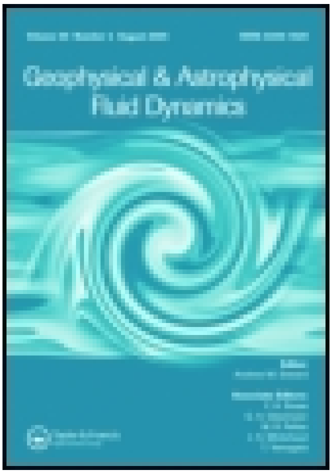

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The generation of nonaxisymmetric magnetic fields in the giant planets

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THE GENERATION OF NONAXISYMMETRIC MAGNETIC FIELDS IN THE GIANT PLANETS

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We consider mean-field dynamo models in uniformly rotating spheres and spherical shells, with anisotropic alpha and magnetic diffusivity tensors which are functions of the inverse Rossby number, Ω^*. When we include an α-quenching nonlinearity we show that, for all values of Ω^* considered, nonaxisymmetric magnetic fields are stable to axisymmetric perturbations. However the stability of nonaxisymmetric magnetic fields is weakened for large values of Ω^*, which would make the generation of axisymmetric magnetic fields more probable. When a small amount of differential rotation is introduced, only axisymmetric dipole-like solutions are stable. We draw attention to the possibility that the nonaxisymmetric magnetic fields of Uranus and Neptune could be the result of anisotropic alpha and magnetic diffusivity tensors. The more nearly axisymmetric magnetic fields of Jupiter and Saturn could result from their more rapid rotation, or possibly because of internal differential rotation.

KEY WORDS: MHD, magnetic fields, dynamo theory, giant planets.

1. INTRODUCTION

The dipole moments of the magnetic fields of Jupiter and Saturn are quite closely aligned with the rotation axes, in contrast to the magnetic fields of Uranus and Neptune which are highly nonaxisymmetric with dipole moments almost perpendicular to the rotation axes. The origin of this difference is very puzzling. Possible explanations include that Uranus and Neptune just happen to be in a state of field reversal, similar to the reversals found in geological records of the earth's magnetic field (Schultz and Paulikas, 1990; Rädler and Ness, 1990), or that the types of dynamos that operate in Uranus and Neptune are fundamentally different from those of Jupiter and Saturn (Connerney *et al.*, 1991), in that the fields of Uranus and Neptune are generated close to the surfaces of these planets, whereas the fields of Jupiter and Saturn are generated in deeper layers. For example, Stevenson (1982) proposed that the very high

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degree of axisymmetry of Saturn is related to the presence of significant differential rotation (see also Charbonneau and Bagenal, 1995), which may be absent or less strong in the other planets. For a brief overview of these matters see the first section of Bagenal (1992). Ruzmaikin and Starchenko (1991) suggested that in Uranus and Neptune, dynamo action occurs in a thin shell at the bottom of the ocean of water, liquid methane and ammonia, where the electrical conductivity is high. They argue that both axisymmetric and nonaxisymmetric fields are excited, and that their superposition leads to a field with an inclination angle close to the observed value (58° for Uranus and 47° for Neptune).

The generation of large scale magnetic fields in planets is generally explained in terms of dynamo theory. (Primordial magnetic fields will have decayed because of Ohmic dissipation.) However, from Cowling's (1934) 'anti-dynamo' theorem it is clear that the generated magnetic fields have to be nonaxisymmetric at some scale. Another complication is that a large range of length scales is involved. In order to make this fully three-dimensional, multi-scale problem tractable, statistical or mean-field theories have been developed. These theories permit solutions for the mean magnetic field by introducing statistical or in some cases spatial and/or temporal averages (Braginsky, 1964; Steenbeck *et al.*, 1966). They have in common that the effects of small scale magnetic and velocity fields are subsumed in the so-called α-effect (due to rotation and the resulting lack of mirror symmetry) and a small scale (or turbulent) magnetic diffusivity (Moffatt, 1978; Parker, 1979; Krause and Rädler, 1980; Zeldovich *et al.*, 1983). Cowling's theorem is satisfied by the intrinsic lack of axisymmetry of the small scale fields, and the resulting mean magnetic fields are usually axisymmetric, but nonaxisymmetric mean fields are also possible (provided, of course, that one does not average over azimuth). However, nonaxisymmetric fields are generally less easily excited, especially when differential rotation is present (Rädler, 1986a).

In the framework of mean-field dynamo theory there are two important mechanisms that can be responsible for a preference for nonaxisymmetric magnetic fields. These are anisotropy of the α-effect (Rüdiger, 1980; Rüdiger and Elstner, 1994), and the interaction between self-consistently generated large scale motions and the magnetic field (Barker and Moss, 1993, 1994). It is natural that the two effects together may operate in spherical bodies like planets and stars. Under special conditions, when the two induction effects (α-effect and differential rotation) operate in separate layers, there is a narrow strip in parameter space where a weak differential rotation can also lead to a preference of nonaxisymmetric modes (Roberts and Stix, 1972; Moss *et al.*, 1991). The possibility of the dynamo active layers being very thin (Ruzmaikin and Starchenko, 1991) does bring the excitation conditions for axisymmetric and nonaxisymmetric modes closer together (Brandenburg *et al.*, 1989a), but this effect alone does not lead to a preference for nonaxisymmetric modes. The lifetimes of planetary magnetic fields are long compared to their turbulent diffusion times, and therefore transient magnetic fields will hardly survive. This means that, in the nonlinear regime, we are mostly interested in stable solutions. It is important to note that nonlinear stability cannot be predicted from linear theory (Brandenburg *et al.*, 1989b; Jennings and Weiss, 1991): in particular, a superposition of linear solutions is then meaningless. Thus, it appears to us that we are still far away from a firmly based understanding of the origin of the nonaxisymmetric magnetic fields present in some of the giant planets. One potentially important factor, which has not yet been emphasized in this context, is the possibility of anisotropies in the α-effect, i.e. that α is really a tensor.

Thus, in the present paper we focus attention on the anisotropy of the α-effect, and assume that the effects of large scale flows are negligible. Our motivation is two-fold. Firstly, a recent analysis by Rüdiger and Kitchatinov (1993) yields, under certain approximations and restrictions, the complete expressions for α_{ij} and η_{ijk} (the α-effect and diffusion tensors) for arbitrary rotation rates. However, a systematic parameter study for spherical, nonlinear models is still lacking. Secondly, the rotation periods of the outer planets, $2\pi/\Omega$, are very short compared to the turbulent correlation times, $\tau_{\rm corr}$. This plausibly leads to model independent features, including a high anisotropy of the α-effect with a very small component along the direction of rotation, and an enhanced small scale magnetic diffusivity along the direction of rotation. This is reflected in the expressions for α_{ij} and η_{ijk} given by Rüdiger and Kitchatinov (1993), which we apply below to the giant planets. Although the detailed tensorial structure may still be uncertain, we feel that by adopting such expressions for α_{ij} and η_{ijk} we, at least, introduce some of the major effects resulting from the inherent anisotropy of the small scale or turbulent motions.

In the present paper we study a sequence of dynamo models in spheres with different inverse Rossby numbers, $\Omega^* = 2\Omega\tau_{\rm corr}$, where $\tau_{\rm corr}$ is a correlation time of the small scale motions, and compute the magnetic field evolution for certain initial conditions. Physically, the parameter Ω^* measures the tendency of the Coriolis force to introduce anisotropy in otherwise isotropic turbulence. In order to isolate the effects of large inverse Rossby numbers, we ignore the details of the internal structure of the giant planets. However, in some cases we do go a step further by also considering dynamo action in a spherical shell, which may be more appropriate for Uranus and Neptune. We show that the basic consequences of the anisotropies of α_{ij} and η_{ijk} are indeed also present when the dynamo action is confined to a shell. In most of these calculations we assume the rotation to be uniform, i.e. we neglect differential rotation, but we present one calculation in which a modest differential rotation has been included.

2. THE MODEL

The evolution of the mean magnetic field $\langle \mathbf{B} \rangle$ is governed by the dynamo equation

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{u} \rangle \times \langle \mathbf{B} \rangle + \mathcal{E} - \eta \nabla \times \langle \mathbf{B} \rangle), \tag{1}$$

where $\langle \mathbf{u} \rangle$ is the mean velocity, $\mathcal{E}$ is the turbulent electromotive force resulting from the small scale motions (Krause and Rädler, 1980), and η the microscopic magnetic diffusivity (inversely proportional to the electrical conductivity). We solve (1) inside a sphere of radius R, and match the solution to an external potential field that vanishes at infinity. As usual, we express the turbulent electromotive force as $\mathcal{E}_i = \alpha_{ij}\langle B_j \rangle + \eta_{ijk}\langle B_{j,k} \rangle$. We adopt the expressions for the tensors α_{ij} and η_{ijk} that have been derived by Rüdiger and Kitchatinov (1993), and are summarized in the paper by Rüdiger and Brandenburg (1995) in the context of a spherical model. In their model the anisotropy results primarily from rotation via the Coriolis force, but also from the vertical stratification. The inverse Rossby number, Ω^*, is the governing parameter determining the strength of the α-effect, and we shall use Ω^* as the bifurcation parameter. The tensor structure of α_{ij} and η_{ijk} is given by

$$\alpha_{ij}=\tfrac{3}{4}\Omega^*[-\psi(\hat{z}_k U_k)\delta_{ij}+\psi_{\omega}(\hat{z}_i U_j+\hat{z}_j U_i)+(2\psi_z+\psi-2\psi_{\omega})(\hat{z}_k U_k)\hat{z}_i\hat{z}_j]$$

$$-\varepsilon_{ikl}U_k[\phi_1\delta_{jl}+(\phi_2-\phi_1)\hat{z}_j\hat{z}_l], \tag{2}$$

$$\eta_{ijk}=(\phi_1+\phi_2)\eta_0\varepsilon_{ijk}+(\phi_1-\phi_2)\eta_0\varepsilon_{ijl}\hat{z}_l\hat{z}_k, \tag{3}$$

where $\psi, \psi_{\omega}, \psi_z, \phi_1, \phi_2$ are known functions of Ω^* (cf. Rüdiger and Brandenburg, 1995). (For illustration, when $\Omega^*=40$, the numerical values of $\psi, \psi_{\omega}, \psi_z, \phi_1$ and ϕ_2 are respectively 3.9×10^{-2}, 6.9×10^{-5}, -2.2×10^{-5}, 2.9×10^{-2}, 9.0×10^{-4}. These numbers show that the dominant contribution comes from ψ and ϕ_1.) For $\Omega^*\gg1$, we have $\psi\approx\pi/(2\Omega^*)$ and $\phi_1\approx3\pi/(8\Omega^*)$, and the expressions (2) and (3) then take the approximate forms

$$\alpha_{ij}=-\frac{3\pi}{8}(\hat{z}_k U_k)(\delta_{ij}-\hat{z}_i\hat{z}_j)-\frac{3\pi}{8\Omega^*}\varepsilon_{ikl}U_k(\delta_{jl}-\hat{z}_j\hat{z}_l), \tag{4}$$

$$\eta_{ijk}=\frac{3\pi}{8\Omega^*}\eta_0(\varepsilon_{ijk}+\varepsilon_{ijl}\hat{z}_l\hat{z}_k). \tag{5}$$

In these formulae, $\hat{\mathbf{z}}$ is the unit vector in the direction along the rotation axis, and $\mathbf{U}=\nabla\eta_0$ is the gradient of turbulent intensity, which is measured by the basic value of the turbulent diffusivity η_1; see below. The simplified expressions (4) and (5) show that the z-component of the α tensor tends to vanish almost completely, while the turbulent diffusion along z can be almost doubled, compared with the value in the absence of rotation. These features appear to be robust and model independent. [For the actual computations we still used the full expressions (2) and (3).] In our model, the α-effect is due to the stratification in the turbulent intensity u_t, which is quantified in terms of a reference value of the turbulent magnetic diffusivity, $\eta_1=\frac{1}{3}u_t^2\tau_{\mathrm{corr}}$, assumed to be much larger than the microscopic value. Somewhat arbitrarily, we adopt a profile for η_0 with a constant gradient, $\eta_0(r)=\eta_1(1+r/R)$, where R is the radius of the sphere, and use spherical polar coordinates (r,θ,ϕ). The components of the α-effect are then proportional to $U_k=\hat{r}_k\partial\eta_0/\partial r=\hat{r}_k\eta_1/R$. We introduce α-quenching as the nonlinearity that causes the magnetic field to saturate, multiplying ψ, ψ_{ω} and ψ_z by $1/[1+\alpha_B\langle\mathbf{B}(r,\theta,\phi,t)\rangle^2]$, with $\alpha_B=1$ defining our field scaling. The effect of the Lorentz force on the large scale motions would also provide a back reaction on the magnetic field via the Taylor constraint (e.g. Kirk and Stevenson, 1987), but this is ignored in the present paper.

The computational method and the code adopted are described by Barker and Moss (1994). We use second order finite differences in the meridional plane and spectral derivatives in the azimuthal direction. The equations are advanced in time using a modified DuFort-Frankel scheme, which is of second order and unconditionally stable with respect to the diffusion operator. The interior field fits on to a potential field in $r > R$. At the centre, $r = 0$, regularity conditions are imposed. In the one case where we solve the equations in a shell, $0.5 \leq r/R \leq 1$, we use a perfect conductor boundary condition at $r = 0.5\,R$.

In most of the cases described below, the number of mesh points was 31×61 in $0 \leq r/R \leq 1$, $0 \leq \theta \leq \pi$, and the number of collocation points in the ϕ-direction was 4. We did check that the computation at $\Omega^* = 200$ was satisfactorily reproduced at a resolution of 61×121. Based on earlier comparisons we feel that the relatively low number of collocation points is adequate, even for $\Omega^* = 200$. It is indeed quite typical for nonaxisymmetric mean-field dynamo models that the energy contained in the higher Fourier modes decays rather rapidly (see also Rädler *et al.*, 1990).

3. CRITICAL VALUES OF Ω^*

We begin our investigation by determining the critical values of Ω^*, above which linear modes of different magnetic field geometry become excited. We consider dynamo action both in a full sphere as well as in a spherical shell with an inner radius of $0.5\,R$. The basic modes are the axisymmetric solutions that are either symmetric or antisymmetric with respect to the equatorial plane, S0 and A0, respectively, and the corresponding nonaxisymmetric counterparts with azimuthal dependence $e^{im\phi}$ with $m = 1$: S1 and A1, respectively. The results for the critical values $\Omega^*_{\rm crit}$ for the onset of dynamo action are given in Table 1. (A study of somewhat similar solutions, for nonlinear and anisotropic α-effect, but isotropic and uniform η, has been performed by Rädler *et al.* (1990), where further properties of those different types of solutions are also discussed.)

For each of the two models described above, the S1 mode is the easiest to excite. The topology of this solution is that of a dipole lying in the equatorial plane. For the complete sphere, the mode next excited is A1, followed by S0 and finally A0. In the case of a spherical shell the critical values for S0 and A0 are indistinguishable, which is a well known property of shell dynamos and results from the close similarity of the two field geometries. Note also that the critical values of Ω^* are rather large (of the order of 30). In other cases studied previously, where differential rotation also contributes to the magnetic field generation, the critical values are between 1 and 5.

Table 1 Critical values $\Omega^*_{\rm crit}$ for the onset of dynamo action

Mode	A0	S0	A1	S1
Full sphere	37.0	36.4	33.1	**27.9**
Spherical shell	35.0	35.0	35.8	**32.3**

4. NONLINEAR MODELS

We now turn to the investigation of a sequence of nonlinear models in a complete sphere as a function of Ω^*. In all cases investigated ($\Omega^* \leq 200$), we find that the stable solutions have S1 topology (Figures 1 and 2). There are marked deviations from a simple perpendicular dipole type configuration, with significant contributions from higher multipoles leading to a more complicated latitudinal structure. However, since the solution is matched to an external potential field, higher spherical harmonics decay more rapidly with radius than the lowest dipole mode. Thus, the field visible from a space craft would still resemble that of a perpendicular dipole; see Figure 3.

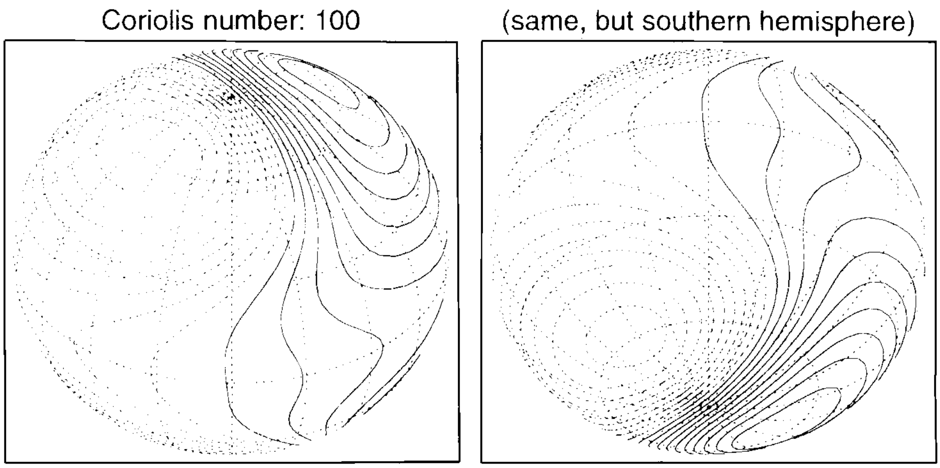

Figure 1 Contours of the radial magnetic field component on the surface of the sphere. Dotted contours refer to negative values. $\Omega^* = 100$.

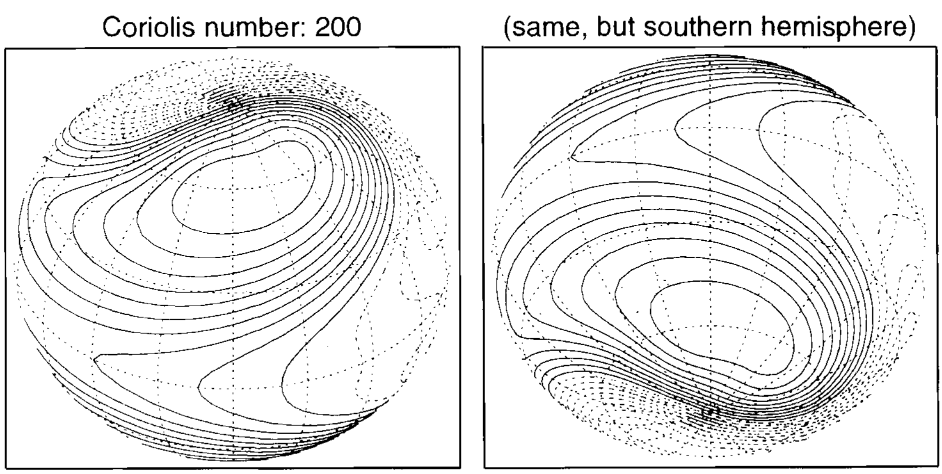

Figure 2 Same as Figure 1, but for $\Omega^* = 200$.

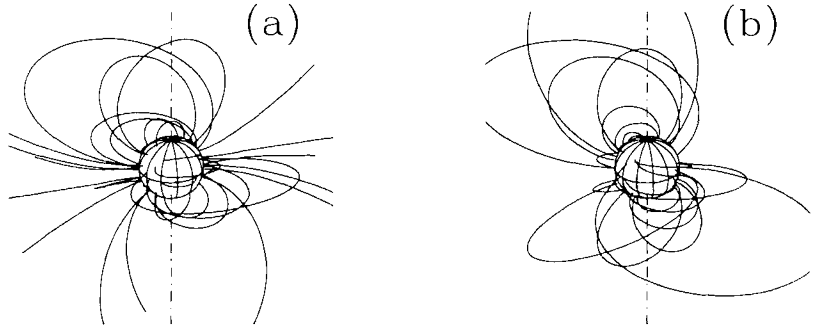

Figure 3 Three-dimensional visualization of field lines outside the dynamo active sphere, $r > R$. (a) $\Omega^* = 100$, (b) $\Omega^* = 200$.

For large values of Ω^*, the time for an initially almost axisymmetric solution to evolve to S1 becomes progressively longer as Ω^* increases. For $\Omega^* = 200$, when we start the computation from an arbitrary configuration the solution first evolves rapidly to near the A0 state, and then the global parity increases slowly to near $P = +1$. Here $P = (E^{(S)} - E^{(A)})/(E^{(S)} + E^{(A)})$ is a measure of the parity (± 1 for pure symmetry/antisymmetry), where $E^{(S)}$ and $E^{(A)}$ are the energies in the field components of the same parity as pure quadrupole/dipole fields. Only after more than 50 diffusion times ($T_{\text{diff}} = R^2/\eta_1$) do the nonaxisymmetric fields begin to contribute even 1% of the total energy. Their strength then steadily, but very slowly, increases. We did not follow this solution to its final equilibrium state, but instead constructed directly the S1 solution (by using suitable initial conditions) and showed that it was stable to an arbitrary perturbation. We deduce from these experiments that the stability of the nonaxisymmetric state becomes weaker as Ω^* increases and expect (although we have not proved it) that for large enough Ω^* an axisymmetric state will be stable. This behavior in the far nonlinear regime is counter-intuitive and could not have been guessed from linear theory. The nonlinear feedback apparently drives the distribution of α into a state that is more favorable for axisymmetric modes.

It is well known that a modest differential rotation can destabilize nonaxisymmetric solutions (Rädler, 1986b; Moss *et al.*, 1990, 1991). In order to test this we now include a weak differential rotation with a profile resembling the zonal flows of Jupiter and Saturn. For this purpose we chose the profile

$$\delta\Omega = \delta\Omega_0 \cos(12\theta)(\sin\theta)^5, \tag{6}$$

where θ is colatitude. This profile looks similar to those given by Ingersoll and Pollard (1982). We continued the same profile into the interior such that Ω is only a function of the distance from the axis. Assuming typical zonal velocities of 100–500 m/s, and an outer radius $R = 60\,000$ km and a rotation period $P = 10^{\text{h}}$, we have $\delta\Omega_0/\Omega_0 \approx VP/(2\pi R) \approx 0.01$–$0.05$. This ratio is rather small (at least compared to the solar dynamo!– but see below), and is consistent with planetary dynamos often being considered to be so-called α^2 dynamos, as opposed to the $\alpha\Omega$ dynamo which is believed to operate in the sun. For $\delta\Omega_0/\Omega_0 = 0.02$ and $\Omega^* = 100$ we now find only the A0 configuration to be stable to arbitrary perturbations (Figure 4).

5. APPLICATION TO PLANETS

Our results clearly demonstrate the possibility of the excitation of stable nonaxisymmetric fields over a broad range of inverse Rossby numbers, Ω^*. When Ω^* becomes large (~ 200) such nonaxisymmetric states are approached very slowly, suggesting that nonaxisymmetric fields would eventually lose their stability if Ω^* increased further. The inclusion of effects such as differential rotation also favors axisymmetric fields for larger values of Ω^*. Although we have not performed separate nonlinear calculations for a spherical shell, the similarity of the linear results of Table 1 for the sphere and spherical shell do not lead us to expect significant differences in nonlinear behavior.

It is tempting to identify the sequence of models with increasing values of Ω^* with the occurrence of different field geometries observed for the outer planets (cf. Rädler and Ness, 1990). Of course, the stable fields in our models are either strictly axisymmetric or strictly nonaxisymmetric, whereas the fields of Uranus and Neptune can be approximated by a combination of $m = 0$ and $m = 1$ components. But we can hardly expect to reproduce detailed planetary field structures with such a simple model: our major point is that the introduction of anisotropies in the α-effect can remove the preference for strictly axisymmetric fields. Ingersoll and Pollard (1982) give typical time scales for Jupiter and Saturn of around 5 days, and they also quote a similar value for the earth. This implies values of the inverse Rossby number around 160. The rotation periods of Uranus and Nepture are longer (approximately 17^h and 16^h, respectively, instead of about 10^h and 11^h for Jupiter and Saturn, e.g. Bagenal, 1992) and, although we do not

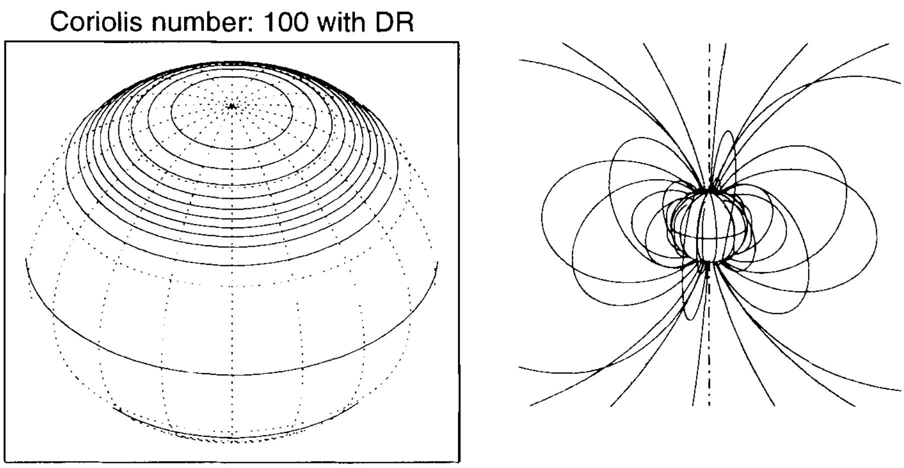

Figure 4 The radial magnetic field component and field lines for a model with weak differential rotation included. $\delta\Omega_0/\Omega_0 = 0.02$, $\Omega^* = 100$.

know the values of τ_{corr} in those cases, we expect that Ω^* is also smaller. Thus, the general picture seems to be that fields with strong nonaxisymmetries are to be expected for small and intermediate values of Ω^*, and that the fields could be predominantly axisymmetric for very large values of Ω^*. Our limited experimentation also suggests that a modest differential rotation may play a role in determining the stable field configuration.

So far we have only considered the value of Ω^*. In order to see how other aspects of our model compare with the real physical situation, we now discuss the values of the turbulent velocity, the alpha effect and the turbulent diffusivity. The turbulent velocity u_t is usually estimated from mixing length theory, where the total thermal flux is of order ρu_t^3 (cf. Torbett and Smoluchowski, 1980; Stevenson, 1983; Ruzmaikin and Starchenko, 1991). This leads to values for u_t of the order of 1 cm/s, with that for Jupiter being probably rather larger than for Uranus. The corresponding value of α is much smaller than the differential velocity, $\delta\Omega R$, adopted in the previous section. It is therefore not surprising that global differential rotation would have a very strong effect, if it were present. (Further, our chosen value of $\delta\Omega/\Omega$ is now seen to imply a differential rotation that is large compared with our assumed α-effect.) However, we know that the turbulence effects that lead to nonuniform rotation in stellar convection zones are strongly reduced for rapid rotation (Kitchatinov and Rüdiger, 1993). It is therefore plausible that, in the interior of giant planets, differential rotation is virtually zero. Indeed, it seems that this may be required on quite general grounds, if the nonaxisymmetric magnetic fields of the outer planets are to be explained as a product of dynamo action. Moreover, the ratio of toroidal to poloidal magnetic fields, which is of the order of $(\delta\Omega R/\alpha)^{1/2}$, would, even for modest differential rotation, be so large that the internal toroidal field strength would greatly exceed the equipartition value.

The above estimate for u_t implies that the correlation length of the small scale motions, $\ell = u_t\tau_{corr}$, is much smaller than the outer radius, i.e. $\ell/R = O(10^{-4})$. The value of α is about $u_t^2\tau_{corr}/L$, where L is the scale height of the turbulent intensity. In our model we have $L = R$, because we assumed a linear profile for η_0, which is the simplest assumption. Therefore, α is much smaller than u_t. Nevertheless, even in this 'worst' case the dynamo would still be excited, because the turbulent diffusivity is also very small ($\approx 10^4\,\mathrm{cm^2/s}$). For the metallic hydrogen layer of Jupiter, the microscopic value of η is, at $4 \times 10^2\,\mathrm{cm^2/s}$ (Stevenson, 1983), still below the turbulent value. However, for the metallic water layer of Uranus, where $\eta \approx 10^4\,\mathrm{cm^2/s}$ (Torbett and Smoluchowski, 1980), this is no longer the case and the microscopic contribution has to be included. In that case the dynamo number, $\alpha R/(\eta + \eta_t)$, is about 10 for Uranus. This is approximately the critical value above which dynamo action sets in, although a little smaller. The corresponding value for Jupiter is about 400, which is a highly supercritical value. In reality, since $L/R < 1$, those values have to be multiplied by $(L/R)^{-1}$, and this would make the dynamo in Uranus also clearly supercritical.

With the values given in the paragraph above, we can estimate the maximum admissable value of $\delta\Omega/\Omega$ below which stable nonaxisymmetric magnetic fields can still be found. Experience with various classes of $\alpha\Omega$ dynamo models (e.g. Rädler, 1986a; Rüdiger and Elstner, 1994; Moss, unpublished) shows that this is case as long as

$C_\Omega \equiv \delta\Omega R^2/\eta_t$ is less than about 10^3, i.e.

$$\left(\frac{\delta\Omega}{\Omega}\right)_{\max} \approx \frac{10^3}{\Omega R^2/\eta_t}. \tag{7}$$

For Uranus this yields $(\delta\Omega/\Omega)_{\max} = 3 \times 10^{-8}$, whereas for Jupiter the value is smaller by a factor of thirty (10^{-9}). This difference is mainly due to the larger radius. These values may sound rather small, but we should note that if the westward drift of the Earth's field were to be caused by differential rotation, the value of $\delta\Omega/\Omega$ is then also estimated to be very small, at about 10^{-6} (but note that nonaxisymmetric dynamo fields normally exhibit longitudinal drift even in the absence of differential rotation, e.g. Rädler, 1986a).

It cannot be ruled out that the value of the correlation time is really one or two orders of magnitude larger that the value adopted above. This would increase the values of α and Ω^*, i.e. the dynamo would become even more supercritical. For very large values of Ω^* nonaxisymmetric magnetic fields would tend to be destabilized. The precise value of the correlation time, above which this would happen, is probably very model dependent and therefore uncertain.

There are further obvious problems in applying our models to actual planets. In Saturn and Jupiter dynamos presumably operate in their metallic cores, whereas in Uranus and Neptune a dynamo may instead operate in the outer layers, consisting of liquid water, ammonia, and methane (e.g. Hubbard, 1984), so the physical and geometrical conditions are therefore rather different. (However our results do suggest that geometrical differences may not be very important.) Thus we can consider the present investigation as only a first step towards understanding the fundamental properties of rapidly rotating bodies with relatively weak or no differential rotation.

6. CONCLUSIONS

According to our current understanding, planetary interiors where magnetic field are generated by dynamo action, are likely to be almost rigidly rotating. However, given the very high rotation rates, the small scale motions are likely to be highly anisotropic. Consequently, a model for the magnetic fields of the giant planets must include these anisotropy effects. We have therefore computed a sequence of dynamo models for rigidly rotating spheres and spherical shells, with emphasis on the effects of anisotropy.

Our main conclusion is that, both in the linear and in the nonlinear regime, nonaxisymmetric magnetic fields are a natural consequence of such a model. As the angular velocity increases, however, the stability of the nonaxisymmetric magnetic fields appears to weaken. This trend agrees with the observed behavior of the magnetic fields of the outer planets: as the rotation rate increases, the fields tend to be more nearly axisymmetric. We cannot exclude the possibility that this could be coincidence, but it is worthwhile noting that this is qualitatively the behavior one should expect from the point of dynamo theory. We would reiterate that our treatment of the α-tensor is fairly rudimentary, in that we have adopted a rather simple model, and have ignored the contributions of density stratification. If we included the effects of density stratification, the structure of the α-tensor would not change, but only the magnitude of its components (see Rüdiger and Kitchatinov, 1993). Thus, we feel that we have introduced a key element into our model (and it appears that anisotropy quite generally favors nonaxisymmetric field generation). There are a number of other obvious improvements that might be made to our model, but their implementation would go far beyond the scope of the present work. Our main intention is to make the point that anisotropy in the α effect is a hitherto neglected, but potentially important, ingredient of planetary dynamo models.

Finally, we should like to reemphasize a key point: a rather small amount of differential rotation in an electrically conducting fluid *always* tends to destroy nonaxisymmetric magnetic fields, by winding them up and so bringing together field lines of opposing direction (Rädler, 1986b; Moss *et al.*, 1990). This appears to be a quite general result. Nevertheless, in the surface layers where the electrical conductivity is low, differential rotation will have almost no effect. However, Charbonneau and Bagenal (1995) argue that Saturn's envelope is sufficiently deep for differential rotation actually to extend into deeper regions where the conductivity is large enough to destroy nonaxisymmetric magnetic field components. In Uranus and Neptune, on the other hand, the heat fluxes and convective velocities are smaller than in Jupiter and Saturn, and they are therefore less likely to develop significant differential rotation. Clearly, all planets should really be considered individually. Nevertheless, due to their rapid rotation, they all have the potential to show nonaxisymmetric magnetic fields, but the degree to which this is realized may vary.

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