

A. Because of cooling only from the top
B. Because density increases downward
C. Because the gas spreads over large scales
D. Because temperature increases with depth
E. Because sound speed increases with depth

… from lecture 27

Enthalpy flux

$$F_{\rm conv} = \overline{\rho u c_p \delta T}$$

Mixing length approximation

$$u^2 / \ell \sim g \delta T / T$$

Scaling behavior

$$F_{\rm conv} = \overline{\rho}\, u_{\rm rms}^3$$

→ Slower with depth

Radiative transfer solution

Radiative transfer

$$\mu \frac{dI}{d\tau} = I - S$$

non-gray $\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$

Thermal equilibrium
(energy equation)

$$\rho \frac{de}{dt} + P\nabla \cdot \mathbf{u} = -\nabla \cdot \mathbf{F}$$

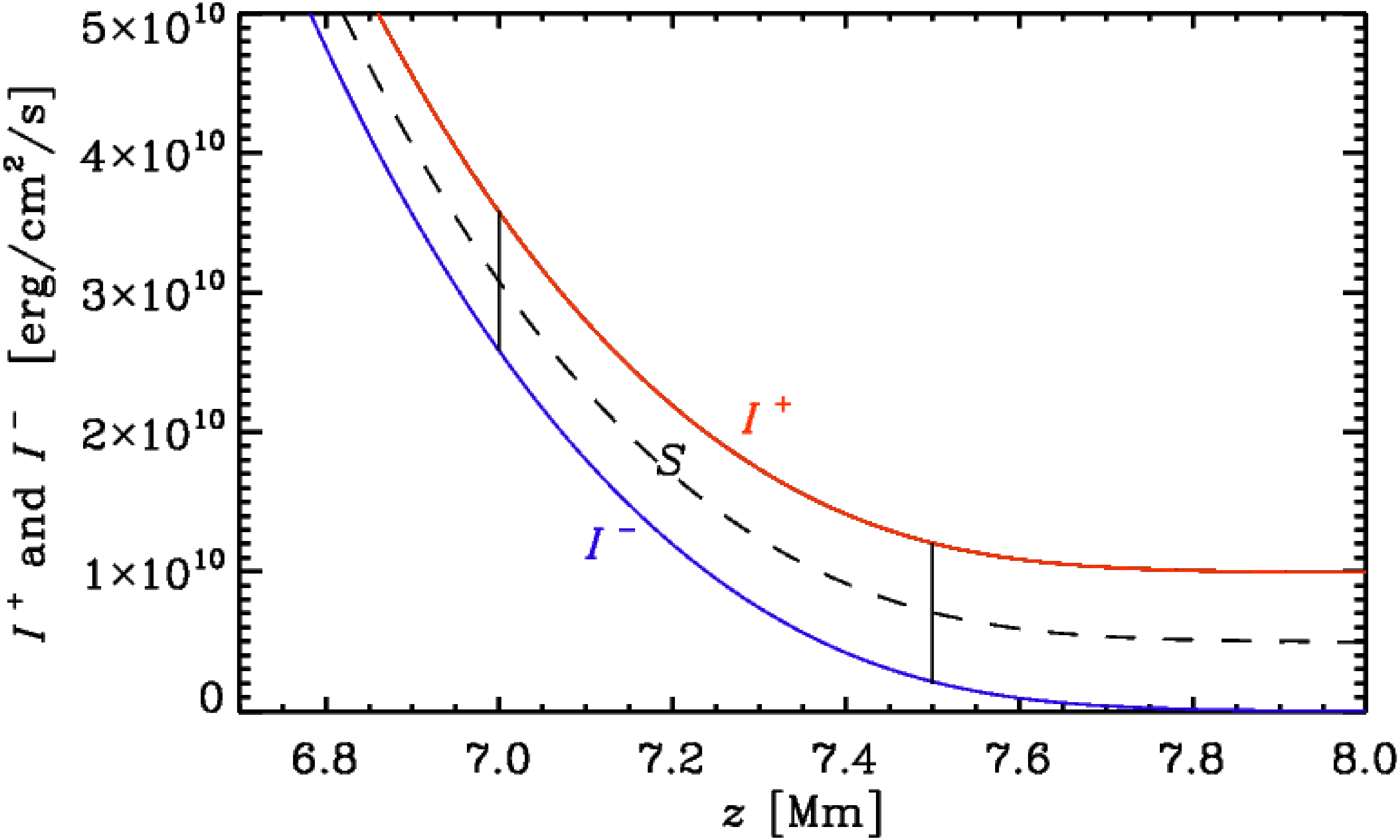
$$\Rightarrow F_z = \int_{4\pi} n_z I \, d\Omega = 2\pi \int_{-1}^{1} \mu I \, d\mu = \text{const}$$

Boundary condition
at the top (τ=0)

$$F_z = \int_{\substack{2\pi \\ \text{upperhalf}}} n_z I \, d\Omega = 2\pi \int_{0}^{1} \mu I \, d\mu = \text{const}$$

In sinulations: Feautrier technique $I = I_+ + I_-$

Two-stream approximation



2-stream approximation

Because of $\frac{dP}{d\tau} = Q = \frac{1}{4\pi} F$

we have $P = P_0 + P_1 \tau$ $\frac{dP}{d\tau} = P_1 = Q = \frac{1}{4\pi} F$

at τ=0 we have $P = P_0 = \frac{1}{2} I_+ = \frac{1}{4\pi} F_z$

$$\frac{\sigma_{\mathrm{B}}}{\pi} T^4 = S = P = \frac{1}{4\pi} F(1+\tau) = \frac{\sigma_{\mathrm{B}}}{4\pi} T_{\mathrm{eff}}^4 (1+\tau)$$

$$\Rightarrow T^4 = \frac{1}{4} T_{\mathrm{eff}}^4 (1+\tau)$$

Eddington approximation

Radiative transfer

$$\mu \frac{dI}{d\tau} = I - S$$

non-gray $\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$

Thermal equilibrium

$$F_z = \int_{4\pi} n_z I \, d\Omega = 2\pi \int_{-1}^{1} \mu I \, d\mu = \text{const}$$

Boundary condition at the top (τ=0)

$$F_z = \int_{\substack{2\pi \\ \text{upperhalf}}} n_z I \, d\Omega = 2\pi \int_{0}^{1} \mu I \, d\mu = \text{const}$$

ansatz (trial solution)

$$I = I_0 + \mu I_1$$

Eddington approximation

Note that

$$\int_0^1 \mu I \, d\mu = \int_0^1 \mu(I_0 + \mu I_1)\, d\mu = \tfrac{1}{2} I_0 + \tfrac{1}{3} I_1$$

$$\int_{-1}^1 \mu I \, d\mu = \int_{-1}^1 \mu(I_0 + \mu I_1)\, d\mu = \tfrac{2}{3} I_1$$

$$\int_{-1}^1 I \, d\mu = \int_{-1}^1 (I_0 + \mu I_1)\, d\mu = 2 I_0$$

integrate over μ

$$\mu \frac{dI}{d\tau} = I - S \qquad \frac{1}{3}\frac{dI_1}{d\tau} = I_0 - S$$

integrate over μ

$$\mu^2 \frac{dI}{d\tau} = \mu(I - S) \qquad \frac{1}{3}\frac{dI_0}{d\tau} = \frac{1}{3} I_1 = \text{const}$$

Eddington approximation

Because of $\frac{dI_0}{d\tau} = I_1 = \text{const}$

we have $I_0 = a\tau + b$

$$\int_0^1 \mu I \, d\mu = \tfrac{1}{2} I_0 + \tfrac{1}{3} I_1 = \tfrac{1}{2\pi} F_z \qquad \Rightarrow b = \tfrac{2}{3} I_1 = \tfrac{1}{2\pi} F$$

$$\int_{-1}^1 \mu I \, d\mu = \tfrac{2}{3} I_1 = \tfrac{1}{2\pi} F \qquad \Rightarrow a = \tfrac{3}{4\pi} F$$

$$I_0 = \tfrac{1}{2\pi} F\left(\tfrac{3}{2}\tau + 1\right) \qquad I_1 = \tfrac{3}{4\pi} F \qquad \Rightarrow I = \tfrac{3}{4\pi} F\left(\tau + \tfrac{2}{3} + \mu\right)$$

$$\tfrac{\sigma_B}{\pi} T^4 = S = I_0 = \tfrac{1}{2\pi} F\left(\tfrac{3}{2}\tau + 1\right) = \tfrac{\sigma_B}{2\pi} T_{\text{eff}}^4 \left(\tfrac{3}{2}\tau + 1\right) \qquad \Rightarrow T^4 = T_{\text{eff}}^4 \left(\tfrac{3}{4}\tau + \tfrac{1}{2}\right)$$

What we learned today

- Eddington approximation
  - gives I proportional to $\tau+2/3$
- Two-stream approximation
  - gives I proportional to $\tau+1$