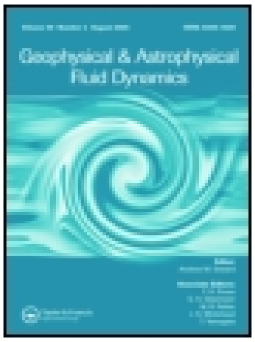

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MAGNETIC FIELD STRUCTURE IN DIFFERENTIALLY ROTATING DISCS

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In order to gain a better understanding of the processes that may give rise to non-axisymmetric magnetic fields in galaxies, we have calculated field decay rates for models with a realistic galactic rotation curve and including the effects of a locally enhanced turbulent magnetic diffusivity within the disc. In all cases we have studied, the differential rotation increases the decay rate of non-axisymmetric modes, whereas axisymmetric ones are unaffected. A stronger magnetic diffusivity inside the disc does not lead to a significant preference for non-axisymmetric modes. Although Elsasser's antidynamo theorem has not yet been proved for the present case of a non-spherical distribution of the magnetic diffusivity, we do not find any evidence for the theorem not to be valid in general.

KEY WORDS: Galactic magnetic fields, hydromagnetics, disc dynamos, Elsasser theorem.

1. INTRODUCTION

During the last decade observational techniques for magnetic field measurements in galaxies have been improved considerably. The observations seem to indicate that the magnetic field of an appreciable fraction of all spiral galaxies with a global spiral pattern is of a bisymmetric nature, that is, the magnetic field is pointing in opposite directions in alternate spiral arms (Sofue *et al.*, 1988; Beck, 1990). These observations pose a considerable challenge to dynamo theories to find the origin of the galactic magnetic field. In fact, in all dynamo models known to us the fastest growing (or most slowly decaying) mode is axisymmetric (Brandenburg *et al.*, 1990). One purpose of our work is to see how realistic models for gas motions in galaxies will affect the growth or decay of galactic fields.

This work was inspired by a study by Skaley (1985), who found dynamo action already due to the combined effect of a non-spherically symmetric distribution of the magnetic diffusivity η and a differential rotation. This is a special case for which Elsasser's anti-dynamo theorem (1946) does *not* apply. This theorem says that a magnetic field cannot be maintained by a purely toroidal flow. However, it has been proved so far only for the case of a constant or, at most, for a spherically symmetric distribution of η. The situation for a latitude-dependent η is therefore open, as was earlier pointed out by Rädler (1983, p. 30). If Skaley's "$\eta\omega$-mechanism" works then it can provide a way for explaining the non-axisymmetric magnetic fields in galaxies. Whether the galactic magnetic field is axisymmetric or bisymmetric would depend on the relative importance of the $\alpha\omega$- and $\eta\omega$-effects.

The analytical models of the disc dynamo (Ruzmaikin *et al.*, 1985; Baryshnikova *et al.*, 1987) assume a thin disc and a tightly wound spiral field. Since the field is essentially dependent on global boundary conditions, asymptotic approximations leading to a local equation for the field have to be treated with some caution, as has been stressed by Krause (1990) and Rädler and Wiedemann (1990). In our models such restrictions will not be made and more open spiral patterns are also allowed.

2. BASIC MODEL

We shall take as our basic model of the galaxy a turbulent differentially rotating gas disc embedded in a spherical halo. Since the mean density of ionized gas is probably lower in the halo than in the disc we assume generally a small conductivity in the halo, that is, the turbulent magnetic diffusivity is stronger in the halo than in the disc. On the other hand the turbulence in the disc may also give rise to an enhanced effective magnetic diffusivity in the disc. Because of this uncertainty we shall also consider the possibility of a highly conducting halo. A similar model has been used by Stepinski and Levy (1988) for accretion discs. However, these authors only studied axisymmetric modes.

In the following we shall use spherical coordinates (r, θ, ϕ), because well tested numerical procedures can be adopted in this case. In particular the boundary conditions can be easily formulated on a sphere $r = L$ such that the magnetic field matches to a potential field in the outer space $r > L$. In our calculations we have also taken L to be the radius of the disc. The idea of avoiding problems with the boundary conditions by embedding a dynamo in a conducting domain has been previously proposed by Elstner *et al.* (1990).

The rotation curve will be assumed to be of the Brandt (1960) form, modified to give constant rotational velocity at large radii:

$$v = v_0 \frac{r}{r_0} \left[1 + \left(\frac{r}{r_0} \right)^n \right]^{-1/n}, \tag{1}$$

where v_0 is the value of the circular velocity v at large radii, r_0 is roughly the radius at which differential rotation begins, and n describes how rapid is the transition from rigid body to differential rotation. Mostly we shall take $n = 2$. For simplicity, in this paper we have assumed that the angular velocity $\omega = v/r \sin\theta$ is constant on spherical shells. Most of the observed rotation curves (see Rubin *et al.*, 1985 and references therein) can be approximately represented by (1) by varying v_0 and r_0.

Equation (1) may give a better representation of actual rotation curves if r is replaced by the distance from the axis, $r \sin\theta$. In our formalism this would give rise to further terms in the expansion of ω, but we shall neglect these in the present paper.

We shall take a turbulent magnetic diffusivity η of the form

$$\eta=\eta_h-\eta_d, \tag{2}$$

where the halo diffusivity η_h is assumed constant and the disc diffusivity η_d is chosen to be

$$\eta_d=\eta_0\begin{cases}1-\left(\dfrac{z}{z_0}\right)^2 & \text{if } |z|\leqq z_0\\ 0 & \text{if } |z|\geqq z_0,\end{cases} \tag{3}$$

where $z=r\cos\theta$ is the coordinate in the direction parallel to the rotation axis. In the present calculations the half-thickness of the disc, z_0, was 0.2 times the disc radius.

Little is known about the thickness of the ionized gas layer in real galaxies. A higher conductivity inside the disc will correspond to $\eta_0>0$. The representation (3) of the disc diffusivity is merely a first guess; we intend to try other functions in the future.

The relative strength of the differential rotation against magnetic diffusion is measured by the Reynolds number, which we define as

$$R_m=v_0L/\eta_h. \tag{4}$$

No α-effect has been included in the calculations reported in this paper. This is because we wanted to see how the free decay modes of the field are affected by galactic kinematics, without introducing the additional uncertainty of some specific model for the α-effect. Results with a flat distribution of α but with a spherically symmetric η are given in Brandenburg *et al.* (1990).

3. NUMERICAL METHOD

The basic equation governing the time development of the mean magnetic field is the induction equation

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times(\mathbf{u}\times\mathbf{B}-\eta\nabla\times\mathbf{B}), \tag{5}$$

where $\mathbf{u}=\hat{\boldsymbol{\phi}}v$.

Equation (5) poses an eigenvalue problem for the complex eigenvalue λ, where $\mathcal{R}e\,\lambda$ is the growth rate and $\mathcal{I}m\,\lambda$ the eigenfrequency. We determine λ using the so-called Bullard–Gellman formalism (see e.g. Roberts and Stix, 1972). The field is divided into a poloidal and a toroidal part according to

Table 1 The results for the functions $d_l(r)$ for $l=0$, 2, and 4. The abbreviation $\zeta \equiv z_0/r$ is used

	$r \leqq z_0$	$r > z_0$
d_0	$1-\frac{1}{3}\zeta^{-2}$	$\frac{2}{3}\zeta$
d_2	$-\frac{2}{3}\zeta^{-2}$	$\frac{1}{3}\zeta[3\zeta^2-5]$
d_4	0	$\frac{9}{4}\zeta[\zeta^4-2\zeta^2+1]$

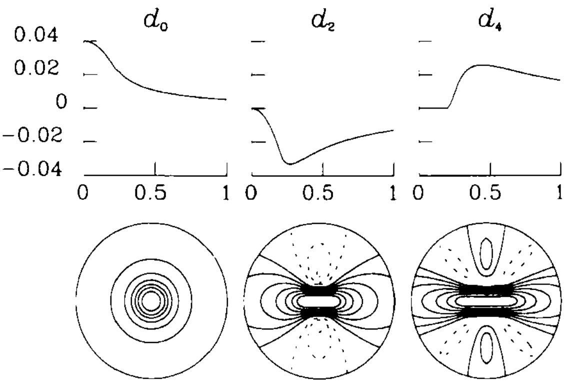

Figure 1 *Above*: Radial dependence of the three first coefficients in the diffusivity expansion (7). *Below*: Diffusivity distribution in a meridional plane when contributions up to the given order are included.

$$\mathbf{B}=\nabla\times\nabla\times(\hat{\mathbf{r}}S)+\nabla\times(\hat{\mathbf{r}}T). \tag{6}$$

The defining scalars S and T are then expanded in terms of spherical harmonics. Using the orthogonality of the different (l,m) components the diffusion equation reduces to an infinite set of ordinary differential equations for the functions S_l^m and T_l^m. This is solved numerically by truncating the system to N_l equations using N_r grid points in the radial direction. For further details on the numerical treatment of the resulting eigenvalue problem, see Roberts and Stix (1972) and Brandenburg *et al.* (1989).

In our scheme the diffusivity has to be expanded in terms of spherical harmonics. We have included the three first non-zero terms in the expansion of (3) in terms of Legendre polynomials:

$$\eta_d=\eta_0[d_0(r)P_0(\cos\theta)+d_2(r)P_2(\cos\theta)+d_4(r)P_4(\cos\theta)]. \tag{7}$$

Explicit expressions for the coefficients d_l obtained from (3) be setting $z=r\cos\theta$ and expanding are given in Table 1. Contours of constant η for the resulting expansion (7) are plotted in Figure 1.

4. RESULTS

The most interesting eigenmodes are the symmetric and antisymmetric ones with $m=1$, S1 and A1. The modes are all decaying. Figures 2–5 show how the magnetic field structure is affected by varying the relevant parameters. In Figure 3 an example of an A1 mode is shown. By definition the antisymmetric modes have the property that the field is vertical in the disc plane. They are therefore unlikely to be relevant for galaxies, where the field lines are generally parallel to the plane.

The differential rotation influences the field in two ways. First, the shear generates a toroidal field from a poloidal one. Thus the field strength will be large where the shear is largest, i.e. around the radius r_0. Secondly, differential rotation will draw the frozen-in field into a spiral shape. Strong differential rotation will give rise to a more tightly wound spiral (compare Figures 2 and 4). To get a pitch angle in agreement with observations requires a magnetic Reynolds number of a few hundred.

The effect of the diffusivity is to destroy the field. If the diffusivity is lower in the disc, the field will be strongest in the disc. On the other hand, if there is a quiescent, high-conductivity halo, the field expelled from the disc is concentrated in the halo. These qualitative expectations are confirmed by our calculations (see Figure 5).

From the viewpoint of applications to real galaxies the most interesting parameter is the magnetic Reynolds number. We have seen that R_m has to be of the order of a few hundred to give a similar spiral pitch angle as is observed. A very rough estimate of the turbulent magnetic diffusivity would be $\eta \approx az_0$, where a is the velocity dispersion in the gas. The magnetic Reynolds number is then

$$R_m = \frac{v_0 L}{a z_0}, \tag{8}$$

with $v_0 = 200\,\mathrm{km s^{-1}}$, $a = 7\,\mathrm{km s^{-1}}$, $L = 10\,\mathrm{kpc}$ and $z_0 = 1\,\mathrm{kpc}$, we obtain $R_m = 300$. Thus reasonable galactic parameters lead to a Reynolds number in the interesting range.

Let us finally note that the decay time of the most long-lived mode is about 0.1 T_d, where $T_d = L^2/\eta_h$ is the diffusion timescale. In terms of the galactic crossing time T_c this is $0.1 \times R_m T_c$, i.e. 10 T_c for $R_m = 100$. Our modes are thus fairly long-lived on a dynamical time scale.

Turning now to the question of preference for bisymmetric modes we may take as our typical case $R_m = 100$, $\eta_0/\eta_h = 0.75$ and $r_0 = 0.3\,L$. The most long-lived bisymmetric modes then have the decay rates -11.5 (S1) and -18.4 (A1), whereas the decay rates for the corresponding axisymmetric modes are -6.9 (A0) and -15.3 (S0). We therefore should be looking for circumstances where the S1 mode decays more slowly than the A0 mode.

In Tables 2 and 3 we give the decay rate and frequency for S1 modes as a function of Reynolds number R_m and of disc scale r_0. The decay rate of A0 is unaffected by rotation. Clearly, varying the rotation curve does not lead to any preference for the bisymmetric modes.

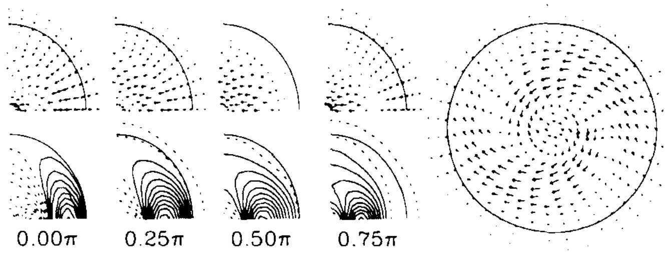

Figure 2 Field configuration for the first S1 mode. The parameter values are $\eta_0/\eta_h = 0.75$, $R_m = 100$, $r_0 = 0.3L$. *Left upper row*: projection of field vectors onto a meridional plane at different phases. *Lower row*: Contours of constant azimuthal field in a meridional plane for the same phases as in the upper row. Dotted lines denote negative values. *Right*: Field vectors in the plane of the disc. The vertical field component is zero in the disc plane.

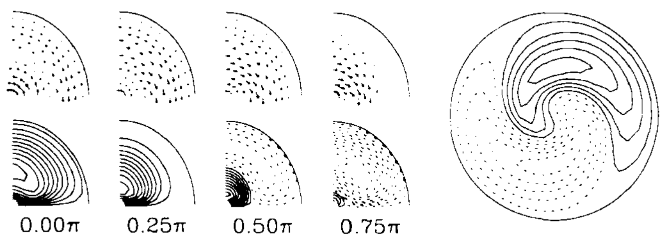

Figure 3 Field configuration for the first A1 mode. On the right are shown level contours of the vertical field component in the disc plane. The radial and azimuthal fields are zero in the plane. Otherwise the same as Figure 2.

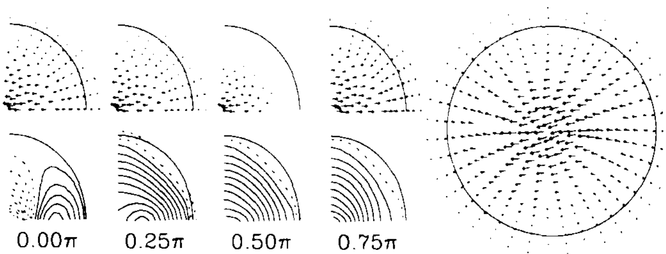

Figure 4 Same as Figure 2 but for $R_m = 10$.

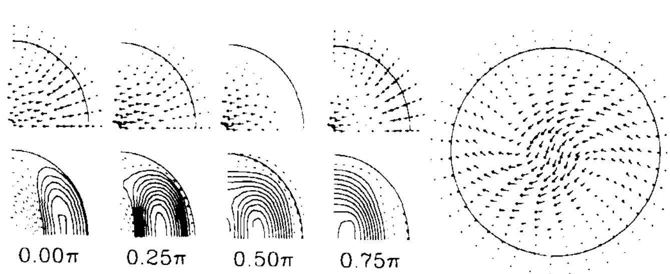

Figure 5 Same as Figure 2 but with the diffusivity in the disc larger than in the corona, $\eta_0/\eta_h = -0.75$.

Table 2 Eigenvalues of the first S1 mode for different Reynolds numbers. λ is measured in units of η_h/L^2. $\eta_0/\eta_h=0.75$ and $r_0=0.3L$. The decay rate of the first A0 mode is -7.2

R_m	$-\mathcal{R}e\,\lambda$	$-\mathcal{I}m\,\lambda$
0	8.6	0
10	8.7	4.0
10^2	11.5	37.3
10^3	30	327

Table 3 Eigenvalues for the first S1 mode for different values of r_0. $\eta_0/\eta_h=0.75$, $R_m=100$. λ is measured in units of η_h/L^2

r_0/L	$-\mathcal{R}e\,\lambda$	$-\mathcal{I}m\,\lambda$
0.1	9.5	14
0.3	11.5	37
0.5	12.1	55
0.7	11.7	68
0.9	10.9	77

Table 4 Eigenvalues for the first S1 and A0 modes for different values of the disc strength. The Reynolds number based on the diffusivity at the centre is constant$=400$. $r_0=0.3L$. λ is measured in units of η_h/L^2

η_0/η_h	*S1*		*A0*		$\mathcal{R}e\,\lambda(S1)/\mathcal{R}e\,\lambda(A0)$
	$-\mathcal{R}e\,\lambda$	$-\mathcal{I}m\,\lambda$	$-\mathcal{R}e\,\lambda$	$-\mathcal{I}m\,\lambda$	
0.75	46	150	28	0	1.77
0.8	54	151	33	0	1.51
0.9	92	157	59	0	1.42
0.95	169	160	109	0	1.40

The effect of the disc diffusivity on the eigenvalues for A0 and S1 modes is shown in Table 4. The decay rates of both A0 and S1 are enhanced for larger η_0, but the ratio $\mathcal{R}e\,\lambda(\mathrm{S1})/\mathcal{R}e\,\lambda(\mathrm{A0})$ becomes smaller as η_0/η_h increases. However, the effect is not large enough to explain the observations.

There are of course many further modifications of the turbulent viscosity that might favour bisymmetric fields. We are currently pursuing some of these.

It should be noted that the results are not in agreement with what we have expected from the studies by Skaley (1985). We do not find any evidence for the possibility of an "$\eta\omega$-dynamo mechanism". Therefore we have made also a direct comparison using for η and ω the profiles of Skaley ($\eta=r^2[1+\varepsilon P_2(\cos\theta)]$ and $\omega=10R_m(1-r^2)$). The results, documented in Table 5, show that (i) a flatter

Table 5 The growth rates and eigenfrequencies for the A1-mode using for η and ω the profiles of Skaley. λ is measured in units of η_h/L^2. The level of truncation N_r and N_l is included as well

ε	R_m	$-\mathcal{R}e\,\lambda$	$-\mathcal{I}m\,\lambda$	N_r	N_l
0	0	20.2	0	50	2
0	10	29.9	33.0	50	2
1	0	17.4	0	30	10
1	10	29.94	59.7	30	10
1	10	29.92	59.7	30	6
1	10	30.0	59.7	20	6
2	0	19.5	0	30	10
2	10	24.8	33.5	30	12
2	10	24.5	35.0	30	10
2	10	24.7	40.7	30	6

distribution of η (larger values of ε) decreases the decay, but (ii) even a weak differential rotation (small values of R_m) gives rise to an enhanced dissipation. The last property is explained by a general argument by Rädler (1986), who gave an estimate for the increase of the negative growth rate with the magnetic Reynolds number: $-\mathcal{R}e\,\lambda \propto R_m^{2/3}$. This relation is well confirmed by the calculations of Rädler *et al.* (1989), but it is not in agreement with the findings of Skaley.

5. CONCLUSIONS

The important result of this paper is that there is no evidence for a generation of non-axisymmetric magnetic fields by the combined effects of differential rotation and a non-radial distribution of magnetic diffusivity. The results are, of course, only preliminary, since we have not covered the full range of diffusivity distributions that could occur. Furthermore only a single rotation profile has been studied so far. Nevertheless, clearly the mechanism envisaged in the present paper does not look very promising for explaining the bisymmetric magnetic field configurations observed in some galaxies. It seems therefore that Elsasser's antidynamo theorem may be valid also in the case of a general (non-spherical) distribution of the magnetic diffusivity.

Typical galactic rotation curves do not seriously impair the chance of survival of non-axisymmetric fields. The field structures we have obtained are qualitatively similar to those observed. Although all our modes are decaying, the inclusion of an α-term could produce steady or growing modes without significantly altering the field structure (cf. Brandenburg, 1990). It should also be stressed that the decay times of bisymmetric fields are fairly large on a dynamical time scale. Thus, if they can be generated e.g. in tidal interactions or galaxy mergers, they might survive when all dynamical traces of the disturbance have disappeared. More detailed comparisons with observed galactic fields might therfore be worthwhile, but this will require a more careful consideration of the turbulent gas motions in different types of galaxies.

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