

# *Last time...*

- Fourier synthesis
  - Tophat function
- Center-to-limb variation
  - Connection with  $\mu$
  - Other pleasures on the way
- What else can go into the report
  - Relation to other work
  - Where to go from here

# *Lecture 38*

- Notes on Homework 4+5 resit
  - Numerical integration of moments of intensity
  - Iteration steps in solar wind equation
- Center-to-limb variation
  - Connection with  $\mu$
- About final report
  - Relation to other work (introduction)
  - Where to go from here (conclusions)

# Current Conditions

## Solar wind

speed: **512.1** km/sec

density: **2.7** protons/cm<sup>3</sup>

[explanation](#) | [more data](#)

Updated: Today at 1358 UT

## X-ray Solar Flares

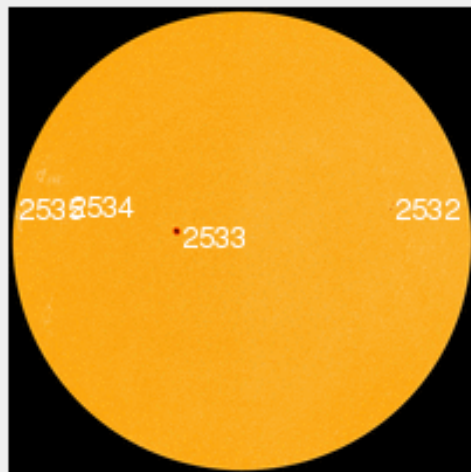
6-hr max: **B2** 0821 UT Apr25

24-hr: **B2** 0821 UT Apr25

[explanation](#) | [more data](#)

Updated: Today at: 1400 UT

## Daily Sun: 25 Apr 16



Sunspot AR2533 has a stable magnetic field that poses no threat for strong solar flares. Credit: SDO/HMI

## What's up in space

Monday, Apr. 25,

Looking for a unique Mother's Day gift? How about [Space Roses](#)? Proceeds from the sale of these far-out blooms support student space weather research.



**ALMOST NO CHANCE OF FLARES:** Solar activity is very low. There are four sunspots on the solar disk, but not one has the type of unstable magnetic field that poses a threat for explosions. NOAA forecasters say the chance of a strong flare today is no more than 1%. **Solar flare alerts:** [text](#) or [voice](#)

**ASTEROID FLYBY:** Newly-discovered asteroid [2016 FY3](#) is flying past Earth today about 1.5 million miles away. There's no danger of a collision, but the 310 meter-wide space rock is close enough to see through amateur telescopes. Dennis Simmons photographed it this morning from Brisbane, Qld, Australia:



# Numerical integration

Discretize: 
$$\int_{-1}^1 I(\mu) d\mu \approx \sum_{i=1}^{n_\mu} I(\mu_i) \Delta\mu_i$$

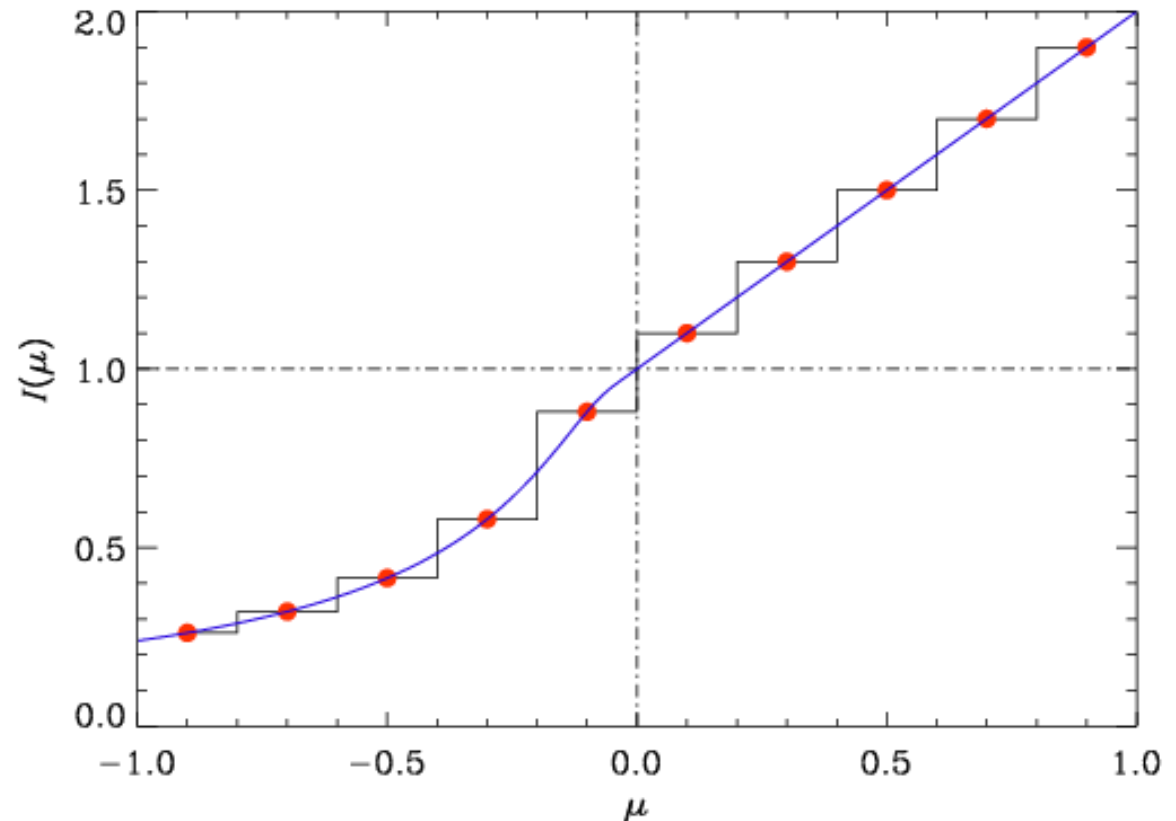
Here:

$$\tau=1/3$$

constant intervals

$$\Delta\mu_i = 2 / n_\mu$$

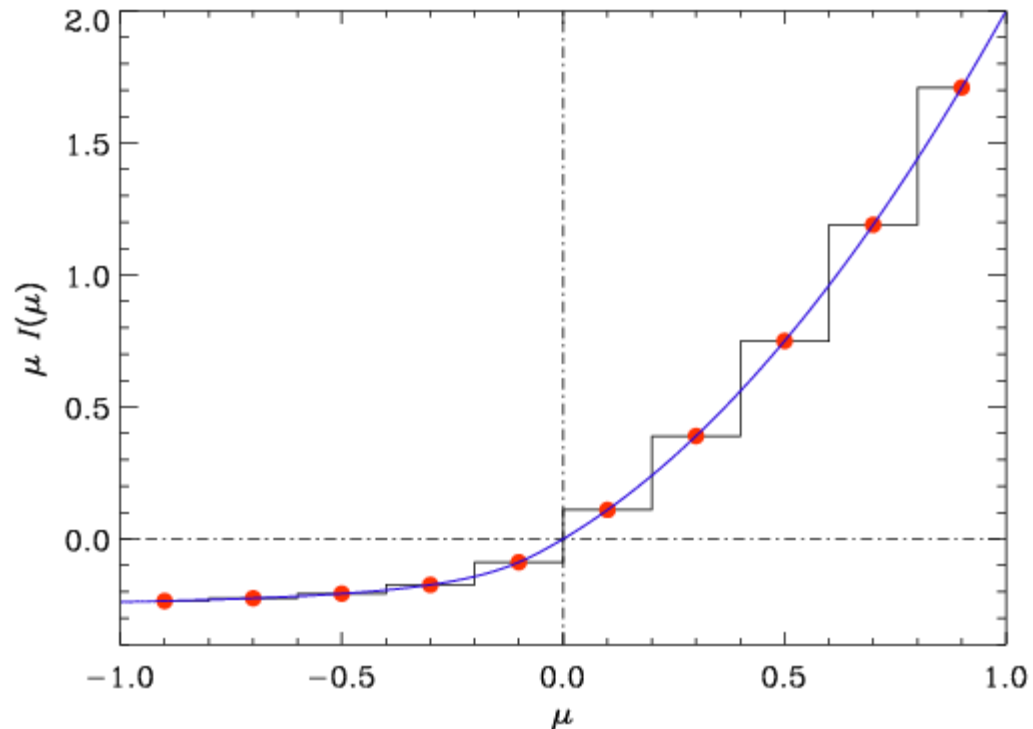
origin excluded  
and  $I(0)=1$



# 1<sup>st</sup> moment

Discretize:

$$\int_{-1}^1 \mu I(\mu) d\mu \approx \sum_{i=1}^{n_\mu} \mu_i I(\mu_i) \Delta\mu_i$$



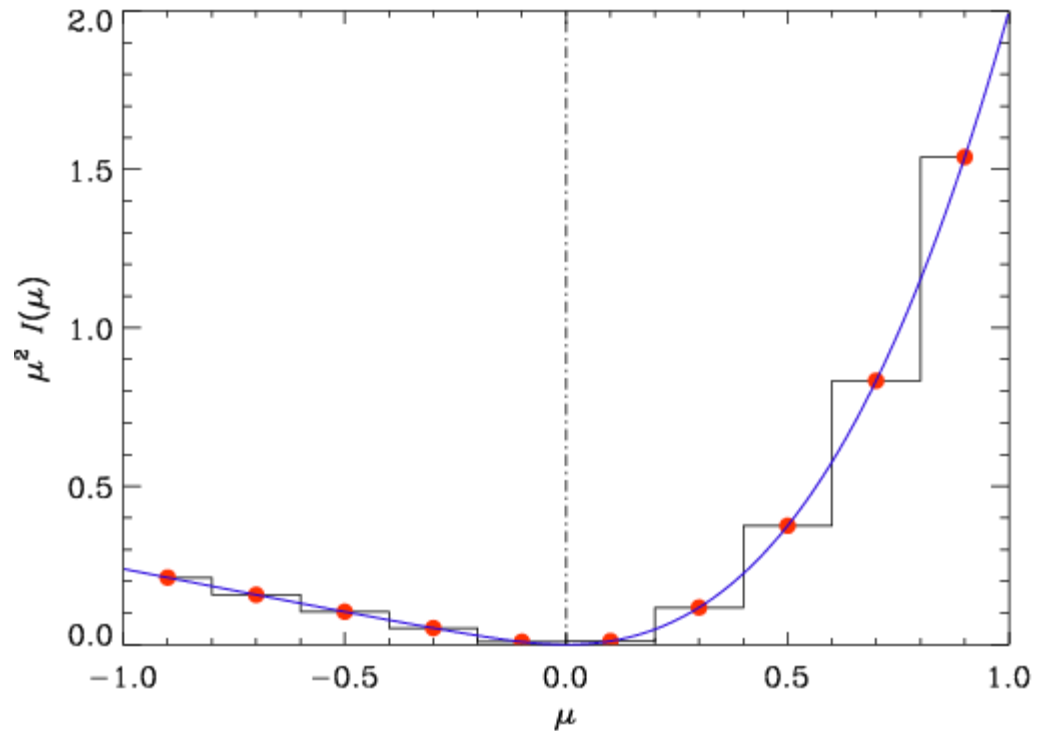
now  $I(0)=0$

# 2<sup>nd</sup> moment

Discretize:

$$\int_{-1}^1 \mu^2 I(\mu) d\mu \approx \sum_{i=1}^{n_\mu} \mu_i^2 I(\mu_i) \Delta\mu_i$$

again  $I(0)=0$



# 2<sup>nd</sup> moment

Discretize:

$$\int_{-1}^1 \mu^2 I(\mu) d\mu \approx \sum_{i=1}^{n_\mu} \mu_i^2 I(\mu_i) \Delta\mu_i$$

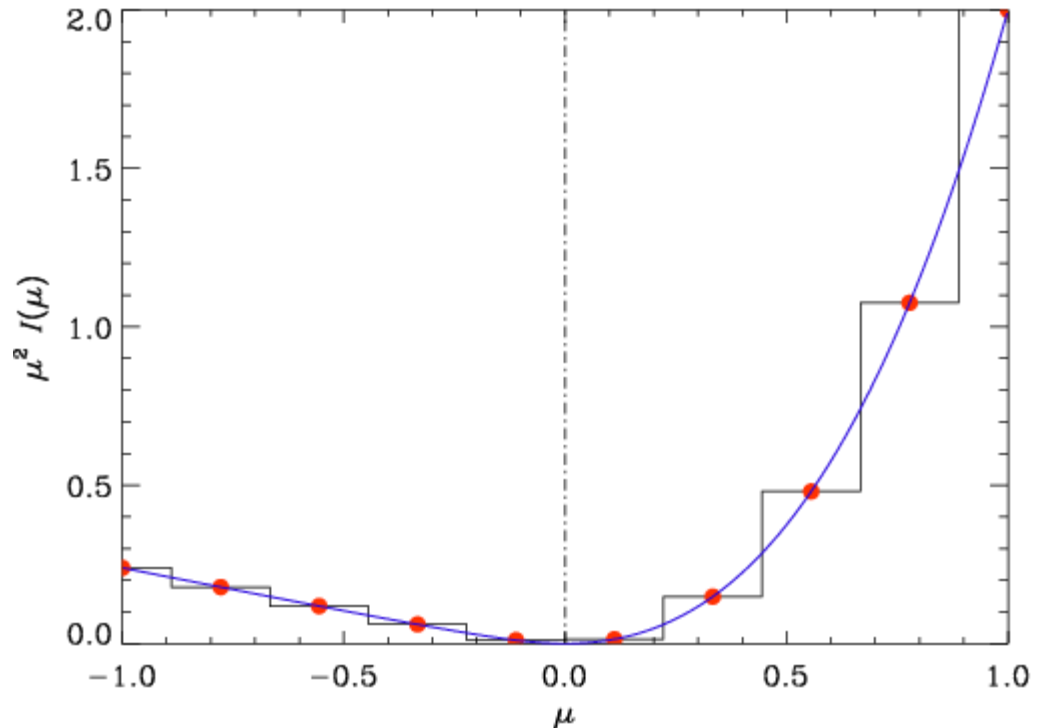
Nonuniform  
intervals

$$\Delta\mu_i = 2 / (n_\mu - 1)$$

Except for:

$$i = 1 \text{ or } i = n_\mu$$

$$\Delta\mu_i = 1 / n_\mu$$

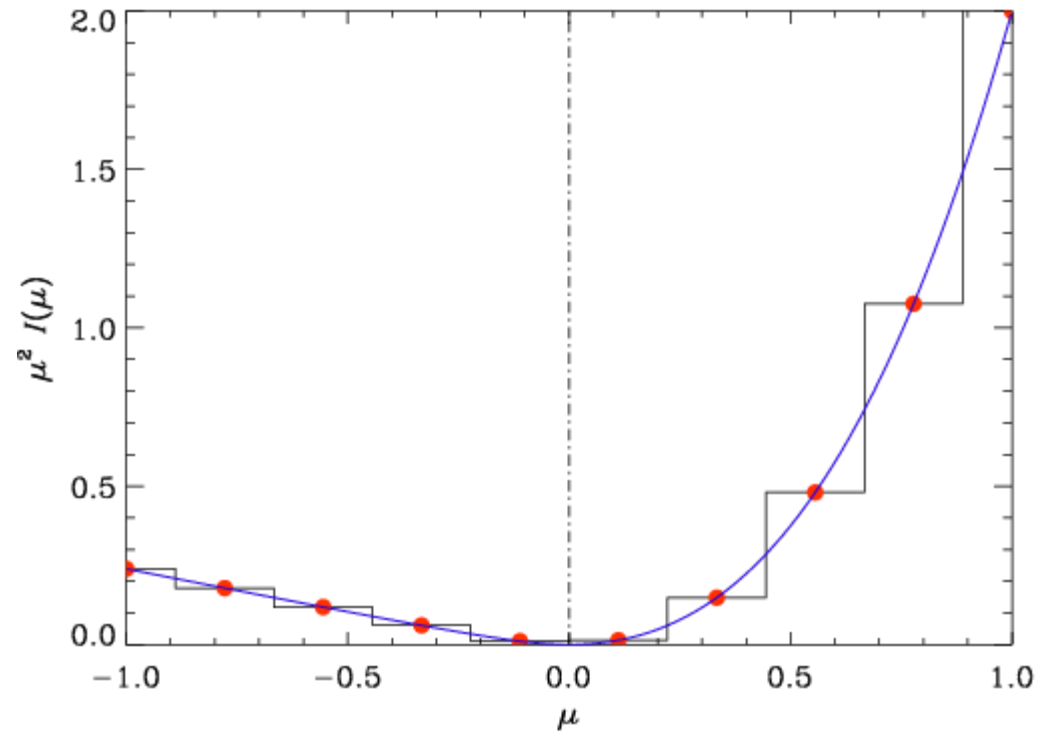


# Why the factor 2

Nonuniform  
intervals

$$\Delta\mu_i = 2/(n_\mu - 1)$$

- A. Boundary points now included
- B. Length of interval
- C. Even number of points

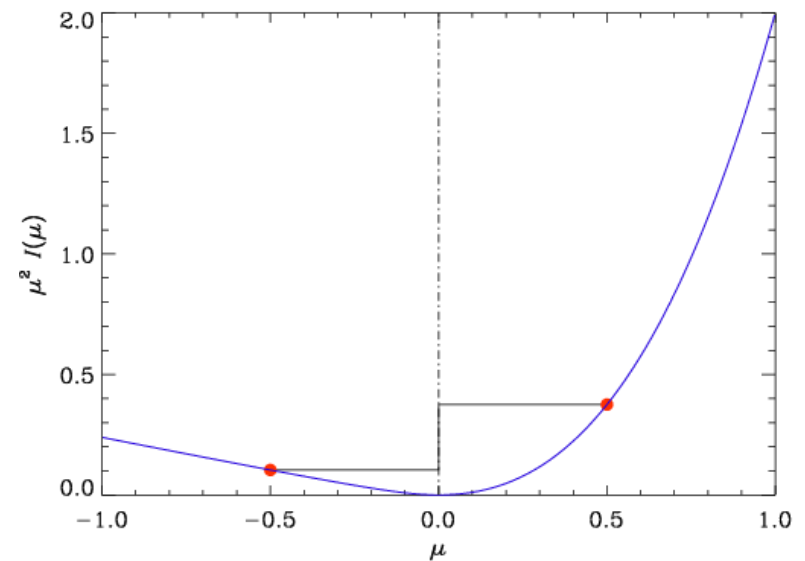
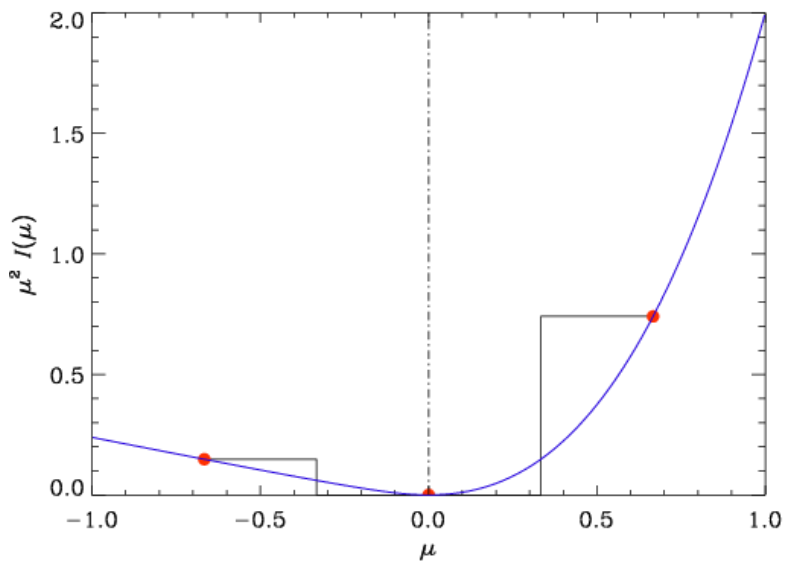
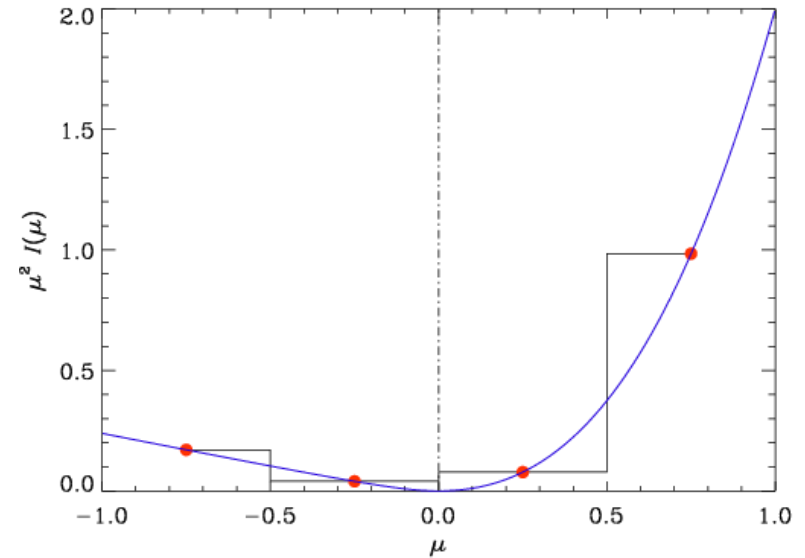




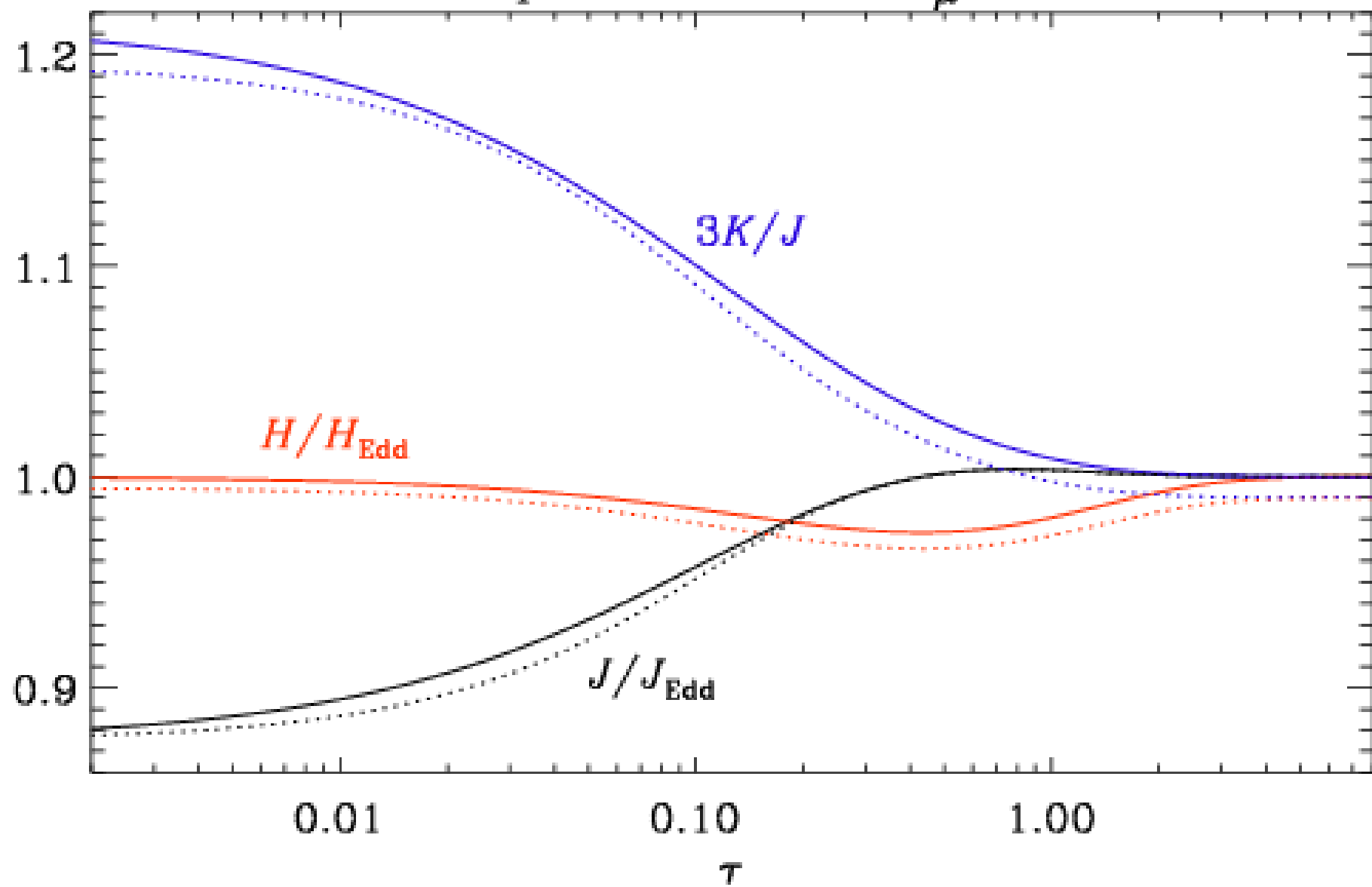
# Fewer points

Convergence for  $K$  (2<sup>nd</sup> mom):

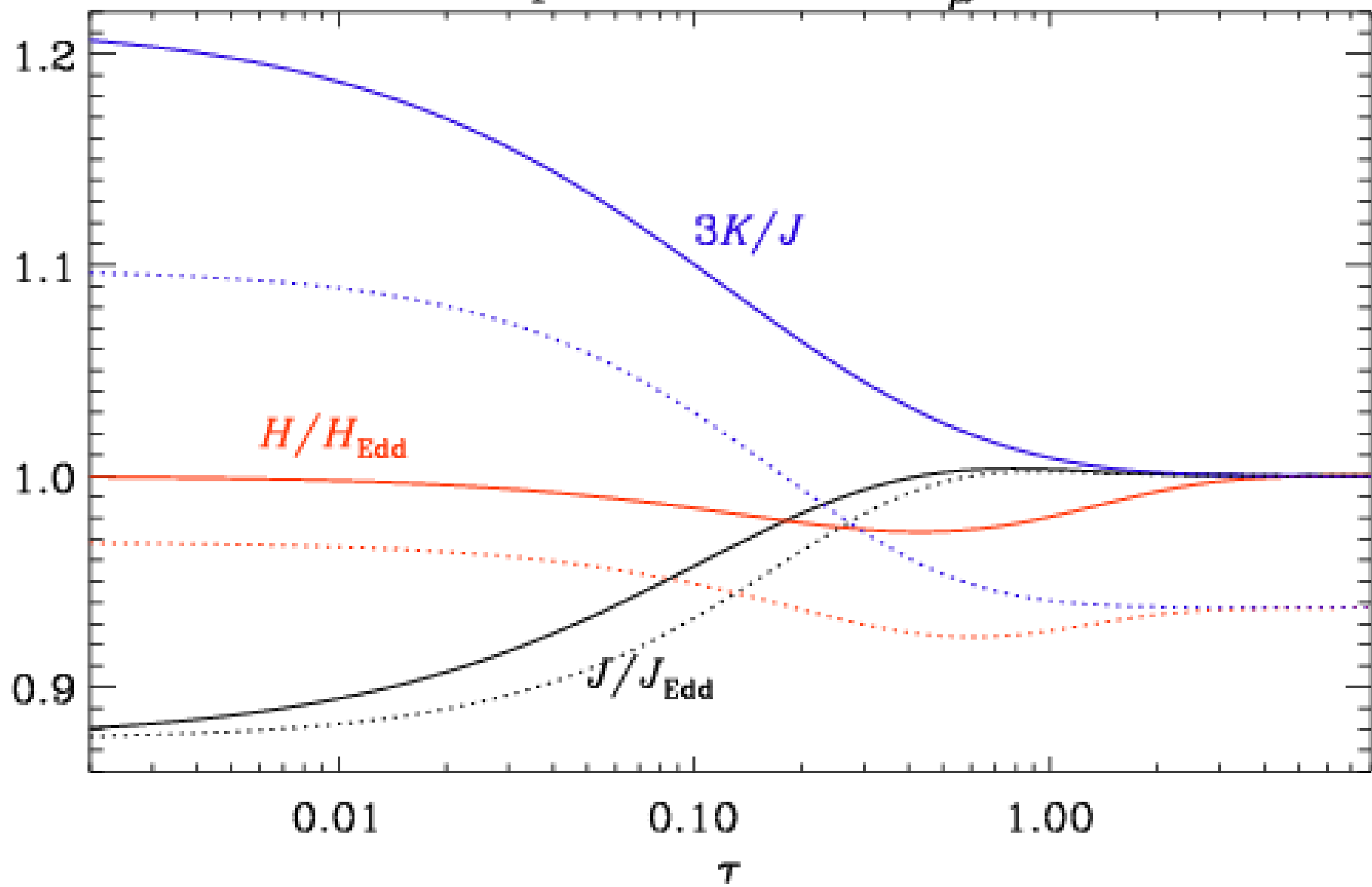
N	Const	Half
3	0.30	0.56
4	0.32	0.44
5	0.328	0.40
10	0.341	0.356
500	0.345	0.345



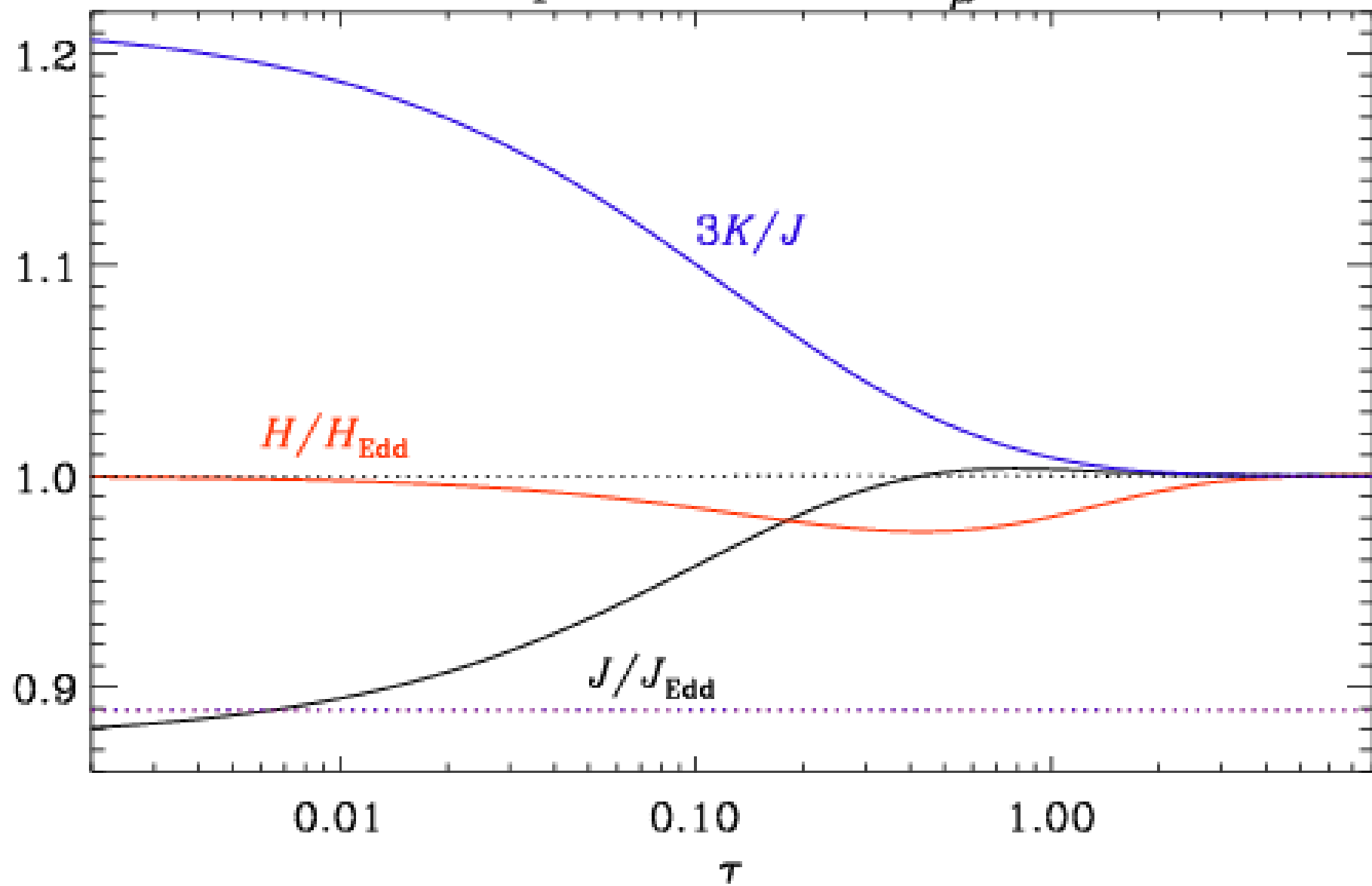
comparison with  $\tau_\mu = 10$



comparison with  $n_\mu=4$



comparison with  $n_\mu=3$



# Integrating wind equations

Continuity eqn

$$\frac{d}{dr}(r^2 \rho u_r) = 0$$

so  $r^2 \rho u_r = \text{const} = \dot{M} / 4\pi$

or  $\ln r^2 + \ln \rho + \ln u_r = \ln(\dot{M} / 4\pi)$

Momentum eqn

$$u_r \frac{du_r}{dr} = -c_s^2 \frac{d \ln \rho}{dr} - \frac{GM}{r^2} \quad \frac{d}{dr} \left( \frac{1}{2} u_r^2 + c_s^2 \ln \rho - \frac{GM}{r} \right) = 0$$

insert

$$\frac{1}{2} u_r^2 - c_s^2 \ln u_r - c_s^2 \ln r^2 - \frac{GM}{r} = \text{const}$$

Just plot contours of this in the  $u_r - r$  plane!



Next

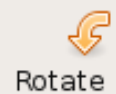
12

of 72

Fit Page Width



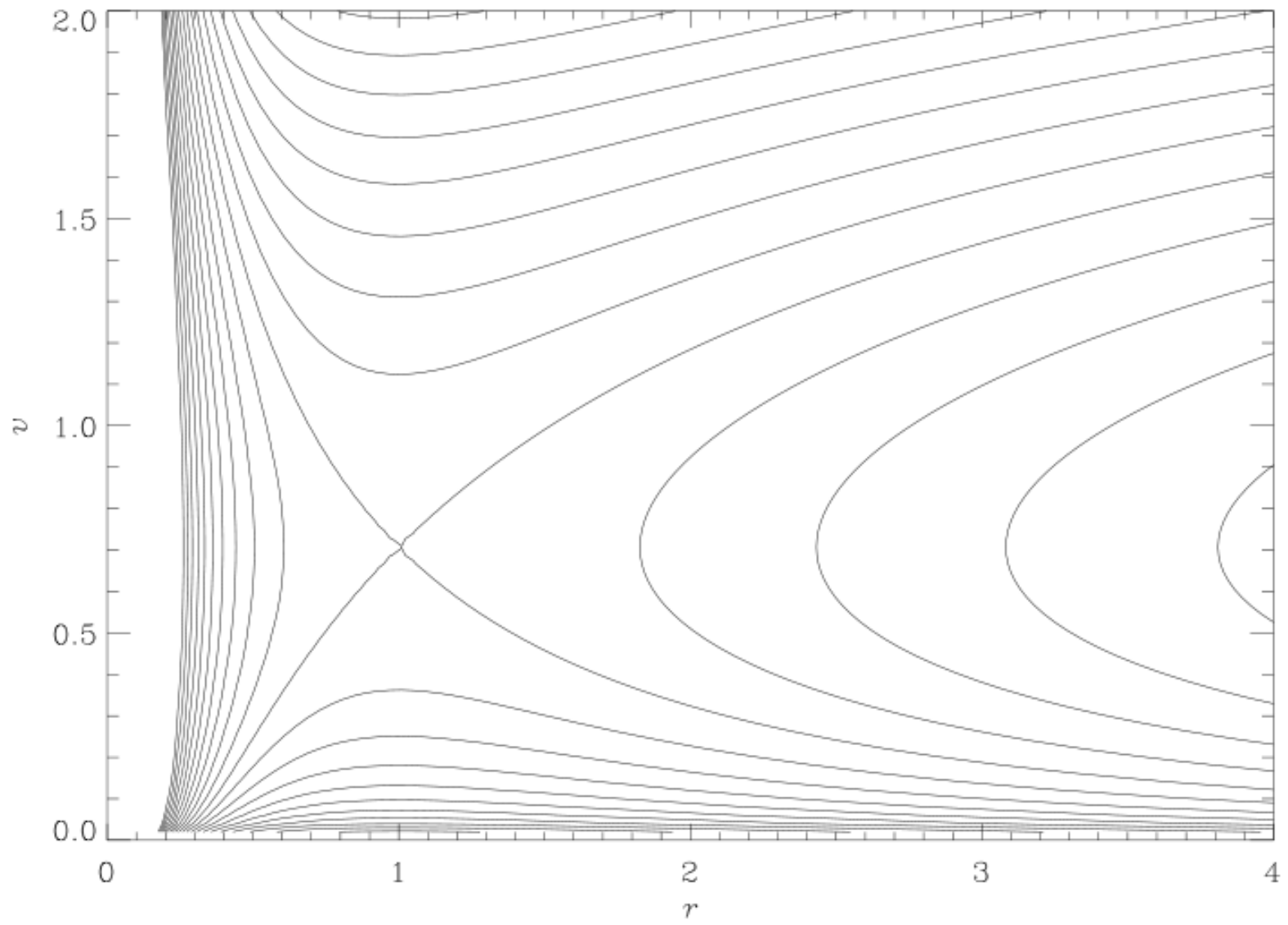
Reload



Rotate Left



First Page



# Resit homework 4+5, problem 6

(c) Show that Eq. (3) can be written as

$$\mathcal{M} = \sqrt{C + 2 \ln \mathcal{M}}, \quad (4)$$

where  $\mathcal{M} = |u_r|/c_s$  and

$$C = 4 \left( \ln \tilde{r} + \frac{1}{\tilde{r}} \right) - 3,$$

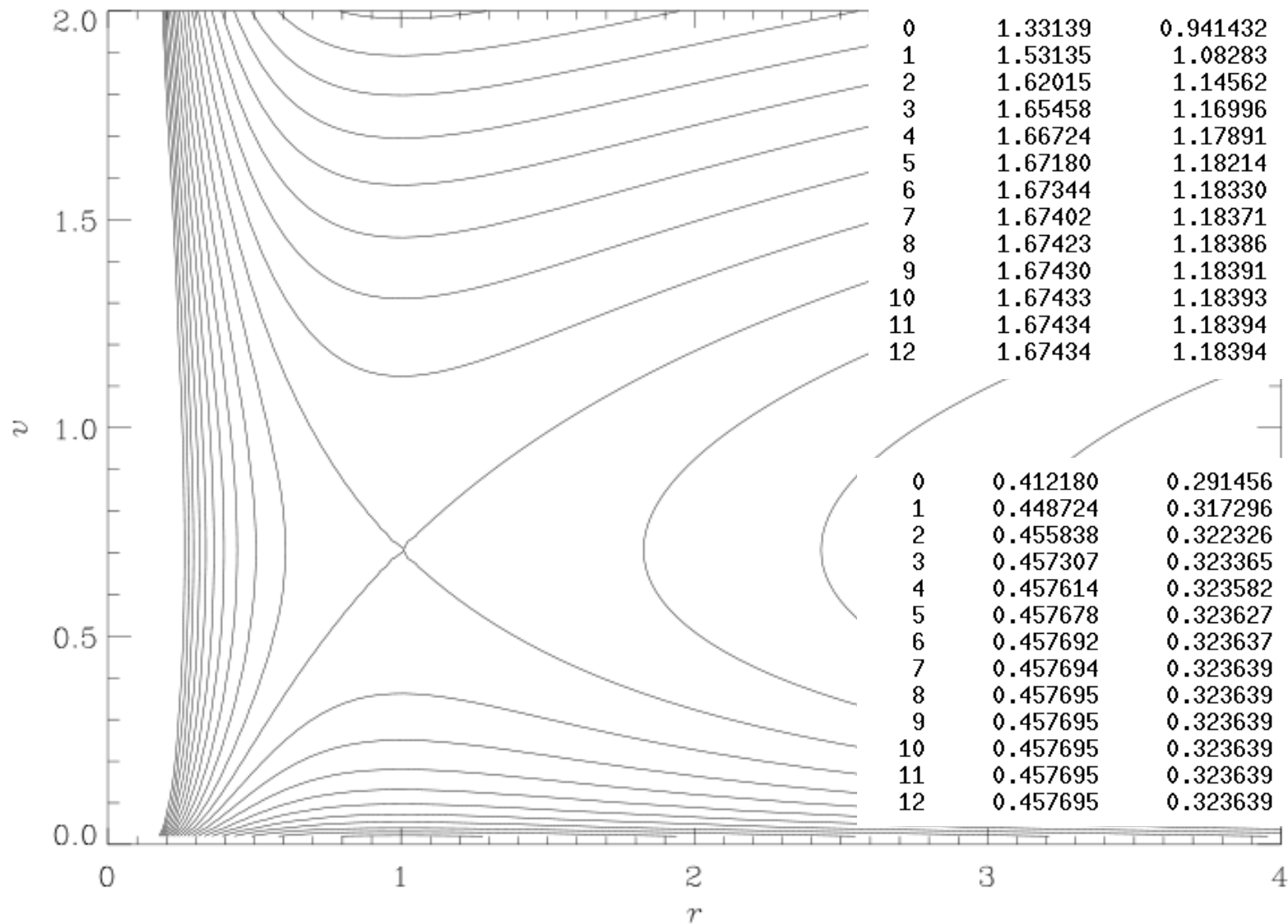
with  $\tilde{r} = r/r_*$ .

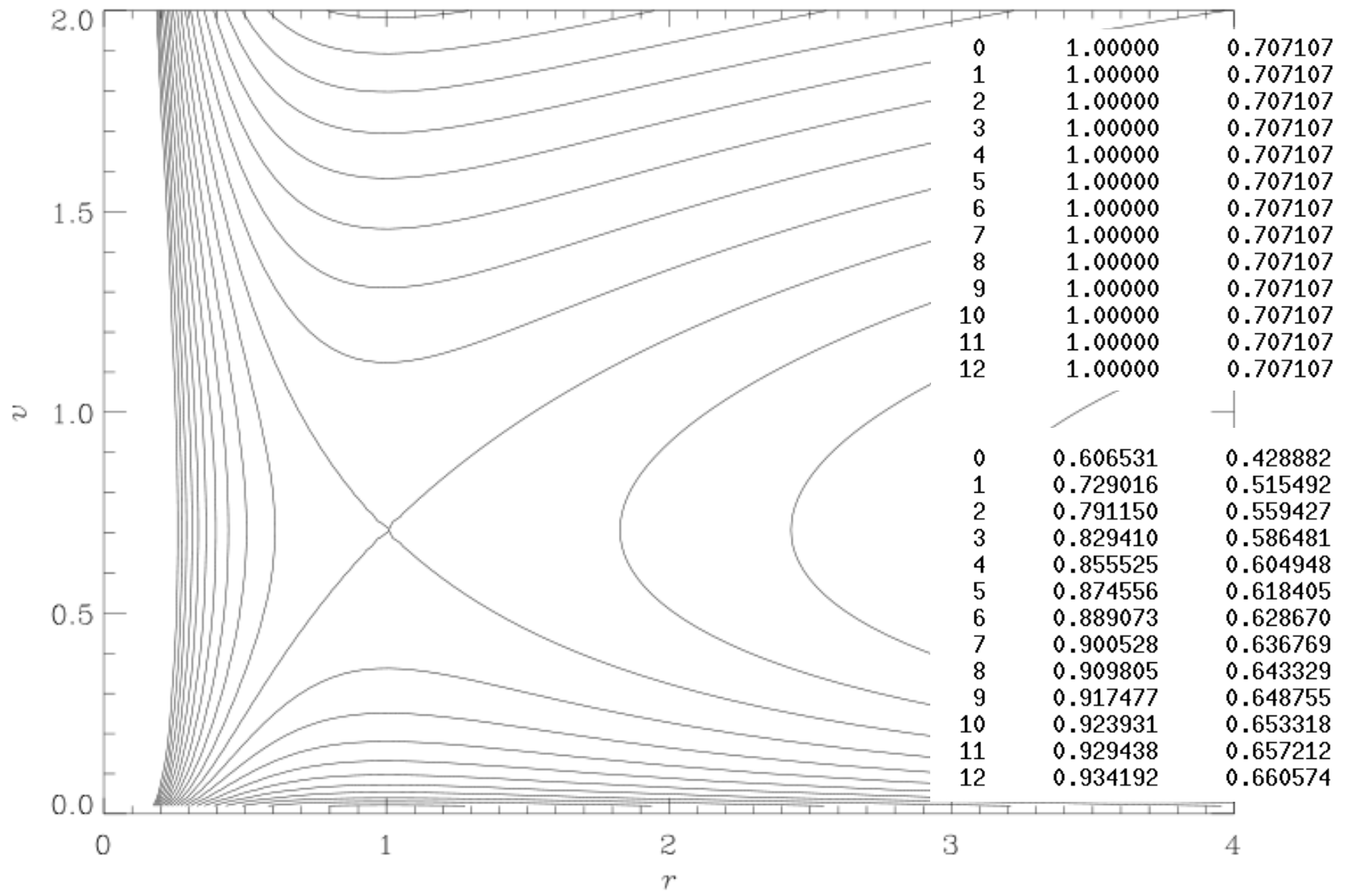
- (d) Calculate the value of  $C$  for  $\tilde{r} = 10$ , and find the corresponding value of  $\mathcal{M}$  using three iteration steps starting with  $\mathcal{M} = 1$ . Show your working in all intermediate steps. Sketch the solution for  $\mathcal{M}$  against  $\tilde{r}$ , and indicate the points where  $\tilde{r} = 1$  and 10.
- (e) Show that Eq. (4) can also be written as  $\mathcal{M} = \exp \left[ \frac{1}{2}(\mathcal{M}^2 - C) \right]$ , and, for the same value of  $C$ , iterate for  $\mathcal{M}$  starting again with  $\mathcal{M} = 1$  (use three iterations, show your working). Again, sketch the solution of  $\mathcal{M}$  against  $\tilde{r}$ , indicate the points where  $\tilde{r} = 1$  and 10, and show the direction of the flow. In what area of stellar physics can this model be applied?

# *Iteration steps for $r=2$*

	$m$	$m/\sqrt{2}$
0	1.33139	0.941432
1	1.53135	1.08283
2	1.62015	1.14562
3	1.65458	1.16996
4	1.66724	1.17891
5	1.67180	1.18214
6	1.67344	1.18330
7	1.67402	1.18371
8	1.67423	1.18386
9	1.67430	1.18391
10	1.67433	1.18393
11	1.67434	1.18394
12	1.67434	1.18394







# Webpage updates

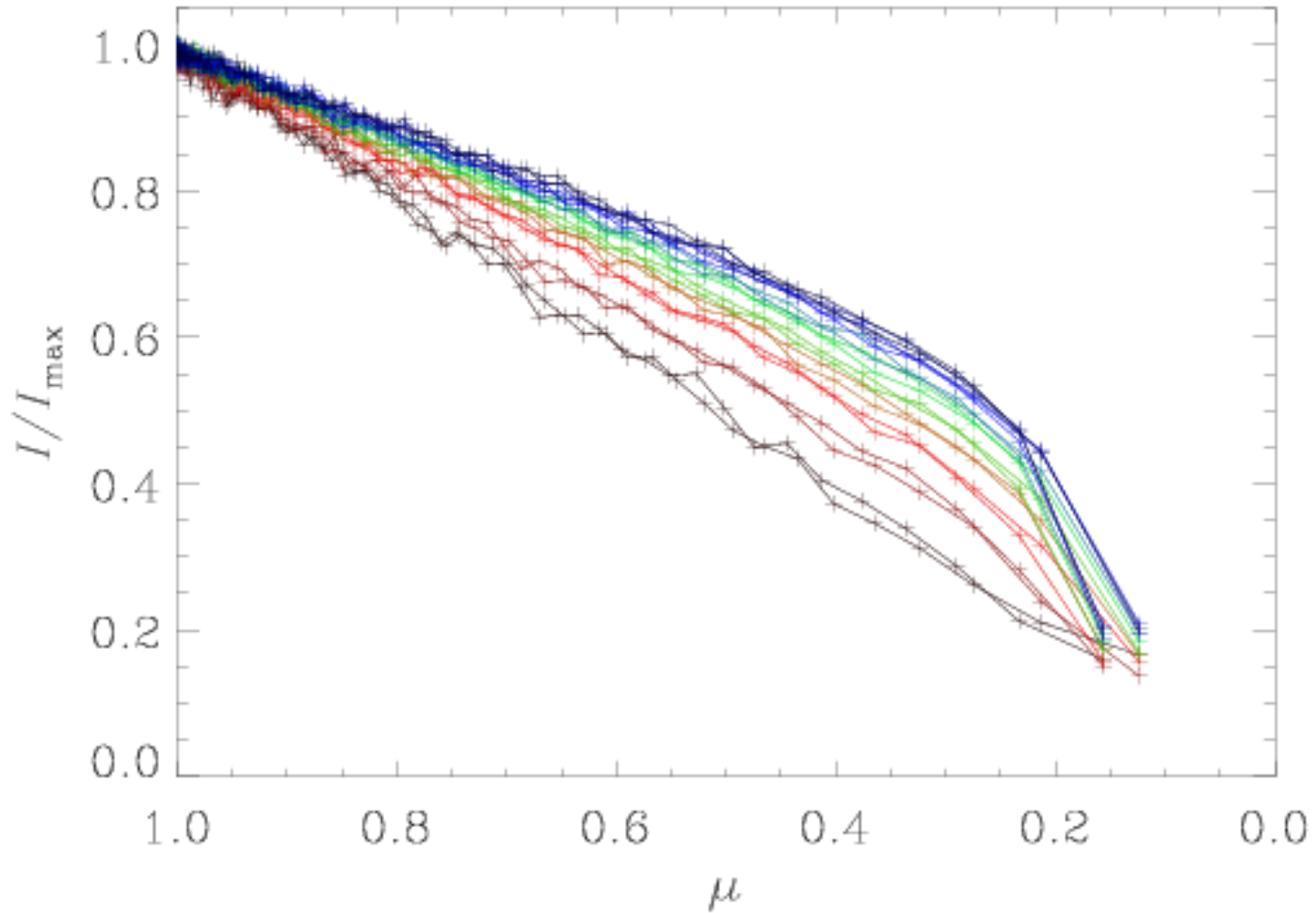
- Take pictures and describe the apparatus briefly. Again this will be part of your project report.
- Determine  $I_\lambda(\mu)$  by scanning with the spectro-photometer across the solar disk. Make sure you cover reasonably well both limbs. Check this before taking data for `production runs'. Record spectra in fine enough intervals (either 13 or 26 points corresponding to 10 or 5 revolutions on this little `crank'. Make sure you can translate the revolutions into  $\mu$  and estimate an error for this procedure.
- Repeat measurements *at least* 3 to 5 times. Double-check that the weather is ok all the time. If small clouds appear, make a clear note of this. Such data are in principle useless, but maybe you can learn something from them anyway. Write down any irregularities in the measurements. This might help in tracing the course of systematic changes that might occur over the 1.5 months period that measurements are taken.
- If there is time, perform additional experiments that you can think of. This will give you **extra points** and the five best innovative ideas will be rewarded at the end.
- Determine the solar spectrum in integral light without telescope by going outside with the with the detector and computer.
- Determine the angular characteristics (angular dependence) of the detector. Try to fit this dependence locally around the maximum to a  $\cos^n\theta$  dependence, where  $\theta$  is the angle from the normal.

## Regarding the theoretical interpretation

For the case of a gray atmosphere (no  $\lambda$  dependence), see [Homework key 2](#).

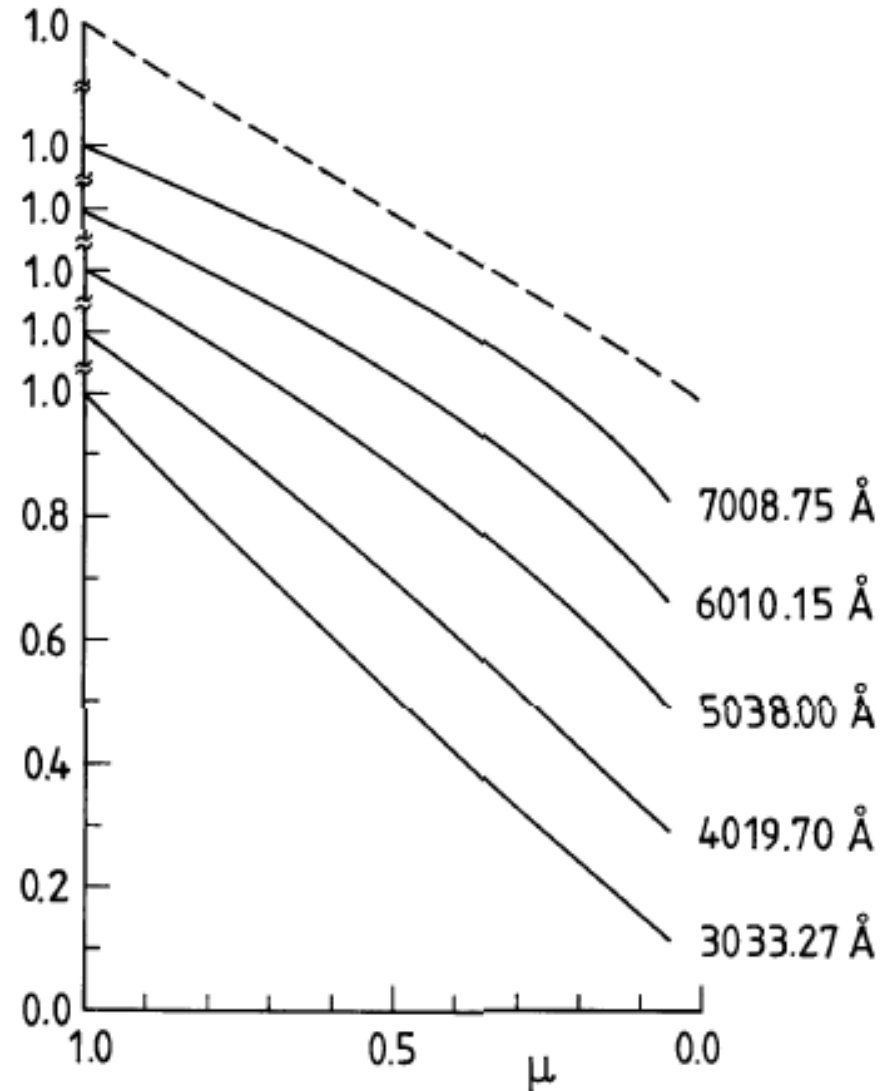
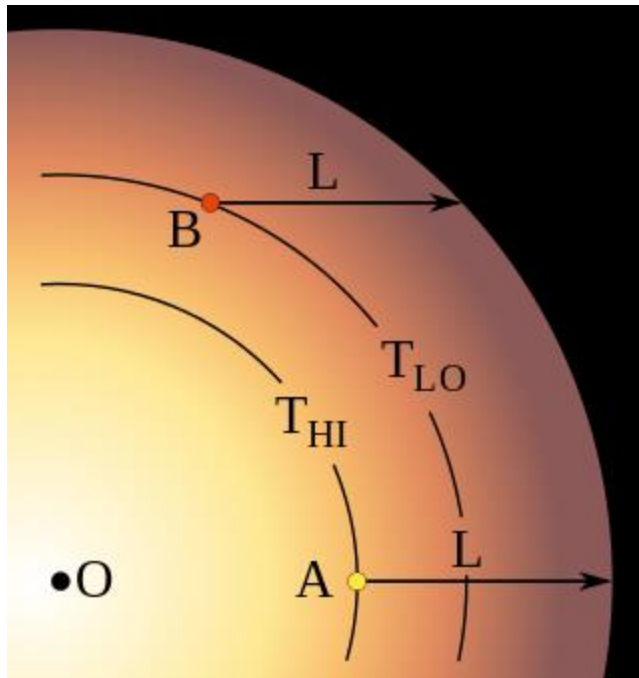
- [Back to main course page](#)
- [notes on the analysis](#)
- See [Lecture 37](#) for further details
- Link to [Axel's idl directory](#)
  
- [Schedule \(past+future\)](#)

# $\mu$ dependence



# Limb darkening

- Stix Sect. 4.3.1
- See deeper



# *What else can go into report?*

- Literature search
  - History
  - earlier work
- Setup, description
- Results, details
- Conclusions, where to go from here
- References



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Databases to query:  Astronomy  Physics  arXiv e-prints

Authors: (Last, First M, one per line)  SIMBAD  NED  ADS Objects

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# Lect.3, relation to opacity

Leading order  $I_\nu = B_\nu$

Insert

$$\cos \theta \frac{dB_\nu}{dr} = -\rho \kappa_\nu (I_\nu - B_\nu)$$

so

$$I_\nu = B_\nu - \frac{\cos \theta}{\rho \kappa_\nu} \frac{dB_\nu}{dr}$$

Interested in flux

$$\int_{4\pi} I_\nu \cos \theta d\Omega = 2\pi \int_{-1}^1 I_\nu \cos \theta d \cos \theta$$



# *What we did today*

- Notes on Homework 4+5 resit
  - Numerical integration of moments of intensity
  - Iteration steps in solar wind equation
- Center-to-limb variation
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- About final report
  - Relation to other work (introduction)
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