EQUATIONS - MAGNETIC FUNCTIONS

The induction equation:

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \zeta C_k \nabla^2 \mathbf{B}$$

Use poloidal and toroidal parts to ensure $\nabla \cdot \mathbf{B} = 0$

$$\mathbf{B} = \nabla \times T\hat{\mathbf{z}} + \nabla \times \nabla \times P\hat{\mathbf{z}}$$

i.e.

$$\begin{aligned} B_x &= \partial_x \partial_z P + \partial_y T \\ B_y &= \partial_y \partial_z P - \partial_x T \\ B_z &= -\nabla_h^2 P \end{aligned}$$

For the Poloidal evolution, we take the z component of the induction equation:

$$-\nabla_h^2 \partial_t P = \partial_x (\mathbf{u} \times \mathbf{B})_y - \partial_y (\mathbf{u} \times \mathbf{B})_x + \zeta C_k \nabla^2 (-\nabla_h^2 P)$$

Procedure: Calculate nonlinear terms (configuration space), transform to spectral space, take derivatives for the curl, divide by k^2. Solve.

For the toroidal evolution, we take the z component of the curl of the induction equation:

$$-\nabla_h^2 \partial_t T = -\nabla_h^2 (u \times B)_z + \partial_{xz} (u \times B)_x + \partial_{yz} (u \times B)_y + \zeta C_k \nabla^2 (-\nabla_h^2 T)$$

Procedure: as above.

Notice that poloidal and toroidal parts say nothing about the means. Must solve for these values seperately:

$$\begin{aligned} \partial_t \overline{B_x} &= -\partial_z \overline{(\mathbf{u} \times \mathbf{B})_y} + \partial_z^2 \overline{B_x} \\ \partial_t \overline{B_y} &= +\partial_z \overline{(\mathbf{u} \times \mathbf{B})_x} + \partial_z^2 \overline{B_y} \\ \partial_t \overline{B_z} &= 0 \end{aligned}$$