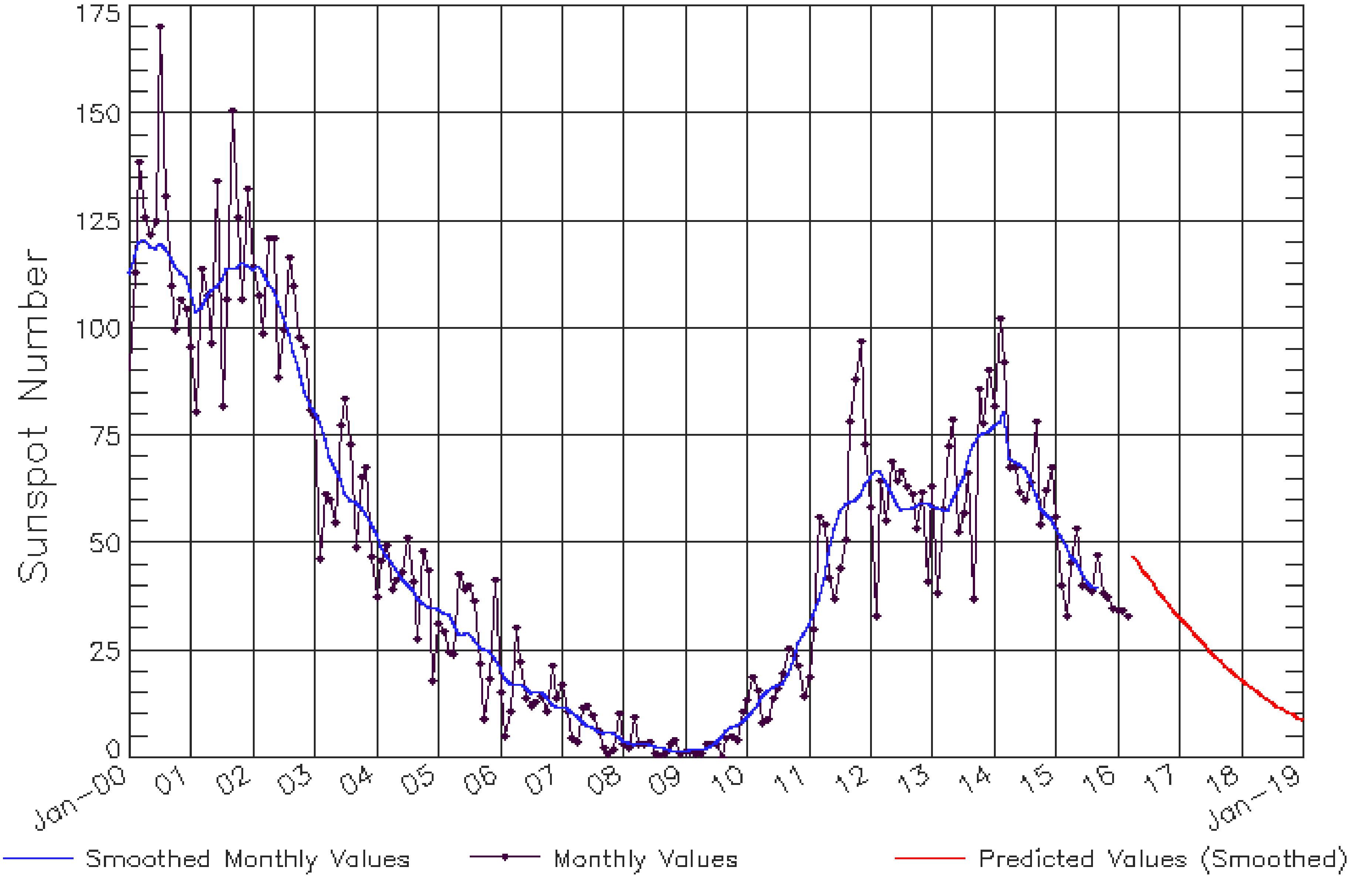
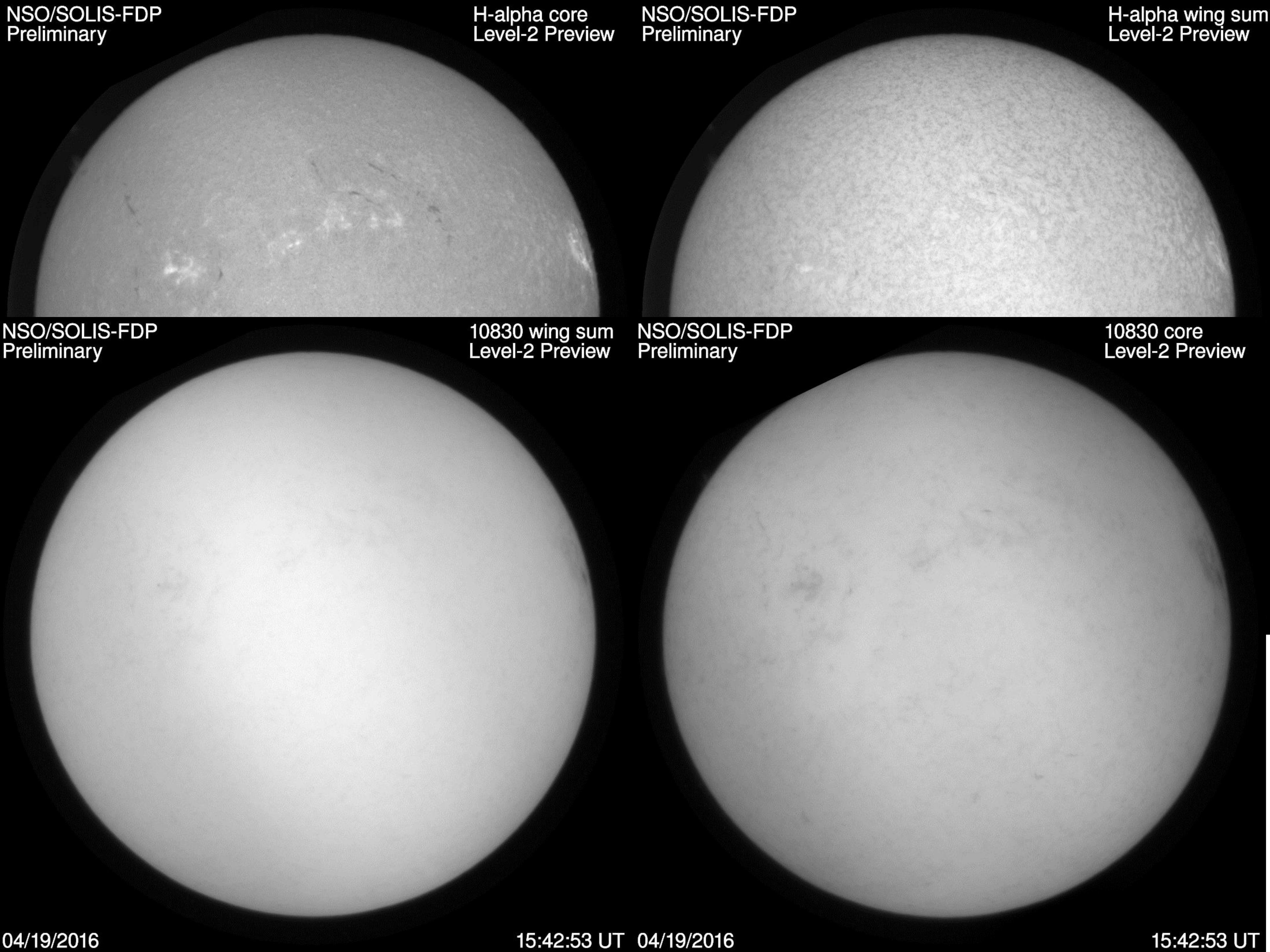
ISES Solar Cycle Sunspot Number Progression

Observed data through Mar 2016



Updated 2016 Apr 4

NOAA/SWPC Boulder,CO USA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H-alpha core
Level-2 Preview
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10830 wing sum
Level-2 Preview
NSO/SOLIS-FDP
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10830 core
Level-2 Preview
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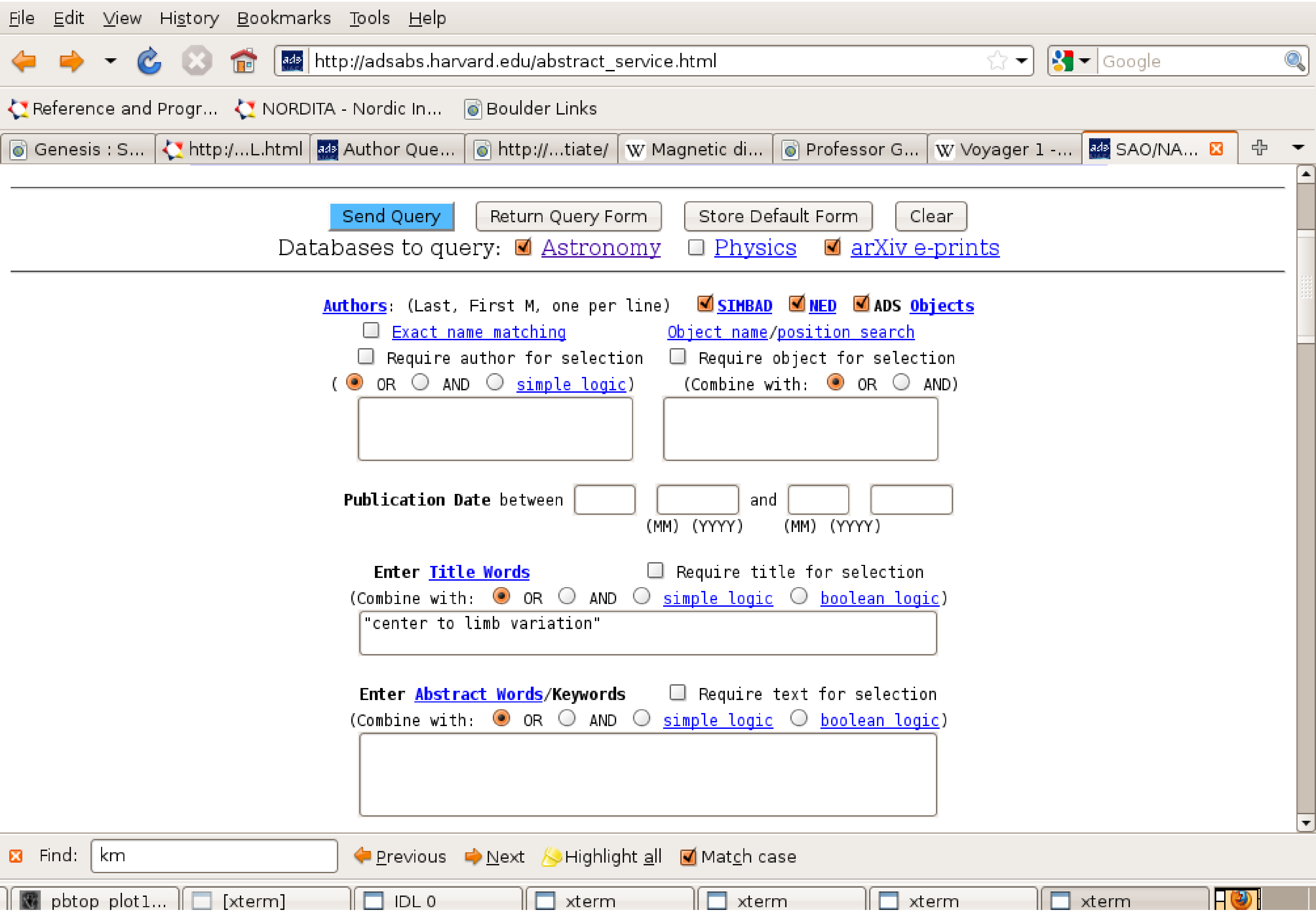
Last time…

- Notes on Homework 4+5 resit
  - Numerical integration of moments of intensity
  - Iteration steps in solar wind equation
- Center-to-limb variation
  - Connection with mu
- About final report
  - Relation to other work (introduction)
  - Where to go from here (conclusions)

Lecture 38

- More on final report
  - Relation to other work (introduction)
  - Where to go from here (conclusions)
- Results so far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SOLAR LIMB DARKENING

I: $\lambda\lambda$(3033–7297)

A. KEITH PIERCE and CHARLES D. SLAUGHTER

Kitt Peak National Observatory, Tucson, Arizona*

(Received 29 June, 1976)

Abstract. The coefficients of several polynomial representations of the limb darkening at 62 wavelengths in the UV and visible portions of the solar spectrum obtained at the McMath Solar Telescope are presented in tabular form. Full corrections for scattered light and seeing have been included in the reductions.

"It is important to draw attention to the simplicity of the theory of the darkening of the Sun's disc towards the limb. It is a consequence simply and solely of the temperature gradient in the outer layers."

E. A. Milne (1930)

Pierce & Slaughter

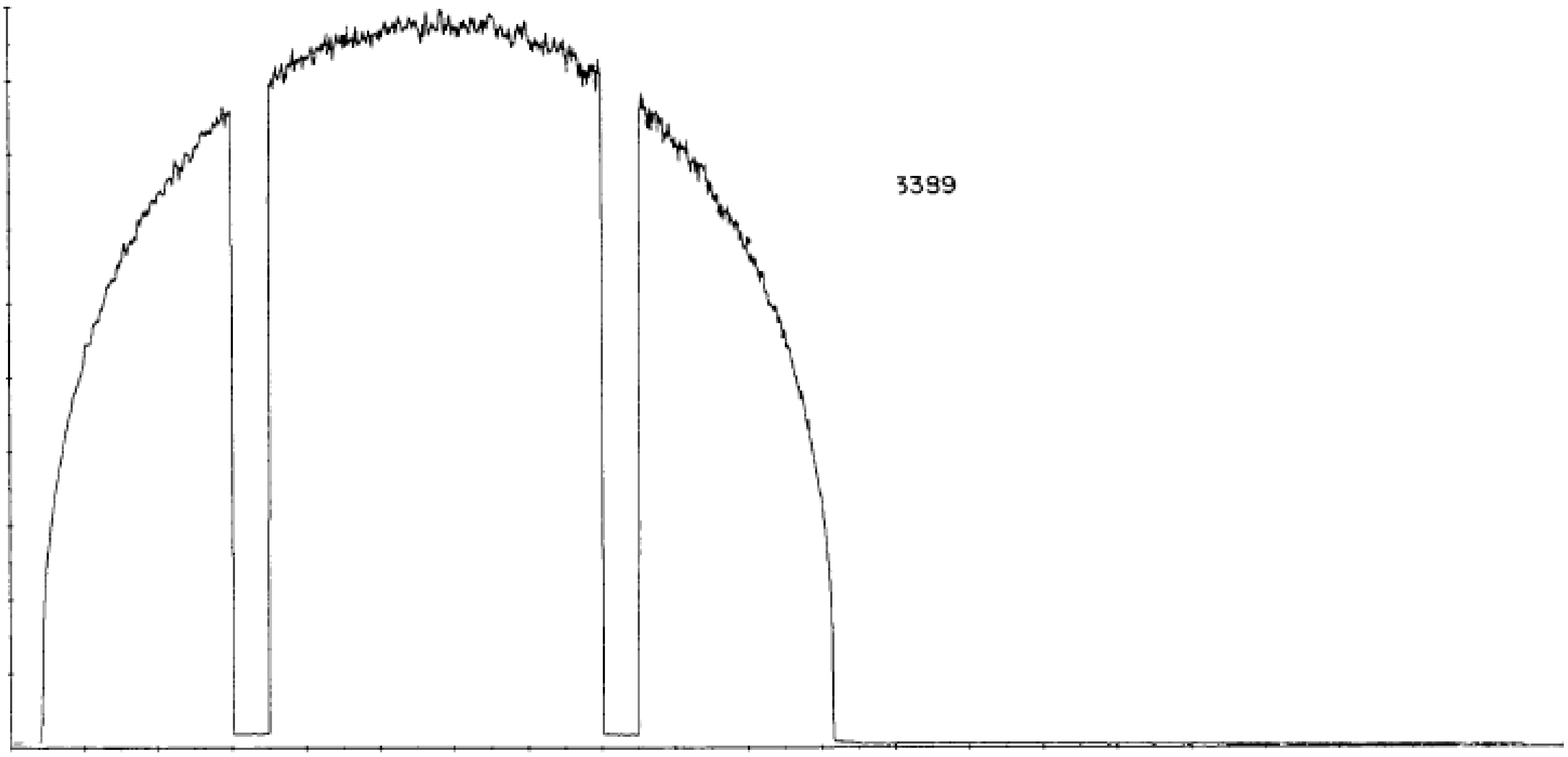
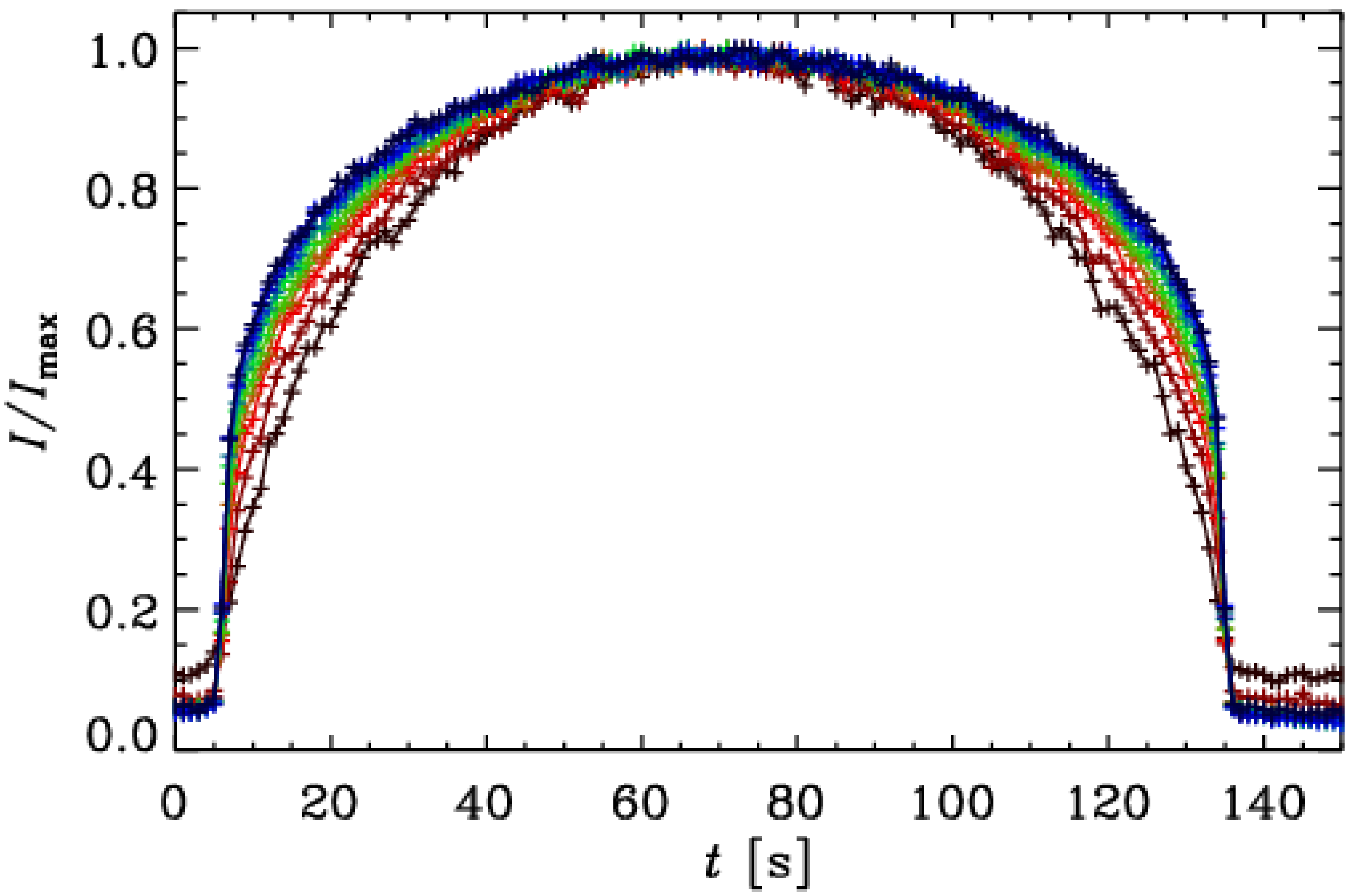


Fig. 1. Drift curve taken at λ6791 Å and λ3389 A.

…from Lecture 37



Several power series have been tried. A good representation to the limb darkening is given by Sykes' (1953) formula:

$$I(\lambda, \xi) = a(2) + b(2)\xi + c(2)\xi^2, \tag{8}$$

where $\xi = \ln \mu$; $\cos\theta = \mu$. The coefficients for each λ are given in Table I. The second

TABLE I

Coefficients of 2nd degree, ln μ, fit to the limb darkening

λ	No.	$a(2)$	$b(2)$	$c(2)$
3033.27	4	1.00000	0.81773	0.21103
3069.82	6	1.00000	0.72281	0.14836
3108.43	6	1.00000	0.70827	0.14319
3204.68	8	1.00000	0.69896	0.14241
3298.97	8	1.00000	0.66442	0.13055
3389.53	8	1.00000	0.63905	0.12218
3499.49	8	1.00000	0.64091	0.12817
3563.52	10	1.00000	0.64312	0.12988
3626.50	6	1.00000	0.62074	0.12442
3658.75	6	1.00000	0.60464	0.11690
3740.88	10	1.00000	0.64219	0.12761
3779.92	5	1.00000	0.63963	0.12675
3852.02	8	1.00000	0.67421	0.14195
3909.28	8	1.00000	0.66030	0.13914
3954.25	8	1.00000	0.65565	0.14273
3988.15	8	1.00000	0.63352	0.13039
4019.70	10	1.00000	0.63598	0.13157
4069.44	8	1.00000	0.62932	0.12916
4117.23	3	1.00000	0.60339	0.11906

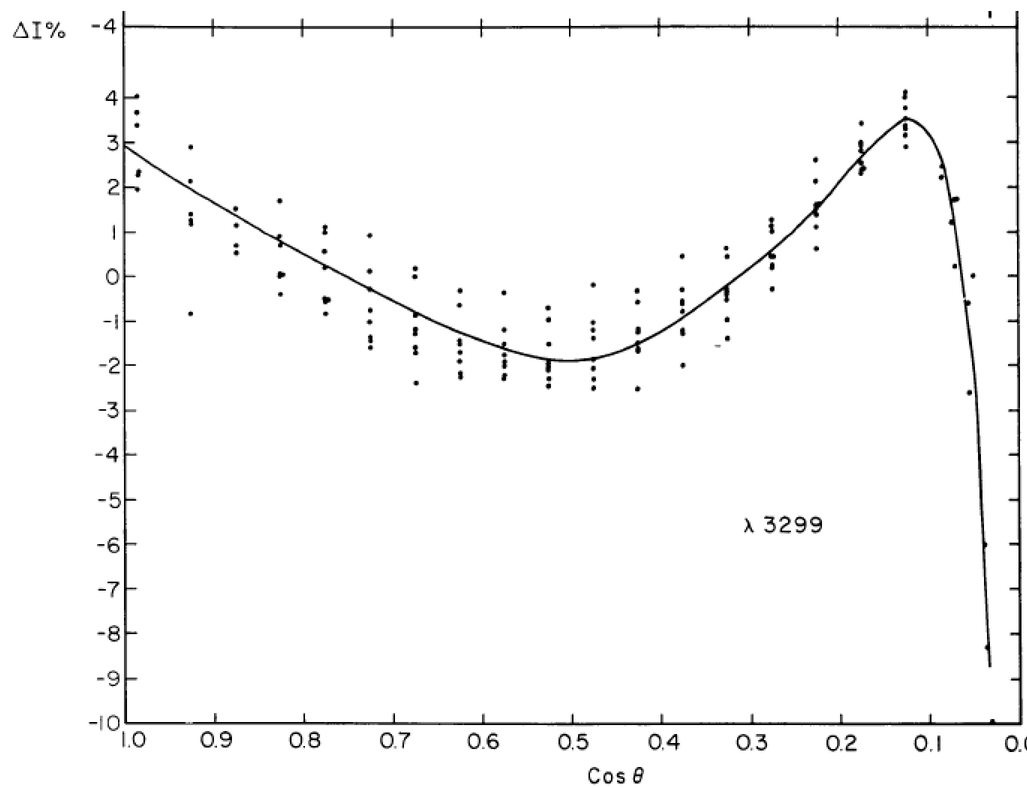


Fig. 5. Residuals, observed minus computed, for the normal points and Equation (8).

$c(2)$ from a least square solution. The residuals when plotted show a systematic trend with $\cos\theta$ as illustrated in Figure 5 suggesting that a higher degree polynomial would have given a better fit. Accordingly a 5th order least squares fit of the observations to the series

$$I(\lambda,\xi)=a(5)+b(5)\xi+c(5)\xi^2+d(5)\xi^3+e(5)\xi^4+f(5)\xi^5 \tag{9}$$

was performed giving the coefficients in Table II. The probable error of a single normal point as obtained from the scatter about Equation (9) is listed in the last column.

Since the representation

$$I(\lambda,\mu)=A(2)+B(2)\mu+C(2)\mu^2 \tag{10}$$

often appears in the literature we list in Table III its coefficients from a least square solution. Table IV gives the coefficients of a 5th order fit to the observations from the equation

$$I(\lambda,\mu)=A(5)+B(5)\mu+C(5)\mu^2+D(5)\mu^3+E(5)\mu^4+F(5)\mu^5 \tag{11}$$

and the probable error of a single normal point.

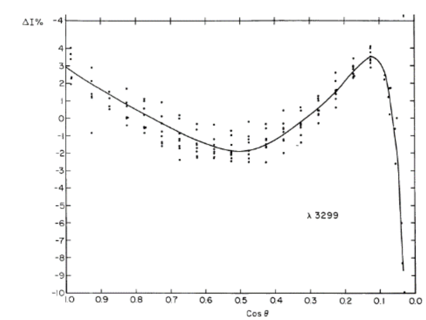
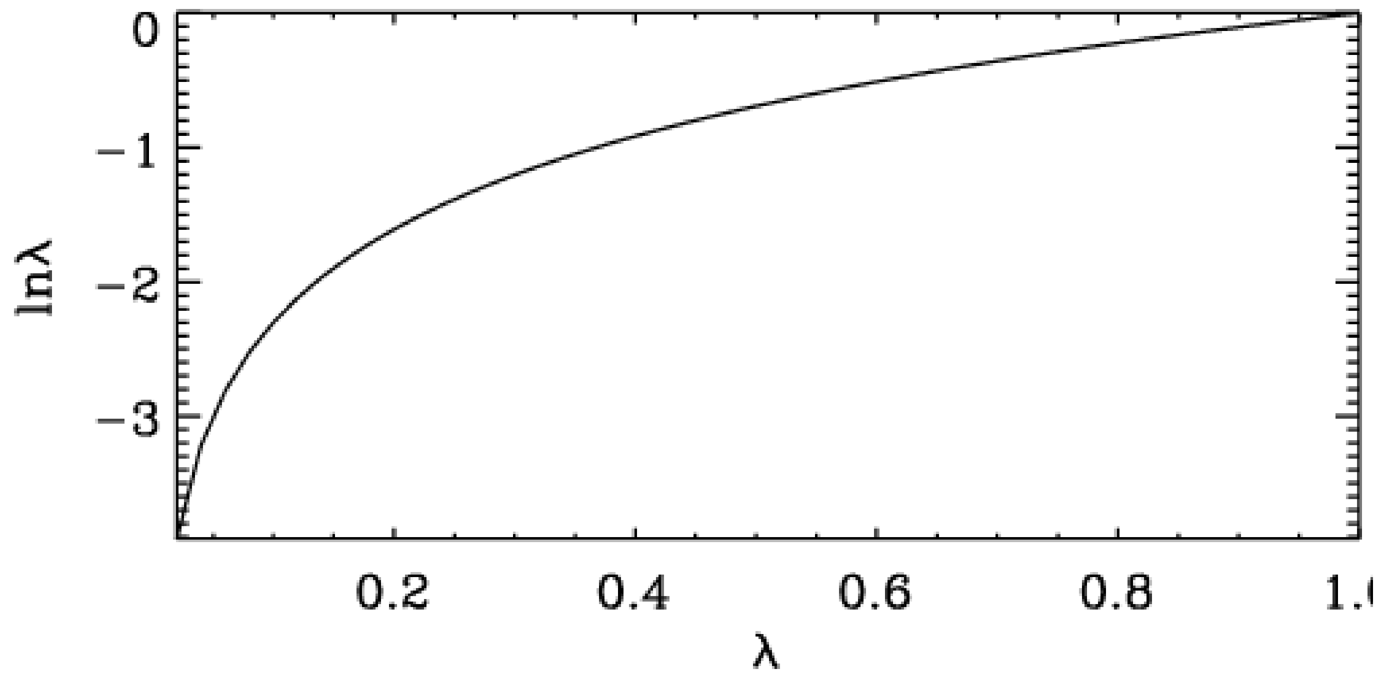
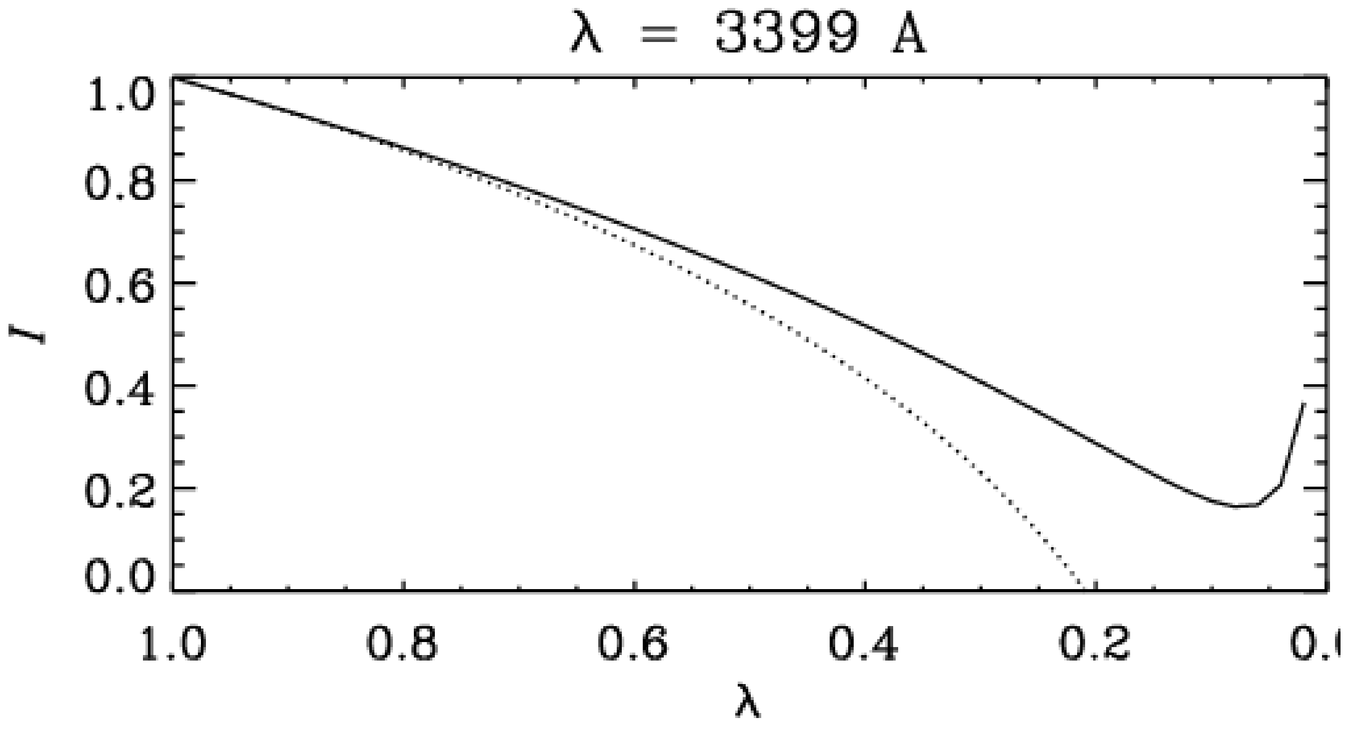


Fig. 5. Residuals, observed minus computed, for the normal points and Equation (8).

Better fit with ln mu



Blue (or rather UV) light



Red light

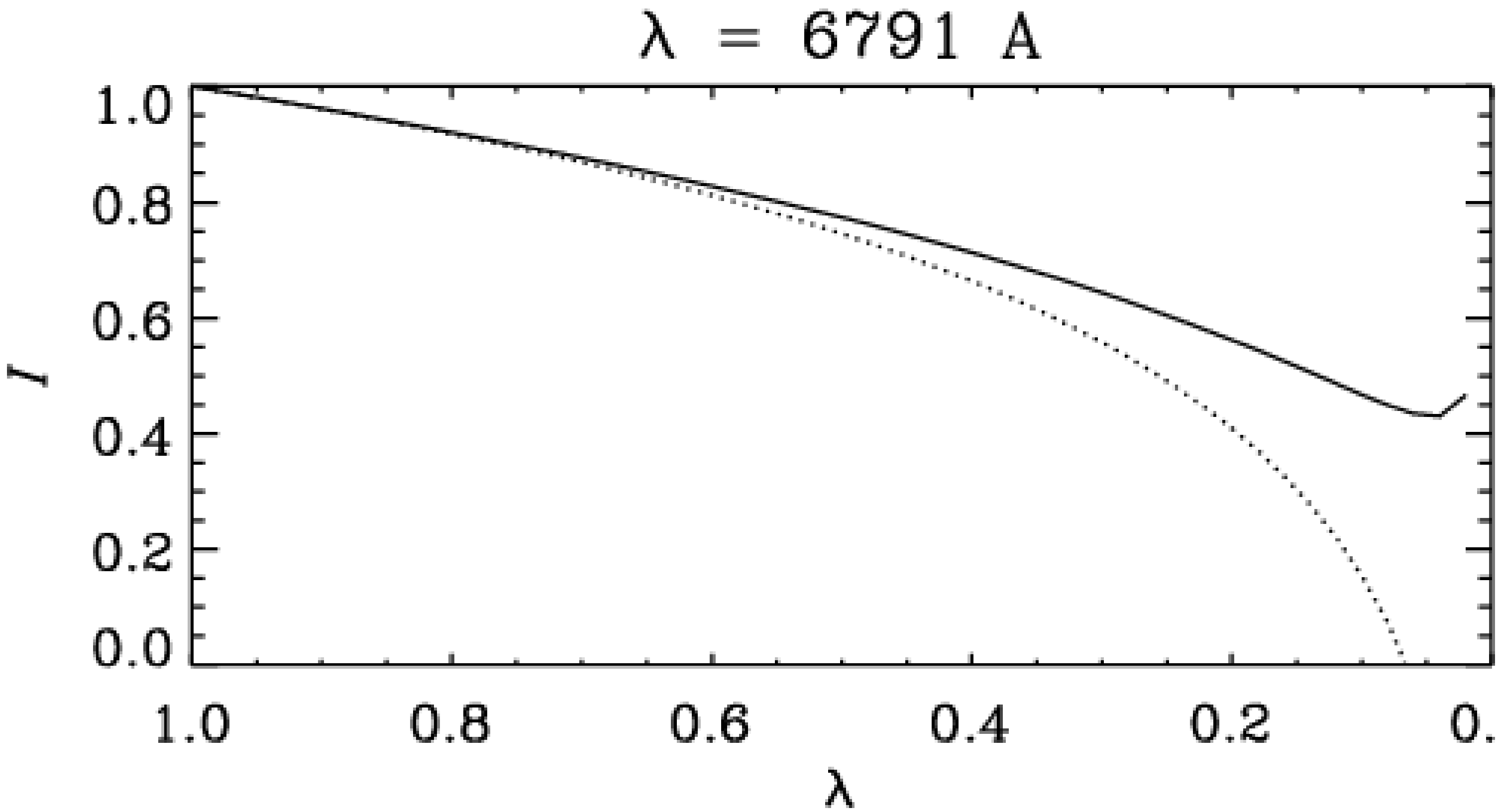


TABLE II
Coefficients of 5th degree, ln μ, fit to the limb darkening, Equation (9)

λ	No.	$a(5)$	$b(5)$	$c(5)$	$d(5)$	$e(5)$	$f(5)$	$Pe \times 10^4$
3033.27	4	1.00000	0.97019	0.52334	0.23153	0.08514	0.01478	79
3069.82	6	1.00000	0.90116	0.35235	0.04708	−0.00600	−0.00164	86
3108.43	6	1.00000	0.90333	0.40472	0.11113	0.02068	0.00195	58
3204.68	8	1.00000	0.87103	0.37246	0.09625	0.01712	0.00157	49
3298.97	8	1.00000	0.81884	0.33404	0.08388	0.01473	0.00135	47
3389.53	8	1.00000	0.77698	0.29530	0.06644	0.01050	0.00090	47
3499.49	8	1.00000	0.75452	0.27592	0.05559	0.00742	0.00061	44
3563.52	10	1.00000	0.74606	0.26455	0.04958	0.00578	0.00044	42
3626.50	6	1.00000	0.72148	0.27406	0.07379	0.01675	0.00198	31
3658.75	6	1.00000	0.68443	0.21916	0.03790	0.00512	0.00048	39
3740.88	10	1.00000	0.72647	0.19643	−0.01482	−0.01775	−0.00255	51
3779.92	5	1.00000	0.72880	0.21602	0.00547	−0.01131	−0.00200	48
3852.02	8	1.00000	0.79182	0.27120	0.01898	−0.01041	−0.00187	66
3909.28	8	1.00000	0.80076	0.33477	0.08431	0.01479	0.00143	37
3954.25	8	1.00000	0.78644	0.31546	0.05927	0.00309	−0.00028	40
3988.15	8	1.00000	0.74422	0.28048	0.05993	0.00881	0.00081	40
4019.70	10	1.00000	0.75222	0.29228	0.06451	0.00821	0.00041	48
4069.44	8	1.00000	0.74615	0.30280	0.08672	0.02051	0.00258	38
4117.23	3	1.00000	0.68191	0.20760	0.01721	−0.00476	−0.00096	52
4163.20	4	1.00000	0.69661	0.22354	0.02358	−0.00398	−0.00098	37
4219.05	4	1.00000	0.70411	0.24501	0.03626	−0.00003	−0.00036	46
4279.30	6	1.00000	0.68623	0.25109	0.05684	0.00879	0.00061	35
4316.45	6	1.00000	0.63476	0.18128	0.01269	−0.00384	−0.00060	51
4438.85	6	1.00000	0.63187	0.15325	−0.03962	−0.03059	−0.00431	58

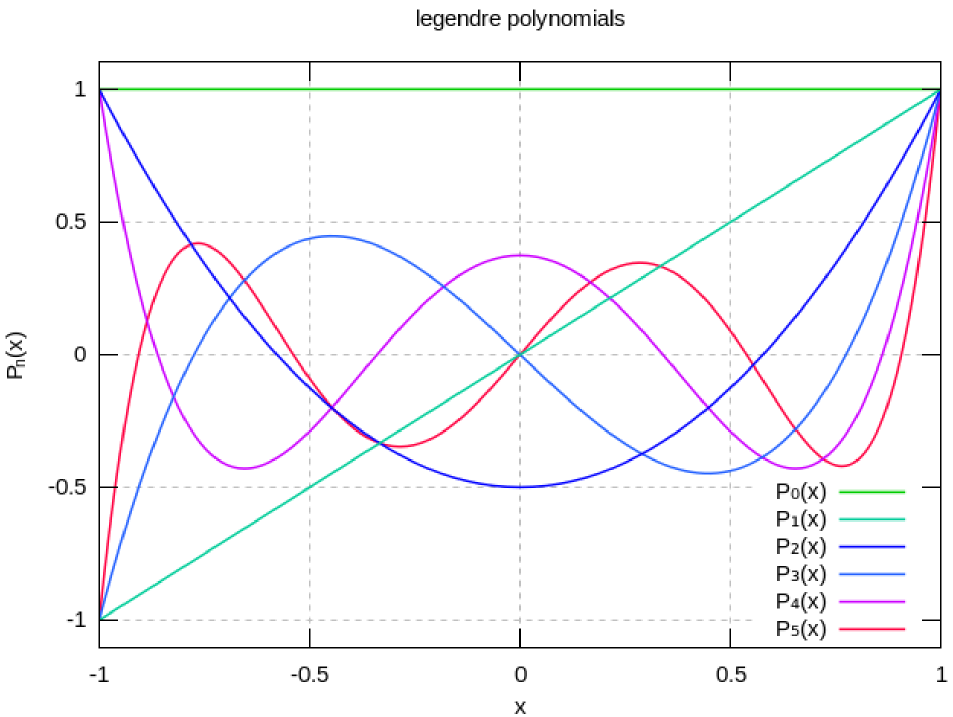
Legendre polynomials

Heard of them before?

A. Sure
B. Maybe
C. Not sure
D. Probably not
E. no

The first few Legendre polynomials are:

n	$P_n(x)$
0	1
1	x
2	$\frac{1}{2}(3x^2-1)$
3	$\frac{1}{2}(5x^3-3x)$
4	$\frac{1}{8}(35x^4-30x^2+3)$
5	$\frac{1}{8}(63x^5-70x^3+15x)$
6	$\frac{1}{16}(231x^6-315x^4+105x^2-5)$
7	$\frac{1}{16}(429x^7-693x^5+315x^3-35x)$
8	$\frac{1}{128}(6435x^8-12012x^6+6930x^4-1260x^2+35)$
9	$\frac{1}{128}(12155x^9-25740x^7+18018x^5-4620x^3+315x)$
10	$\frac{1}{256}(46189x^{10}-109395x^8+90090x^6-30030x^4+3465x^2-63)$



Orthogonality [edit]

An important property of the Legendre polynomials is that they are orthogonal with respect to the L^2 inner product on the interval $-1 \le x \le 1$:

$$\int_{-1}^{1} P_m(x) P_n(x)\, dx = \frac{2}{2n+1} \delta_{mn}$$

(where δ_{mn} denotes the Kronecker delta, equal to 1 if $m = n$ and to 0 otherwise). In fact, an alternative derivation of the Legendre polynomials is by carrying out the Gram-Schmidt process on the polynomials $\{1, x, x^2, \ldots\}$ with respect to this inner product. The reason for this orthogonality property is that the Legendre differential equation can be viewed as a Sturm-Liouville problem, where the Legendre polynomials are eigenfunctions of a Hermitian differential operator:

$$\frac{d}{dx}\left[(1-x^2)\frac{d}{dx}P(x)\right] = -\lambda P(x),$$

where the eigenvalue λ corresponds to $n(n+1)$.

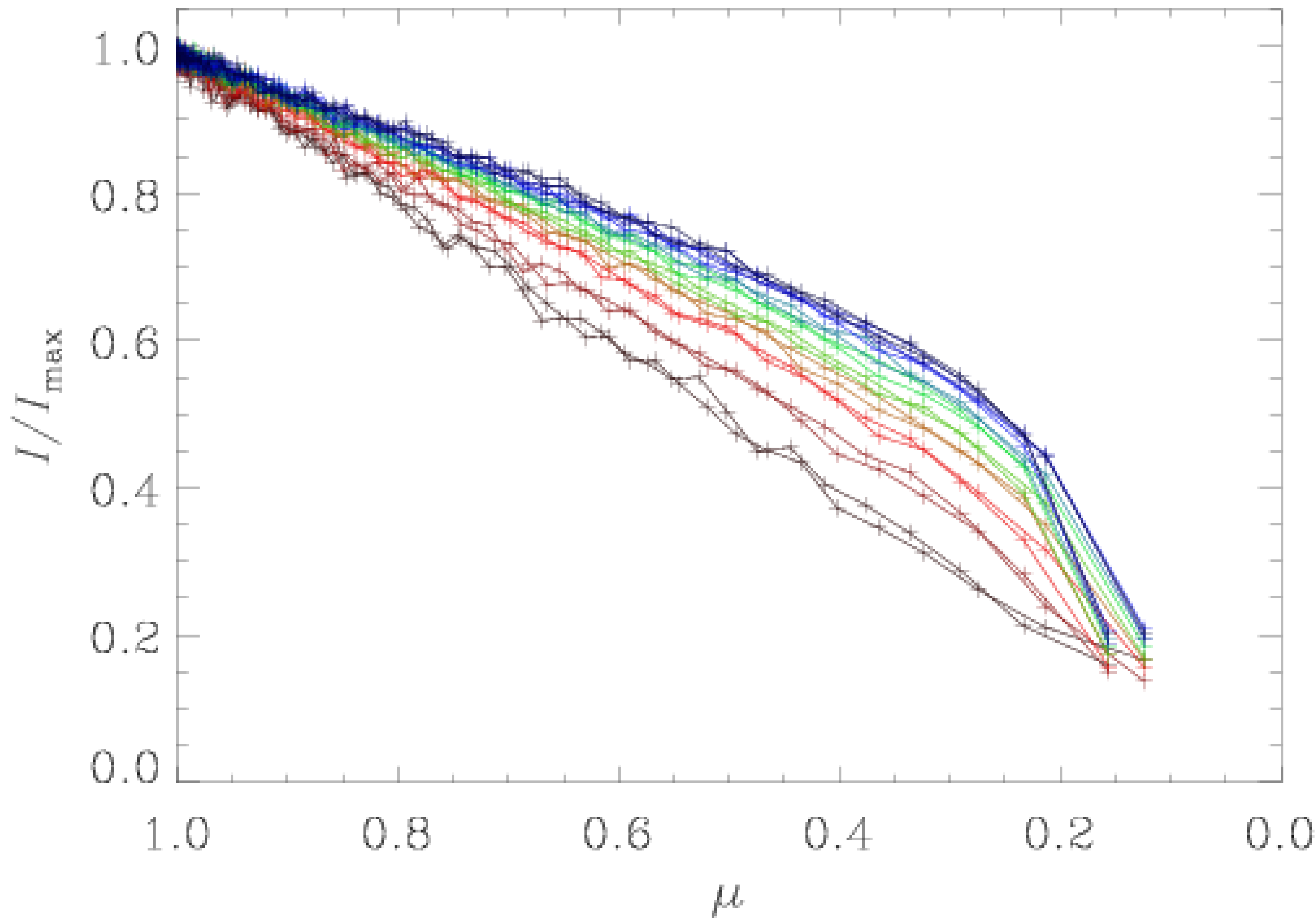
Advantage?

A. Fastest possible convergence

B. Lower orders don't change when extending truncation level

C. Coefficients have physical meaning

μ dependence



What we did today

- More on final report
  - Relation to other work (introduction)
  - Where to go from here (conclusions)
- Results so far
  - Presentations by Olivia, Shashhank, Max
- Discussion
  - Vignetting and correcting for it
  - Collapsing the two halfs on top of each other