

KEPLER'S LAWS

SYNOPSIS: Johannes Kepler formulated three laws that described *how* the planets orbit around the Sun. His work paved the way for Isaac Newton, who derived the underlying physical reasons *why* the planets behaved as Kepler had described. In this exercise, you'll use computer simulations of orbital motions to experiment with the various aspects of Kepler's three laws of motion.

EQUIPMENT: Computer with internet connection to the Solar System Collaboratory.

LENGTH: One lab period.

Getting Started

Here's how you get your computer up and running:

- (1) Launch the **Netscape** browser.
- (2) Do *not* use "maximized" windows - if you don't see the "desktop" in the background, click on the double-window button at the upper right of the Netscape window.
- (3) Go to the website **<http://solarsystem.colorado.edu/cu-astr>**
- (4) Click on "**Enter Website**" at whichever resolution is appropriate for your computer monitor screen.
- (5) Click on the **Modules** option.
- (6) Click on **Kepler's Laws**.

Note: We intentionally do not give you "cook-book" how-to instructions, but instead allow you to explore around the various available windows to come up with the answers to the questions.

But note: use the "applets" on the MAIN window, not the EXTRA window. If you use the EXTRA window, you'll find that the HELP, HINT, and MATH information will be referring to the wrong page.

The window placement is designed to facilitate access with one click of the mouse. Avoid maximizing or moving windows; otherwise, it will only make your life harder!

Part I. Kepler's First Law

Kepler's First Law states that a planet orbits on an ellipse around the Sun.

- I.1 Sketch an ellipse in your lab report. Label the two foci (f_1 and f_2), the semimajor axis (a) and the semiminor axis (b). (*Hint: If you are not familiar with these terms check out SHOW ME THE MATH*).
- I.2 Where is the Sun with respect to that ellipse?
- I.3 What is meant by the eccentricity of an ellipse? Give a description.
On your ellipse, indicate a distance that can be expressed as the semimajor axis (a) times the eccentricity (e) (we will refer to this distance as ae).
- I.4 What happens to the ellipse when the eccentricity becomes zero?
- I.5 What happens to the ellipse when the eccentricity becomes one?
- I.6 (a) Could a planet move on a *circular* orbit?
(b) If your answer is "yes", where would the Sun be with respect to that circle?
- I.7 On planet Blob the *average global* temperature stays exactly constant throughout the planet's year. What can you infer about the eccentricity of Blob's orbit?
- I.8 On planet Blip the *average global* temperature varies dramatically over the planet's year. What can you infer about the eccentricity of Blip's orbit?
- I.9 On your ellipse diagram, draw a line from the position of the sun to some point on the ellipse. Label this line r . This will represent the planet-sun distance. Note that the length of r will change as the planet orbits the sun. When r is at its minimum value, we say the planet is at periapsis (this is a generic term used for an object that orbits any other object). Likewise when r is at its maximum value, we say the planet is at apoapsis.
Now use your ellipse diagram to come up with an equation for r at periapsis and an equation for r at apoapsis.
- I.10 Use the applet to check your equations: For an ellipse of eccentricity $e = 0.9$, find the ratio of periapsis to apoapsis. You can use the tick-marks to read distances directly off the screen (to the nearest half-tick).

- I.11 What is the ratio of periapsis to apoapsis for $e = 0.5$? Show your work.

The following questions pertain to our own Solar System. Remember that the orbits of the different planets are not drawn to scale. We have scaled the diagram to the major axis of each orbit. Note that we are now using the terms **perihelion** and **aphelion** (these terms refer to objects that orbit the Sun – Helios).

- I.12 What is the ratio of perihelion to aphelion for the planet with the largest eccentricity? Be sure to indicate which planet this is.
- I.13 Which planet has the second largest eccentricity? What is the ratio of perihelion to aphelion for that planet?
- I.14 If Saturn's perihelion is 9.0 AU, what is its aphelion?

Part II. Kepler's Second Law

Kepler's Second Law states that, for each planet, the area swept out in space by a line connecting that planet to the Sun is equal in equal intervals of time.

- II.1 For eccentricity $e = 0.7$ use the speedometer to record the speed at perihelion and the speed at aphelion.
- II.2 Do the same for $e = 0.1$.
- II.3 Express the relationship you just found between the planet's distance and its speed. Does this remind you of a conservation principle?
- II.4 Where does the planet spend most of its time, near periapsis or near apoapsis?
- II.5 Using the applet, devise your own way to show that the planet is taking equal amounts of time to cover equal areas (thus proving Kepler's Second Law). Describe all of your steps. *Hint:* You may want to include a diagram.

Part III. Kepler's Third Law

Kepler's Third Law states the relationship between the size of a planet's orbit (given by its semi-major axis), and the time required for that planet to complete one orbit around the Sun (its period).

- III.1 By clicking on the UP and DOWN buttons, run through all the possible combinations of integer exponents available ($1/1$, $1/2$, ..., $1/9$; $2/1$, $2/2$, ..., $2/9$; $3/1$, $3/2$, ... $3/9$; etc).

Which combinations give you a good fit to the data? List **all** combinations that work.

III.2 The period of Halley's comet is 76 years. From the graph, what is its semi-major axis?

III.3 Using decimal exponents, find the exponent of a (the semi-major axis) that produces the best fit to the data for the period p raised to the following powers:

(a) $p^{0.6}$ _____

(b) $p^{5.4}$ _____

(c) p^{78} _____

III.4 Why do you think Kepler chose to phrase his third law as he did, in view of the fact that there are many pairs of exponents that seem to fit the data equally well?

III.5 In the 18th century, two astronomers noticed a relationship between the orbital distances of the planets known at that time.

<u>Planet</u>	<u>a = Distance from Sun in AU</u>
Mercury	0.387
Venus	0.723
Earth	1.000
Mars	1.524
Jupiter	5.203
Saturn	9.539

(a) Recreate that relationship.

Hint: Start with an equation of the form $a \approx (x + 4) / 10$, and find some pattern for x .

(b) Is every orbit you calculate with your relationship occupied by a planet? If not, can this be explained?

(c) Extend your relationship to predict the orbits of the next three planets in our solar system and compare it to their actual distances. Are they the same? Would this relationship be a better tool than Kepler's Third Law? Why or why not?

Part IV. "Dial-an-Orbit" Applet

Here's where you can "play god": create your own planet, and give it a shove to start it into orbit.

IV.1 Start the planet at $X = -80$, $Y = 0$.

(a) Find the initial velocity (both X- and Y-components) that will result in a **circular** orbit (use the tick marks to judge whether the orbit is circular): Hint: in science it is best to try to work with the least number of variables possible, therefore try setting $V_x = 0$ and adjusting only V_y .

$$V_x = \text{_____} \quad V_y = \text{_____} .$$

(b) Using the clock, find the period T of that orbit:

$$T = \text{_____} .$$

IV.2 Now start the planet at $X = -60$, $Y = 0$.

(a) Find the velocity that will result in an **elliptical** orbit of semi-major axis = 80 (*attention*: remember the definition of the semi-major axis!).

(b) Use the clock to find the period for that orbit.

IV.3 Would you expect the period you measured in question IV.2 to be the same as the period you measured in question IV.1? Why?

Part V. Follow-up Questions

V.1 In the *Celestial Motions* lab you learned that the same face of the Moon always faces the Earth (i.e. only 50% of the Moon's surface can be observed from Earth). Actually, we have been able to see up to 59% of the Moon's surface from Earth. Explain how we can see more than half of the Moon's surface.

V.2 In Part I we compared the average global temperatures of two planets and asked how their eccentricities might differ. What is the eccentricity of Earth. Given that value, should the Earth's average global temperature have more in common with Blip or Blob? Does your answer make sense and if not can you think of another factor that might regulate global temperatures?

V.3 In IV.1 a hint was given that you should set $V_x = 0$ in order to get a circular orbit. Intuitively many of you probably thought doing that would just cause the "planet" to race off in one direction and never return. Explain why it worked.

