TIME DISCRETISATION – EXAMPLE

Typical advection-diffusion equation:

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} + \kappa\frac{\partial^2 T}{\partial x^2}$$

Crank-Nicolson on the diffusive parts, Adams-Bashforth on the advective parts:

$$(1 - \kappa\frac{\delta t}{2}D^2)T^{n+1} = (1 + \kappa\frac{\delta t}{2}D^2)T^n - R(u,T)$$
$$R(u,T) = \frac{3}{2}(uDT)^n - \frac{1}{2}(uDT)^{n-1}$$

where $D = \partial/\partial x$ and $T^n = T(n\ \delta t)$.

Adams-Bashforth:

- Explicit
- Weak unconditional instability stabilised by diffusion
- Requires CFL condition for stability

$$\delta t \leq C_1\frac{\delta x}{u}$$

Crank-Nicolson:

- Implicit
- Second order accurate
- Unconditionally stable for any δt
- Requires numerical accuracy condition

$$\delta t \leq C_2\frac{(\delta x)^2}{\kappa}$$