STELLAR CONVECTION : Theory and Models

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The understanding of the structure and evolution of stars is a longstanding problem in astrophysics, first addressed quantitatively by Eddington in 1926. Central to this problem is the description of stellar turbulent convection and how it transports heat and energy, how it redistributes angular momentum to yield large scale flows such as differential rotation and meridional circulation, and how it generates, maintains and organizes magnetic fields (see below).

1) General Astrophysical Context

Stars are dynamic objects, as dramatically demonstrated by our closest example, the Sun. The surface of the Sun seethes with turbulent convective motions and larger-scale flows that also persist deep below the surface as revealed by helioseismology (Gough & Toomre 1991, Christensen-Dalsgaard 2003). Solar convection works together with rotational shear and global meridional circulations to generate patterns of magnetic activity such as the 11-year sunspot cycle. Throughout the solar cycle, magnetic flux continually emerges and often destabilizes, producing eruptive events such as coronal mass ejections and solar flares. Understanding the origins of this diverse magnetic activity provides insight into similar processes occurring throughout the cosmos. In this sense, the Sun is a laboratory for astrophysics; nowhere else can we observe the complex interactions between turbulent plasmas and magnetic fields with comparable detail.

In stars the large size L of the system at hand and the low microscopic viscosity nu of the plasma yields very high Reynolds numbers (Re=v L/nu) even for weak characteristic velocities v, implying that motions must be highly turbulent. Given the complexity of describing turbulent nonlinear processes in rotating and magnetized systems (since all stars rotate and most are magnetically active), various simplified prescriptions have been developed over the last century in order to be able to describe at least qualitatively stellar convection zones (Spiegel 1971). A notable example is the so-called mixing length theory (MLT; see Bohm-Vitense 1958; Cox & Guili 1968, Schatzman & Praderie 1990, Hansen & Kawaler 1995) that is often used to account for convective heat transport in stellar evolution models. In MLT, a convective eddy (or blob, parcel, cell, rising element) is displaced from its equilibrium position over a distance Λ, the so-called mixing length, before releasing totally its heat content. It is generally assumed that Λ is proportional to the local pressure scale height Hp, e.g. $\Lambda = \alpha_M$ Hp, with α_M the mixing length coefficient. This coefficient α_M is of order unity and it is calibrated thanks to accurate 1-D solar standard models whose luminosity, radius and chemical abundances (Z/X ratio) must be within 10^{-5} of the observed solar values. Current 1-D standard solar models all possess α_M being between 1.5 to 2 (Brun et al. 2002, Turck-Chieze et al. 2004, Bahcall et al. 2005, Asplund et al. 2006, Antia & Basu 2005, 2006). While this simple prescription of convection has its merit to describe in one dimension the quasi static structure of stars over secular time, it lacks several important physical properties, such as a turbulent spatial and temporal energy distribution, velocity correlations, non locality etc… that are key to a modern understanding of the (magneto)-hydrodynamics of stars.

2) What is convection?

Convection is an instability occurring in a stratified fluid. It is a mechanism for transporting thermal energy by means of the bulk displacement of a fluid. What is hot goes up, what is cold comes down. A common instance is that of a pan of water set on a heat source (electric heating element, gas ring, etc.). The water heated at the bottom of the pan is lighter, and rises to the surface, where it cools, sinks back, is warmed again, rises once more, and so on. Such convective motion tends to reduce the difference in temperature between pan bottom and surface. In the case of a star, convective motions serve to carry away the nuclear energy generated in its core. Positioning of the convection zones is strongly dependent on the star's mass. Cool stars are fully convective, hot stars have convective cores and an extended radiative envelope, and solar type stars possess a radiative interior surrounded by a convective envelope. When there is steep variation in density, as in the solar convection zone, the fluid's entropy turns out to be the natural variable for the characterization of convection efficiency; highly efficient convection maintains a nearly adiabatic stratification such that the integrated heat flux through the convection zone is small relative to the thermal energy of the plasma. In nature, thermal-energy transport may also occur through conduction (direct contact between a hot and a cold body) or radiation. In main sequence stars, conduction plays a negligible role compared to radiation and convection.

2a) Schwartzschild & Ledoux criteria for convective stability

In the inviscid limit ($\nu=\kappa=0$), the criteria to **not** trigger convection is very simple. Let's displace a fluid element by a small radial distance dr in a stratified media (s=surrounding & e=element), we get:

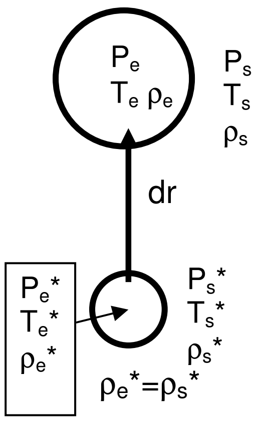

Fig1: vertical displacement of a fluid element in a stratified media

$$\rho_e = \rho_e^* + dr\left(\frac{d\rho}{dr}\right)_e \qquad \text{eq (1)}$$

$$\rho_s = \rho_s^* + dr\left(\frac{d\rho}{dr}\right)_s \qquad \text{eq (2)}$$

Where ρe^*, ρe are respectively, the density of the element before and after the displacement dr, and ρs^*, ρs the density of the surrounding media (and the same convention is used for the pressure P and temperature T). Let's take the difference between equations (1) & (2):

$$\rho_e - \rho_s = dr\left[\left(\frac{d\rho}{dr}\right)_e - \left(\frac{d\rho}{dr}\right)_s\right] > 0$$

If positive, then the equilibria is **stable**

eq (3)

or equivalently $$\left(\frac{d\rho}{dr}\right)_e - \left(\frac{d\rho}{dr}\right)_s > 0 \qquad \text{eq (4)}$$

Now, let's consider the following general equation of state (with μ the mean molecular weight): $\rho=\rho(P,T,\mu)$

$$\frac{d\rho}{\rho} = \alpha\frac{dP}{P} - \delta\frac{dT}{T} + \phi\frac{d\mu}{\mu} \qquad \text{eq (5)}$$

where α, δ, ϕ are thermodynamic coefficients. Substituting equation 5 in equation 4 leads to:

$$\left(\frac{\alpha}{P}\frac{dP}{dr}\right)_e - \left(\frac{\delta}{T}\frac{dT}{dr}\right)_e - \left(\frac{\alpha}{P}\frac{dP}{dr}\right)_s + \left(\frac{\delta}{T}\frac{dT}{dr}\right)_s - \left(\frac{\phi}{\mu}\frac{d\mu}{dr}\right)_s > 0 \qquad \text{eq (6)}$$

Since $P_e = P_s$ & $P_e^* = P_s^*$ (pressure equilibrium) and $d\mu$ null for the element, we get after multiplying by the pressure scale height $Hp = -1/(d \ln P/dr)$, the following stability criteria:

$$\left(\frac{d\ln T}{d\ln P}\right)_s < \left(\frac{d\ln T}{d\ln P}\right)_e + \frac{\phi}{\delta}\left(\frac{d\ln \mu}{d\ln P}\right)_s \quad \text{Stable} \quad \text{eq (7)}$$

or using stellar physics classical gradient notation: $\nabla < \nabla_e + \frac{\phi}{\delta}\nabla_\mu$ eq (8)

Let's assume an atmosphere in which the energy is transported only per radiation, then $\nabla = \nabla_{rad}$. Let's test the stability of this atmosphere by considering the *adiabatic* displacement of an element: $\nabla_e = \nabla_{ad}$

The atmosphere is convectively stable if:
Ledoux:

$$\nabla_{rad} < \nabla_{ad} + \frac{\phi}{\delta}\nabla_\mu \quad \text{eq (9)}$$

Remark : for a perfect gas $P = R\rho T/\mu$ => $\alpha=\delta=\phi=1$

Schwartzschild (if no variation of composition or ionization is assumed):

$$\nabla_{rad} < \nabla_{ad} \quad \text{eq (10)}$$

Remark: The gradient of the specific entropy per unit mass S can easily be related to the difference between ∇ and ∇_{ad} such that (in the case where we neglect variation of composition or ionization):

$$\frac{dS}{dr} = -\frac{c_p}{H_p}\left(\nabla - \nabla_{ad}\right) \quad \text{eq (11)}$$

with c_p the specific heat at constant pressure. This leads to a simple criteria for inviscid convection: There is convection when dS/dr is negative or it is convectively stable for a positive dS/dr.

2b) Rayleigh number

In reality even though atomic viscosity can be very small in a stellar plasma, the threshold to trigger convection will be higher than in the inviscid limit since diffusion effects will suppress the convection instability. The instability criteria for a horizontal layer heated from below is described in detail by Chandrasekhar (1961) and subsequent works have considered rotating spherical shells (Roberts 1968, Busse 1970; Gilman 1975). For illustration, we briefly list the instability criteria for the case of a convective layer heated from below assuming either no slip or stress free boundary conditions for the velocity field. The Rayleigh number is defined as:

$$Ra = \frac{g\alpha\Delta T d^3}{\kappa\nu}$$

in Boussinesq (i.e nearly incompressible) convection.

For stress free conditions, top & bottom, the critical Rayleigh number is : $Ra^c = 658$
For stress free & no slip : $Ra^c = 1100$
For no slip top & bottom: $Ra^c = 1708$

The Rayleigh number has to be above this threshold for convection to start. In stellar convection zones the Rayleigh number can exceed 10^{15} so such instability conditions are easily realized.

The presence of an imposed field generally raises the instability threshold; magnetic fields have a stabilizing influence. For stress-free velocity boundary conditions and a purely radial magnetic field at the boundaries, Ra^c depends on the Hartman number Ha such that $Ra^c = \pi^2 (Ha)^2$ for Ha >> 1.

The Hartman number is defined as: $$Ha = \left(\frac{\sigma B_0^2 d^2}{\rho \nu} \right)$$

So stronger is the imposed magnetic field more difficult it is to trigger convection (check Cattaneo, Emonet & Weiss 2003 for a recent discussion on dynamo and magnetoconvection in an unstable slab with an externally imposed field).

3) Going beyond MLT: 3-D HD & MHD models of stellar convection

While linear stability analysis are very useful to understand the behaviour near the threshold of the instability, the huge Reynolds and Rayleigh number present in stellar convection zones advocate for a nonlinear study of this process. With the arrival of petaflop supercomputers it will become more and more possible to tackle directly the 3-D modelling of stellar turbulent convection zones. Even with the current computers Re numbers as large as 10^{12} are not yet tractable but sustained effort over the last decade has permitted a significant step forward in our understanding of highly nonlinear convection systems under the influence of rotation and magnetism and of the generation of strong magnetic fields through dynamo action (see for instance Brandenburg et al. 1996, Brummell et al. 1996, 1998, Cattaneo 1999, Porter & Woodward 2000, Woodward et al. 2003, Emonet & Cattaneo 2001, Miesch et al. 2000, 2008, Brun & Toomre 2002, Brun 2004, Brun et al. 2004, 2005, Robinson et al. 2003, 2004, Rincon et al. 2005, Stein & Nordlund 1998, 2006, Vogler et al. 2007).

3a) Convection structure and boundary layers

To a good approximation, stellar interiors are nearly in hydrostatic balance so the pressure, density, and temperature decrease outward. Across deep convection zones such as that in the Sun the density decreases by several orders of magnitude. This has a profound influence on the structure of stellar convection as illustrated in Figure 2. Plasma flowing upward expands as its density drops, creating broad convection cells surrounded by an interconnected network of downflow lanes. The convection pattern changes continually as cells are sheared and fragmented. Flow converging into the downflow lanes acquires intense cyclonic vorticity from the Coriolis force, evident in Figure 2 as swirling downflow plumes with a counter-clockwise and clockwise sense in the northern and southerm hemispheres respectively.

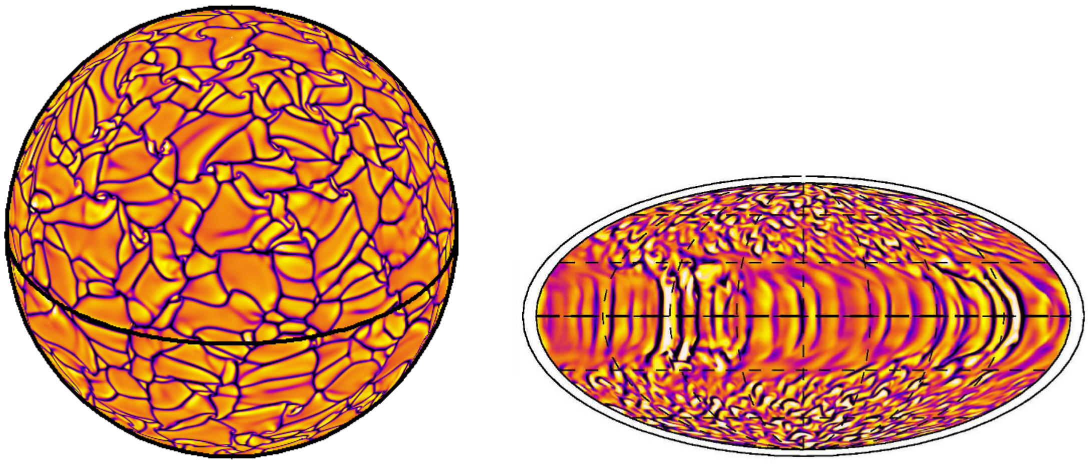

Fig 2: Shown is the radial velocity (downflows dark, upflows yellow/orange) realized in high resolution simulations of solar convection (left panel; Miesch et al. 2008) and of convection in a solar-type star rotating five times faster than the Sun (right panel; Brown et al. 2007).

In Figure 2 the influence of rotation is also evident at low latitudes where downflow lanes tend to align in a north-south orientation in order to mitigate the stabilizing influence of the Coriolis force. Such alignment is barely discernible but becomes much more apparent in simulations of more rapidly rotating stars. Figure 2b shows the simulated convection pattern in a solar-type star rotating five times faster than the Sun. Here there is a clear dichotomy between the small-scale, nearly isotropic convection pattern near the poles and the larger cells near the equator which are elongated in latitude but relatively narrow in longitude. The transition occurs near the so-called tangent cylinder, an imaginary cylindrical surface which is parallel to the rotation axis and tangent to the base of the convection zone.

The simulation shown in Figure 2b also exhibits modulated convection in which vigorous motions occur only in a restricted range of longitudes. Such convection patches can persist for thousands of days or, in deeper convection zones, can come and go in sporadic bursts of activity (Grote et al 2000; Ballot et al 2007; Brown et al 2007). In the limit of very rapid rotation, convection cells become aligned with the rotation axis as is thought to occur in the Earth's outer core (Busse 2000). In RGB and AGB stars the huge extented convective envelope combined with large density contrast results in the presence of only a few broad warm upflows surrounded by a network of narrow downflow lanes over the whole star's surface (Palacios & Brun 2007, Woodward et al. 2003, Freytag 2006). These evolved stars are slow rotators as a direct consequence of their inflated radius. The reduced rotational influence leads to convection patterns that are similar to non rotating convection in spherical shells, with a large l=1 dipole in temperature being established (Chandrasekhar 1961, Woodward et al. 2003, Palacios & Brun 2007). This drives a large meridional circulation. By contrast fast rotator exhibit a banded temperature structure in longitude associated with a self established thermal wind balance (Pedlosky 1987, Miesch et al. 2006, Brown et al. 2007). As the mass of stars increases, core convection progressively broadens and surface convection becomes shallower and shallower until it disappears altogether. In such convective core the density contrast is much smaller, yielding large scale convective patterns with almost no asymmetry between upflows and downflows (Browning et al. 2004).

As in many turbulent flows, boundary layers can play an important role in stellar convection. The upper boundary of the solar convection zone is the photosphere where there is again a transition from convective energy transport to radiation; this is where stars shine. Radiative energy transfer, coupled with ionization and a sharp drop off in density, temperature, and pressure drives another type of stellar convection known as granulation. This is the type of convection one sees in telescopic images of the solar surface which is blanketed by millions of granulation cells, each about 1000 km across.

When modeling relatively small-scale features such as granulation whose physical scale is well below the Sun's Rossby radius (~30,000 km, i.e the size above which Coriolis effects are playing fully), there is no need to consider the full spherical geometry which would require memory and operation counts far exceeding the capabilities of even the most powerful supercomputers. Instead, numerical simulations of granulation are confined to local Cartesian geometries (see Vogler et al. 2007, Stein & Nordlund 1998, 2006 (link scholarpedia article by Stein), Abbett et al. 2001, Abbett 2007). Despite the simplified geometry, granulation simulations face a formidable challenge in accurately capturing the complicated transition region from the solar interior, through the photosphere and chromosphere, and on to the corona. Much of the complexity arises from magnetic fields which continually emerge through this transition region, often destabilizing and triggering explosive events such as coronal mass ejections and solar flares.

The base of the solar convection zone is no less complex. The inner 70% of the Sun by radius is stably stratified, meaning vertical velocity and temperature variations propagate as internal gravity waves rather than overturning as convection cells. However, downward-travelling convective motions can overshoot the base of the convection zone by virtue of their own inertia (Zahn 1991, Rieutord & Zahn 1995). Convective overshoot at the base of stellar envelopes is generally dominated by isolated, intermittent, downflow plumes which are eventually decelerated and dispersed by the buoyancy force. Such overshoot regions are typically very thin, extending less than one percent of the stellar radius.

Furthermore, near the base of the solar convection zone there is a sharp transition from differential rotation above (such that the equator spins about 30% faster than the poles) to nearly uniform rotation below (Thompson et al 2003). This rotational shear layer is known as the solar tachocline (Spiegel & Zahn 1992) and it is thought to play an essential role in the solar dynamo (link to articles by Rudiger, Charbonneau). The solar tachocline and overshoot region support a variety of instabilities and waves that are driven by buoyancy, magnetism, and shear and that mediate the thermal, mechanical, and magnetic coupling between the convection zone and the radiative interior. For a recent overview see Gough & McIntyre (1998), Tobias (2004), Miesch (2005), Miesch et al. (2006), Brun & Zahn (2006), Zahn et al. (2007), Rudiger & Kitchatinov 2007, Kim & Leprovost 2007 and the special volume on the tachocline, Hughes et al. (2007).

Other cool stars likely exhibit boundary layers similar to the Sun. However, some cool stars such as M dwarfs, are convective throughout their interiors so they have a photosphere but no overshoot region or tachocline. Hotter stars such as A stars, on the other hand, are inverted Suns, with convection in their cores and stable outer envelopes. Here overshoot occurs at the outer boundary of the core convection zone, far from the photosphere (although many hot stars also exhibit a thin surface convection zone, from late A to early F stars). Recent simulations of core convective stars reveal that penetrative convection varies significantly with latitude, being stronger near the poles (Browning et al. 2004).

3b) Convective transport and associated large scale mean flows

Helioseismic inversions of large-scale, axisymmetric, time-averaged flows in the Sun (Thompson et al. 2003) currently provide the most important observational constraints on global-scale models of solar convection (BT02, Miesch et al. 2006, 2008). Of particular importance and reasonably well constrained by helioseismology, is the mean longitudinal flow, i.e. the differential rotation Ω (r,θ), which is characterised by a fast equator, slow poles and a profile almost independent of radius at mid latitudes (conical). In the most recent global simulations a fast equator, a monotonic decrease of Ω with latitude and some constancy along radial line at mid latitudes are established, all these attributes being in reasonable agreement with helioseismic inferences. Convection can redistribute angular momentum directly through the Reynolds stress or indirectly by establishing mean (longitudinally-averaged) circulations in the meridional (radius-latitude) plane. In a steady state without magnetism and neglecting the (tiny) viscous effects, these two mechanisms must balance one another. Mean meridional circulations may be maintained by Reynolds stresses, Coriolis forces, or by latitudinal gradients of temperature and entropy through what is known as baroclinicity. In stars like the Sun that possess a substantial rotational influence, baroclinicity tends to establish thermal wind balance such that variations of Ω parallel to the rotation axis are proportional to latitudinal entropy gradients. Global simulations of solar convection do indeed exhibit thermal wind balance in the lower convection zone but this balance breaks down near the surface and in high shearing regions (BT02, MBT06).

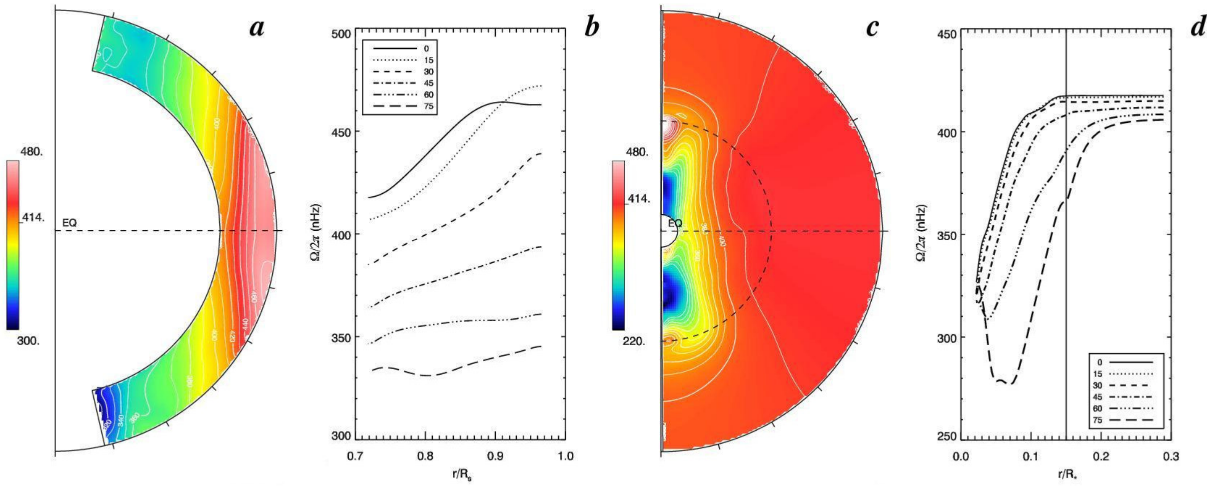

Fig 4: Shown are the angular velocity profiles (in nHz) realized in simulations of convection in the Sun (panels a & b, BT02) and in a 2 solar mass A-star (panels c & d, Browning et al. 2004), averaged over longitude and time. Different curves in panels b, d correspond to different latitudes as indicated.

A study of the redistribution of angular momentum in our convective shells reveals that Reynolds stresses are at the origin of the equatorial acceleration but baroclinicity influences the form of the rotation profile (BT02; MBT06). Recent mean-field models by Rempel (2005) and 3-D simulations by MBT06 suggest that some of this baroclinicity may arise from thermal coupling to the solar tachocline. Thermal gradients in the tachocline may be transmitted to the convection zone by the convective heat flux, promoting a more conical rotation profile through thermal wind balance in better agreement with helioseismic inversions. However convection in rotating system also possesses strong latitudinal (anisotropic) enthalpy (heat) transport that contribute efficiently to latitudinal gradient of entropy and temperature. The near constancy of the isocontours of Ω(r,θ) along radial lines

could be used in turn to assess the radial structure of the tachocline if this boundary layer is assumed to be in strict thermal wind balance (see Brun 2007a). Elucidating the relative roles of convective Reynolds stresses, convective heat flux, and thermal coupling to the tachocline in maintaining the solar differential rotation and meridional circulation remains an area of active study.
In other cool stars the depth of the convection zone and the existence or not of a tachocline (absent for late fully convective M-stars) as well as the rotation rate (slower or faster than the solar rate), yields a large variety of differential rotation profiles. The angular velocity tends to become cylindrical (in agreement with Taylor's constraint of quasi 2-D dynamics, Pedlosky 1987), if the rotation rate becomes too large or the tachocline is not sharp enough to impact the heat redistribution in the convective layer. When considering the dependence of the angular velocity contrast as a function of rotation rate, it is found that the absolute contrast increases with the rotation rate but the relative contrast reduces (Ballot et al. 2007, Brown et al. 2007, 2008). In more massive stars, such as A-stars, a column of fast or slow rotation is found in the convective core depending of the influence of the Coriolis force on the overall dynamics (Browning et al. 2004).

3c) Convective dynamos

The rich display of magnetic activity observed in the solar atmosphere must be maintained by dynamic processes occurring below the surface. Much of the small-scale magnetic flux permeating the solar photosphere is thought to be maintained locally by solar granulation and is quickly replenished on a time scale of a few days (Cattaneo 1999; Schrijver & Title 2003). Turbulent motions in an electrically conducting fluid are known to generate magnetic fields through hydromagnetic dynamo action, sustaining them indefinitely against ohmic decay. However, observed patterns of flux emergence such as the 11-yr sunspot cycle are more enigmatic and although much progress has been made, there are still many open questions about how the global solar dynamo operates. For a thorough discussion see Ossendrijver (2003), Brun et al. (2004), Charbonneau (2005), and Solanki et al (2006), as well as the accompanying scholarpedia articles by Charbonneau and Brandenburg (link to these).

One thing is clear; turbulent convection and differential rotation must play an essential role. Convection generates mean fields directly through dynamo processes and it can convert toroidal flux to poloidal flux through the fragmentation and dispersal of photospheric active regions. We refer to this process generally as the alpha mechanism; within the specific context of mean-field dynamo theory it is known as the alpha-effect (link to Rudiger). Turbulent compressible convection also transports magnetic flux preferentially downward in a phenomenon known as magnetic pumping (Tobias et al 2001; Dorch & Nordlund 2001, Ossendrijver et al 2002; Ziegler & Rudiger 2003). Meanwhile, rotational shear converts poloidal flux to toroidal flux through what is known as the Omega-effect, closing the dynamo loop.

Although there is much debate over precisely where the alpha mechanism occurs, most solar dynamo models place the Omega-effect in or near the solar tachocline (Tobias 2004; Charbonneau 2005). Rotational shear in the tachocline organizes and amplifies toroidal flux until it fragments and rises as a consequence of magnetic buoyancy instabilities (Cline, Brummell, Cattaneo 2003). Rising toroidal flux tubes emerge from the solar photosphere as bipolar active regions. Further hydrodynamical and MHD instabilities seem to play an important role in the dynamical evolution of the inner radiative interior here also with a likely feed back on the surface layers. In so-called flux-transport solar dynamo models, cyclic variability arises from the advection of magnetic flux in the tachocline by an equatorward mean circulation. Similar processes must also occur in other stars although the relative roles of the alpha mechanism, the Omega mechanism, turbulent transport, and meridional circulation may vary. For instance recent work on fastly rotating young suns or solar-like stars tend to indicate that meridional flows are weaker for faster rotation (Ballot et al. 2007, Brown et al. 2007, see however Kuker & Rudiger 2005 for an opposite result using 2-D mean field models). This slower mean MC flow will result in a slower activity cycle if the solar mean field dynamo flux transport framework is left unchanged (Dikpati et al. 2004, Jouve et al. 2007, 2008).

By including a weak seed magnetic field in simulation of turbulent convection, the nonlinear interactions between turbulence, rotation and magnetic fields can be studied in detail. It is found that the magnetic energy (ME) grows by many orders of magnitude through dynamo action if the magnetic Reynolds number ($Rm=vL/\eta$) of the flow is above a critical value (see Gilman 1983, Glatzmaier 1987, Brandenburg et al. 1996, Cattaneo 1999, Brun et al. 2004 (BMT04)). Following the linear phase of exponential growth, ME saturates, due to the nonlinear feed back of the Lorentz forces, to a fraction of the kinetic energy (KE) and retains that level over many Ohmic decay times. Upon saturation, KE has been reduced significantly when compared to the initial hydrodynamical value. In global models, this variation is mostly due to a reduction of the energy contained in the differential rotation. The energy contained in the convective motions is influenced less, which implies an increased contribution of the non-axisymmetric motions to the total kinetic energy balance.

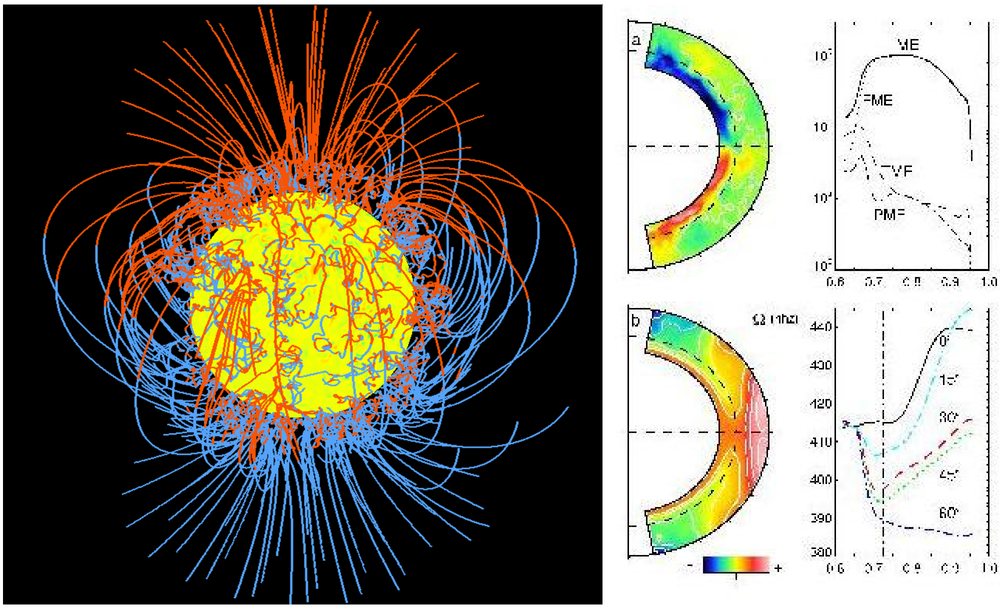

Fig 5: In the left panel we display the magnetic field lines obtained in a model of the magnetized solar convection with potential extrapolation in the solar corona (Brun et al. 2004, red field lines are pointing radially outward). In the right panel we show (top) the mean toroidal field and (bottom) the angular velocity in a simulation of solar convection with a tachocline of rotational shear (Browning et al. 2006).

The radial magnetic field generated through dynamo action is found to be concentrated in the cold downflow lanes, with both polarities coexisting having been swept there by the horizontal diverging motions at the top of the domain. The Lorentz forces in such localised regions have a noticeable dynamical effect on the flow, with ME sometimes being locally bigger than KE, influencing the evolution of the strong downflow lanes via magnetic tension that inhibits vorticity generation and reduces the shear. The magnetic field and the radial velocity possess a high level of intermittency both in time and space, revealed by extended, asymmetric wings in their probability distribution functions (Brandenburg et al. 1996, BMT04). Fast reversals of the poloidal field are observed (~ 400 days) in global 3-D models which are typical of a chaotic dynamical system but are inconsistent with the observed 11-yr solar cycle. In an attempt to resolve that issue Browning et al. (2006) have recently included a stably stratified tachocline of shear in a global magnetic simulation. They confirm through nonlinear simulations that the tachocline plays a crucial role in organising the irregular field produced by the convection zone into intense axisymmetric toroidal structures. The presence of this large scale mean field does seem to influence the nonlinear behaviour of the simulations leading to much less frequent if any magnetic field reversals or excursions.

With fairly strong magnetic fields sustained in the global magnetic simulations, it is to be expected that the differential rotation $\Omega(r,\theta)$ established in the purely hydrodynamical case will respond to the feedback from the Lorentz forces. Indeed Brun (2004) found that the main effect of the Lorentz forces is to extract energy from the differential rotation as the weakening of KE indicates. A careful study of the redistribution of the angular momentum in the shell reveals that the source of the reduction of the latitudinal contrast of $\Omega(r,\theta)$ can be attributed to the poleward transport of angular momentum by Maxwell stresses (see Brun 2004, BMT04).
The large-scale magnetic torques are found to be 2 orders of magnitude smaller, confirming the small dynamical role played by the mean fields in global MHD simulations without a tachocline of shear and a self-established 11-yr cycle.

In other stars, again rotation plays an important role in determining the global properties of their magnetism. In fast rotating solar-type stars, large mean fields are found even without the presence of a tachocline at the base of their convective envelope (Browning et al. 2007b). This is due to a shift in the balance of forces driving the flow between the advection, Coriolis and Lorentz terms. As the rotation rate increases the Lorentz force tends to balance the Coriolis force yielding larger magnetic energy in superequipartion with the kinetic energy of the flow (as in the Earth's iron core, see Rudiger & Hollerbach 2004).
In more massive stars, convective core dynamos are found to be extremely effective. Mega-gauss fields are found in the core at its sheared boundary layer (a kind of "upside down" tachocline, Brun et al. 2005). Such fields must interact with the probable fossil field that the extended radiative envelope of the hot star retained during the star's formation. Recent simulations seem to indicate that dynamo action in such cores will be even more vigorous if the fossil field threads from the stable envelope through to the convective core (Featherstone et al. 2007).

We have shown that numerical simulations of the complex internal solar and stellar magnetohydrodynamics are becoming more and more tractable with today's supercomputers. In particular we have studied how turbulent convection under the influence of rotation can establish a strong differential rotation and weak meridional circulation, generate magnetic fields through dynamo action and how Lorentz forces act to diminish the differential rotation such that poleward angular momentum transport by Maxwell stresses opposes the equatorward transport by Reynolds stresses (see BT02, BMT04, MBT06). Many challenges remain, among them the understanding of the 11-yr solar cycle and of the two shear layers present at the base (the tachocline) and at the top of the solar convection zone, or magnetic coupling to the solar atmosphere is a priority since these layers are directly linked to the solar dynamo and subsurface weather. Another challenge is to get a more accurate and deeper inversion of the meridional circulation present in the solar convection since it plays a crucial role in current mean field solar dynamo models (Dikpati et al. 2004, Jouve & Brun 2007a). Another key element of the solar dynamo is the emergence of magnetic flux from deep within the Sun up to its surface (Jouve & Brun 2007b, Fan 2008). For other stars the relative ordering of the convective and radiative zones, the extent of the convection, the presence of a tachocline of shear, the rotation rate all plays crucial roles in determining the convective patterns, the large scale flows (differential rotation, meridional circulation) and the level of magnetism (Brun et al. 2005, Dobler et al. 2006, Brown et al. 2007b, Featherstone et al. 2007, Browning 2008).

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