# MAINTENANCE OF DIFFERENTIAL ROTATION IN TURBULENT COMPRESSIBLE CONVECTION WITH ROTATION

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ABSTRACT Numerical simulations of 3D compressible convection including $f$-plane rotation show that zonal and meridional mean flows are produced. The properties of such mean flows are different when the convection is turbulent and reflect some inferences from helioseismology about the observed solar differential rotation.

## INTRODUCTION

An apparent dichotomy exists between the inferences drawn from helioseismic observational data concerning the solar differential rotation profile and the predictions of global numerical investigations. The inversion of $p$-mode frequency splittings (cf. Gough & Toomre, 1991) suggest an angular velocity profile that is generally constant on radial lines within the convection zone at least at mid-latitudes, whereas numerical models of convection in rotating, spherical shells (cf. Glatzmaier, 1987) yield profiles that are constant on cylinders concentric to the axis of rotation. One plausible explanation for the discrepancy is that the global convection models are unable to achieve sufficient numerical resolution to attain fully turbulent flows. Other 3D convection studies (Cattaneo *et al.* 1991) have shown that the transport properties of turbulent convection can be very different from those of laminar convection, and so the extension of rotating convection models into a more turbulent regime may provide a different transport mechanism for angular momentum and thus possibly different rotation profiles. Since available computer resources has been the limiting factor in this endeavour, we turn to local rather than global models to be able to study the turbulent regime. We study three-dimensional compressible convection incorporating $f$-plane rotation in a rectilinear slab. The $f$-plane configuration for rotation (see Hathaway & Somerville, 1983) holds the rotation vector constant over the domain studied but allows for the rotation and gravitational vectors to be separated by an angle, $\frac{\pi}{2} - \phi$, where $\phi$ is the latitudinal positioning of the local domain. Solutions are then parameterised by the Taylor number $T_a$, the Rayleigh number $R_a$, and the Prandtl number $\sigma$. A derived parameter,

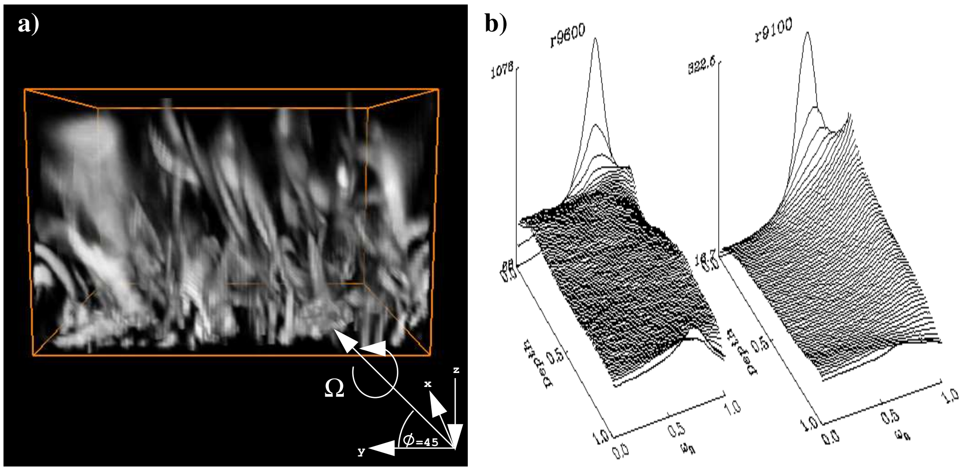


Fig. 1 *a* Enstrophy ($\omega^2$) alignment with the rotation vector, $\boldsymbol{\Omega}$, for $R_o = 0.71$ ($R_a = 5 \times 10^5, T_a = 10^7, \sigma = 0.1, \phi = 45°$). *b* Histograms of the vorticity alignment function $\omega_\Omega(z)$ for r9600 with $R_o = 2.24$ and r9100 with $R_o = 0.71$.

the convective Rossby number, $R_o = \sqrt{R_a/(T_a\sigma)}$ measures the influence of rotation as the ratio of rotational and convective timescales. Flows with $R_o \lesssim 1$ are strongly rotationally-influenced. The code is a version of code 1 of Cattaneo *et al.* (1991) adapted for a massively parallel machine running with resolution up to $192^2 \times 96$ for a domain of aspect ratio $4:4:1$ $(x:y:z)$.

## FLOW STRUCTURE & GENERATION OF DIFFERENTIAL ROTATION.

Highly-nonlinear compressible convection without rotation is characterised by a turbulent interior punctuated by coherent downflows structures associated with the interstices of a superimposed laminar cellular network at the upper surface (Cattaneo *et al.* 1991). Such convection calculations show a domain populated by a few very strong, vertical, spatially- and temporally-coherent vortex structures, or tubes, corresponding to the major downflow sites, with the interior characterised by smaller-scale, weaker but still vertically-orientated tubes. The inclusion of rotation increases the time-dependence of the cellular network, reduces its scale and introduces new mechanisms for cell creation and destruction (Brummell *et al.* 1992, 1994). With rotation, there are more strong vertically-coherent tubes, reflecting the subsequent reduction in cellular scale, and, since the downflows must all be cyclonic due to their convergent nature, vigourous time-dependent vortex interactions take place forming tangled bundles of tubes rather than the simple “chimneys” more typical of the non-rotating case. Rotation also enhances the intensity of the small-scale vortex tubing in the interior and, since the Coriolis term provides a linear mechanism for transfer of kinetic energy between convectively-driven vertical flows and horizontal motions, the interior turbulence becomes more isotropic and more randomly orientated.

Turbulent compressible convection on these rotating $f$-planes generates significant mean horizontal flows. Correlations between upward and horizontal motions are induced by Coriolis forces and provide Reynolds stress source terms for the means motions. Hathaway & Somerville (1983) in laminar Bousinesq calculations explained that a rotation vector not aligned with gravity can immediately supply such correlations, since the action of the Coriolis forces associated with the horizontal component of the rotation vector on vertically-moving fluid elements is to propel them horizontally. In a laminar convective motion, this results in tilted trajectories on the cellular boundaries and thus generates a vertical transport of horizontal momentum. This mechanism is still available for turbulent flows although motions are much less vertically-coherent and it is found that another method of introducing specific correlations exists. When rotation effects are sufficiently strong, the coherent vortex structures tend to align themselves to the axis of rotation in a 'Taylor-Proudman'-like manner. This *turbulent* alignment is easy to see in the visualisations of the flow, e.g. figure 1$a$, but is hard to quantify due to the low filling factor of the coherent structures when combined with spatial averaging, and thus is most obvious in pointwise measures. Figure 1$b$ shows a surface built up of histograms of $\omega_\Omega$, the projection of the vorticity vector onto the rotation direction (suitably normaised) evaluated in planes at each depth (and time-averaged) for two cases with moderate ($R_o = 2.24$) and strong ($R_o = 0.71$) rotational influence. A value of $\omega_\Omega = 1$ indicates perfect alignment with the rotation vector (at 45° in these cases), and it can be seen that whilst the histogram bins of the moderately-rotating case are evenly filled, the strongly-rotating case has its peak where the alignment is with the rotational vector. Note that at $z = 0, 1$ there must be a separate peak at $cos\phi$ enforced by the stress-free boundary conditions.

There are now two distinct mechanisms for generating the velocity correlations required for mean flow maintenance: the *laminar* effect of the horizontal component of rotation, and the *turbulent* effect of vortex structure alignment. The former favours correlation between vertical and East-West (zonal) motion, whereas the latter will generate correlation between vertical and North-South (meridional) motions (since the rotation vector is in that plane).

## PROPERTIES OF MEAN ZONAL AND MERIDIONAL FLOWS.

Figure 2 shows the horizontally and temporally averaged zonal ($u$) and meridional ($v$) mean flows produced in cases each with $R_a = 5 \times 10^5, T_a = 10^7$ and $\sigma = 0.1$ but with varying latitude. The mean zonal velocity in this local model corresponds to differential rotation in the global sense. Notice here that for each latitude, $u$ consists of boundary jets with a near-constant interior. This plateau profile is evident because the small-scale turbulent motions in the interior turnover so quickly that the rotational timescale is not felt (the effective $R_o$ of these scales is large), and the drift velocity is imposed only by the overall alignment correlation introduced by the rotation. It is tempting to link these results to helioseismic inferences despite the tenuous correspondence of this vastly simplified model to the real sun. Since depth in this local model corresponds to radius in the global view, the results in the interior here correspond to an angular velocity profile which is constant with radius across many latitudes, somewhat in accord with the inversions of solar data. Also, the amplitude of the zonal mean flow increases with decreasing latitude consistent with the concept of an equatorial acceleration (albeit retrograde at the surface in this case). This amplification can be understood in terms of the balance between the two alignment mechanisms explained above. At high latitudes, laminar alignment is weakened as the horizontal component of rotation becomes

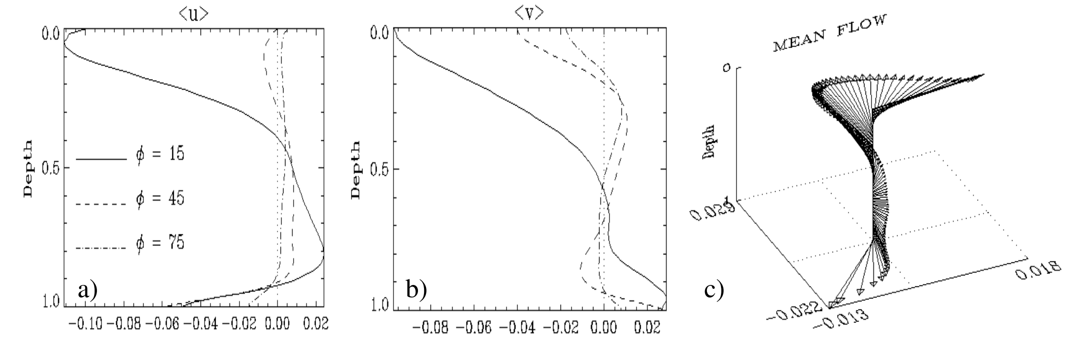


Fig. 2 *a* Mean zonal ($u$) and *b* meridional ($v$) velocity profiles for three cases with varying latitude, $\phi = 15°, 45°, 75°$ and each with $R_a = 5 \times 10^5, T_a = 10^7, \sigma = 0.1$, and *c* an example of spiralling in the vector mean flows.

small, and the turbulent alignment is almost vertical, both effects thereby serving to remove the correlations necessary for the mean flows. At low latitudes, the horizontal component of $\mathbf{\Omega}$ dominates, and the turbulent alignment is nearly horizontal, so that the laminar and turbulent effects act in concert to produce larger zonal and meridional flows. The large meridional flow is not observed in the solar data and is an artifact of the local area model since this formulation cannot allow for deceleration of the meridional flows by convergence at the equator or poles.

If the vector sum of the zonal and meridional flows is plotted as a function of depth with an arrow (like a weathervane) representing the direction and amplitude of the flow (fig. 2*c*), then that vector is typically seen to spiral with depth. Nearly all solutions spiral in a cyclonic manner near the surface as a consequence of the strong effect of compressibility there, with the turbulent interior more constant. This behaviour may be analagous to interpretations of helioseismic ring diagram analyses (Patron *et al.* , 1994).

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