# The turbulent viscosity in accretion discs

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Keplerian shear flows are unstable towards a magnetic shear instability that generates turbulence. The turbulence is considered to be a likely source for viscosity in accretion discs. Recently several groups have simulated this turbulence in order to estimate the strength of the turbulent viscosity. There are however significant quantitative discrepancies between their results. Estimates of the effective (Shakura-Sunyaev) viscosity parameter due to the magnetic field, $\tilde{\alpha}_{SS} = -\langle B_x B_y / \mu_0 p \rangle$ (i.e. the ratio of Maxwell stress to gas pressure) ranges from 0.001 to 0.7. We verify these differences using the same code for all simulations, and show that the higher values of $\tilde{\alpha}_{SS}$ are the result of an applied vertical magnetic field. Without an applied field the typical value is 0.005.

## 1. INTRODUCTION

Velikhov (1959) and Chandrasekhar (1960) have shown that magnetized Couette flows can be unstable even when the analogous non-magnetized flows are stable according to Rayleigh's criterion. In particular a Keplerian flow is stable according to Rayleigh's criterion, but can become unstable if a vertical magnetic field

is applied. Safronov (1972) suggested that this magnetic shearing instability is of importance for the development of turbulence in the protoplanetary disc, but practically nobody paid any attention at the time. The possibility was re-discovered by Balbus & Hawley (1991), and it is now widely recognized as the Balbus-Hawley instability. During the first half of 1995, three papers have been published on three-dimensional numerical simulations of the nonlinear evolution of the instability (Hawley et al. 1995 (hereafter HGB), Matsumoto & Tajima 1995 (hereafter MT), Brandenburg et al. 1995, (hereafter BNST)). In some of these simulations other instabilities, such as the Parker instability (e.g. Matsumoto et al. 1988) or a shearing instability in the toroidal field (Foglizzo & Tagger 1995), also appear.

The magnetic instabilities are important because they make an accretion disc turbulent, which enhances the viscosity in the disc by orders of magnitude. The viscosity in an accretion disc is usually written in the form $\nu_{\text{turb}} = \alpha_{\text{SS}} c_s H_0$ (Shakura & Sunyaev 1973), where $c_s$ is the sound speed, and $H_0$ is the scale height of the disc. $\alpha_{\text{SS}}$ is a dimensionless number, essentially the ratio of the stress to the pressure. ($\alpha_{\text{SS}} \leq 1$ if the turbulence is subsonic and the correlation length is smaller than the scale height.) It is possible to estimate $\alpha_{\text{SS}}$ from the numerical simulations of the magnetic shearing instability. BNST give values between 0.001 and 0.005 for $\alpha_{\text{SS}}$, whereas HGB and MT get values between 0.01 and 0.7 depending on the orientation of the initial field, and its strength relative to the disc pressure (HGB). Part of the difference is due to how the authors define $\alpha_{\text{SS}}$. BNST start from the Shakura-Sunyaev prescription and use the following ansatzes for the Reynolds and Maxwell stresses

$$\langle \rho u_x u_y \rangle = -r \frac{d\Omega}{dr}|_{R_0} \, \nu_t^{\text{kin}} \, \langle \rho \rangle, \tag{1}$$

$$-\frac{1}{\mu_0} \langle B_x B_y \rangle = -r \frac{d\Omega}{dr}|_{R_0} \, \nu_t^{\text{mag}} \, \langle \rho \rangle, \tag{2}$$

where $-r d\Omega_0/dr|_{R_0} = \frac{3}{2}\Omega_0$ and $\langle \rangle$ denotes a volume average. HGB and MT simply give the ratios of the Maxwell and Reynolds stresses to the pressure. We denote these ratios as

$$\tilde{\alpha}_{\text{SS}}^{\text{kin}} = \left\langle \frac{\rho u_x u_y}{p} \right\rangle, \tag{3}$$

$$\tilde{\alpha}_{\text{SS}}^{\text{mag}} = \left\langle -\frac{B_x B_y}{\mu_0 p} \right\rangle, \tag{4}$$

where $p$ is the gas pressure in the disk. This yields $\tilde{\alpha}_{\text{SS}}^{\text{kin}} \approx \frac{3}{2}\sqrt{2}\alpha_{\text{SS}}^{\text{kin}} \approx 2.1\alpha_{\text{SS}}^{\text{kin}}$ and analogously for the magnetic viscosities. The rest of the differences are however due to differences in the initial models and boundary conditions. HGB and MT neglect vertical gravity and stratification of the disc, keep all boundaries (sliding-)periodic, and apply a magnetic flux in either the toroidal or vertical direction on their models. On the other hand BNST include the vertical gravity and stratification, use stress-free upper and lower boundaries, and apply a magnetic field of vanishing flux. The purpose of this paper is to show how these differences affect the results.

In Sect. 2 of the paper we describe our basic model. In Sect. 3 we reproduce the results of HGB and MT and compare them with models with zero net magnetic flux initially. Finally we give the conclusions and a brief discussion in Sect. 4.

## 2. THE MODEL

A general description of our simulations has been published in BNST, so we give only a brief summary here. Unfortunately it is beyond the capacity of current computers to simulate turbulence in the entire accretion disc, so the simulations are restricted to a shearing box (Wisdom & Tremaine 1988) approximating a small part of the accretion disc. We employ Cartesian coordinates orientated so that $\hat{x}$ points in the radial direction away from the central object, $\hat{y}$ along the Keplerian flow, and $\hat{z}$ is parallel to the rotational axis of the disc. The center of the box ($x = y = z = 0$) is located a distance $R_0$ from the central object, and is orbiting it with the angular velocity $\Omega_0$. The Keplerian differential rotation takes the form of a linear shear flow $u_y^{(0)} = -\frac{3}{2}\Omega_0 x$ around this point. We solve the magnetohydrodynamic equations for density, $\rho$, internal energy, $e$, magnetic vector potential, $\mathbf{A}$, and the deviation from the Keplerian flow, $\mathbf{u}$. The azimuthal boundaries are periodic, and the radial boundaries are sliding-periodic because of the differential rotation. The vertical boundaries are stress-free and insulating, and we require the magnetic field to be vertical. When we include the vertical gravitational force we limit the simulations to the upper half of the disc, and therefore require the disc to be symmetric around its midplane.
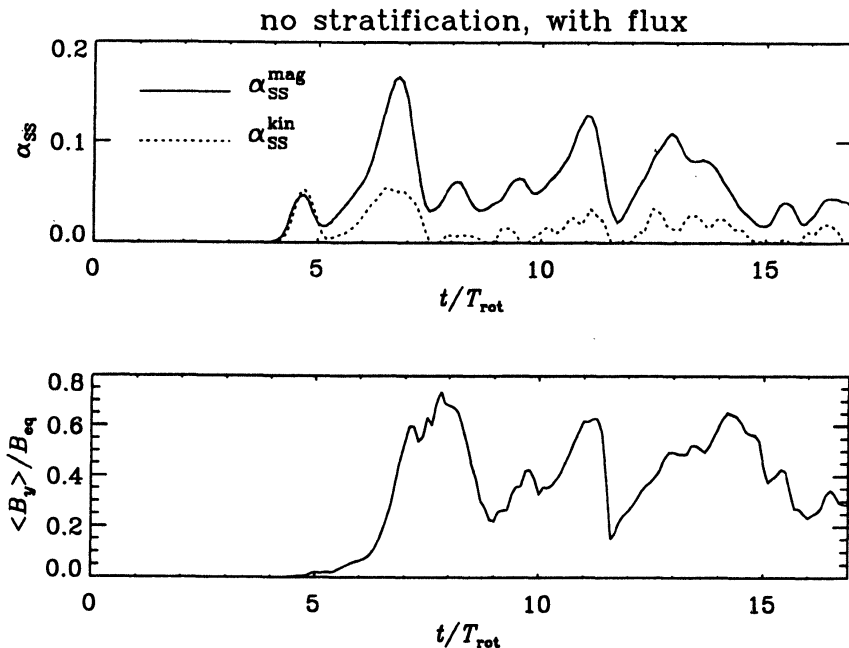


FIGURE 1. Upper part: The variation of $\alpha_{SS}^{mag}$ (solid line) and $\alpha_{SS}^{kin}$ (dashed line) as functions of time for a shearing box without gravity. Lower part: The mean toroidal magnetic field as a function of time.

Initially the disc is isothermal and, in some cases, stratified due to the $z$-component of the gravitational force from the central object, which yields $\rho =$
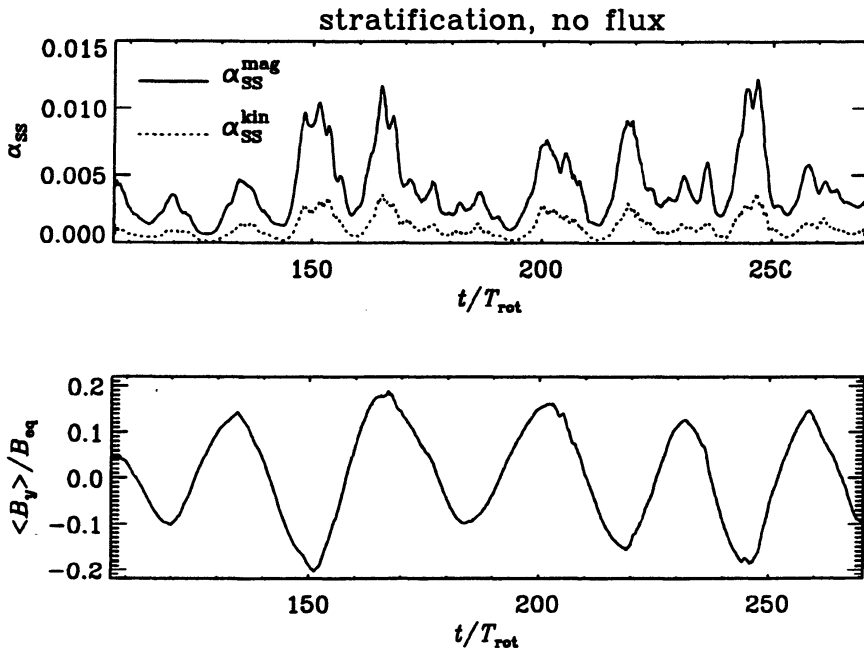
## stratification, no flux



FIGURE 2. Upper part: The variation of $\alpha_{SS}^{mag}$ (solid line) and $\alpha_{SS}^{kin}$ (dashed line) as functions of time for a shearing box with gravity. Lower part: The mean toroidal magnetic field as a function of time.

$\rho_0 e^{-z^2/H_0^2}$. We choose dimensionless variables such that $L_x = GM = \rho_0 = \mu_0 = 1$. $R_0$ is taken as 100, which yields $\Omega_0 = 10^{-3}$, but the final results are independent of $R_0$. The size of the shearing box is $L_x : L_y : L_z = 1 : 2\pi : 2$ with gravity, and $1 : 2\pi : 1$ without gravity, and the number of grid points is $31 \times 63 \times 32$.

For the computations we use a finite difference code that has been described in BNST and Nordlund & Stein (1990). The code employs artificial shock viscosity and hyperviscosity for stability. Some simulations have been continued at twice the resolution without resulting in significant changes of $\alpha_{SS}$.

## 3. RESULTS

We first reproduce model Z4 of HGB; that is a homogeneous box, with no vertical gravity, and an applied vertical magnetic field with $\beta = 2p/B^2 = 400$. In Fig. 1 we plot $\alpha_{SS}^{mag}$ and $\alpha_{SS}^{kin}$ as functions of time. The averages are taken over the full box. $\alpha_{SS}$ fluctuates between 0.02 and 0.17 with an average of 0.08, consistent with HGB and MT. We also plot the average toroidal magnetic field in units of the thermal equipartition field $B_{eq} = (\mu_0 \langle \rho \rangle \langle c_s^2 \rangle)^{1/2}$, where $c_s^2 = (\gamma - 1)e$. The mean toroidal field is 8 times stronger than the applied vertical field, which is different from HGB and MT that do not find any generation of a toroidal net magnetic flux.

Compare this to the model of BNST which includes gravity. The initial field is vertical also in this case, but such that the net flux is zero. The magnetic field
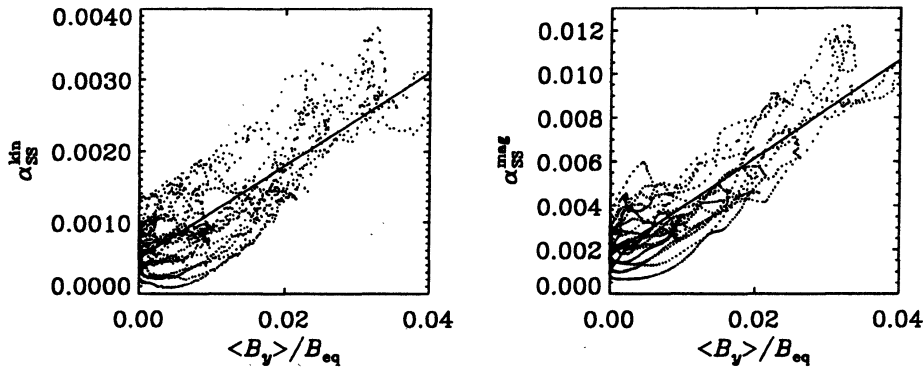
FIGURE 3. The dependence of $\alpha_{SS}^{mag}$ (right) and $\alpha_{SS}^{kin}$ (left) on $\langle B_y \rangle / B_{eq}$ for a shearing box with gravity.
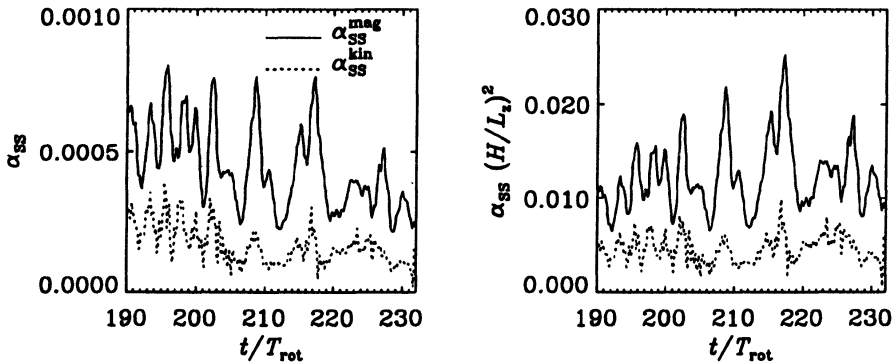


FIGURE 4. Left: The variation of $\alpha_{SS}^{mag}$ (solid line) and $\alpha_{SS}^{kin}$ (dashed line) as functions of time for a shearing box with a scale height increasing gradually to 10, and no net vertical magnetic flux. Right: $\alpha_{SS}^{mag}(H_0/L_z)^2$ (solid line) and $\alpha_{SS}^{kin}(H_0/L_z)^2$ (dashed line) as functions of time for the same model.

is once again dominated by the generated toroidal field, but the field strength is merely half as strong as without stratification and the field appears to be oscillating with a period of about 30 orbital periods (Fig. 2). Of greater importance is that the Maxwell and Reynolds stresses are ten times weaker, and are related to the toroidal field strength via

$$\alpha_{SS}^{mag} \approx 0.0017 + 0.22 \langle B_y \rangle^2 / B_{eq}^2, \tag{5}$$

$$\alpha_{SS}^{kin} \approx 0.0005 + 0.07 \langle B_y \rangle^2 / B_{eq}^2 \tag{6}$$

(Fig. 3).

We have also computed an intermediate model with an initial field of zero net flux and $H_0$ increasing gradually to $\approx 10$ due to Ohmic heating. $\alpha_{SS}$ decreases because of the increasing pressure, but the tension itself is independent of the pressure (Fig. 4).

## 4. CONCLUSIONS

The three models above show that the high viscosities derived by HGB and MT are due to the applied vertical field. Such a vertical field can either be the result of an external source, for instance a magnetized central object or the interstellar medium, or be part of a global field generated by the accretion disc dynamo itself. Unfortunately the periodic horizontal boundary conditions used in all simulations so far enforce conservation of the vertical magnetic flux, and therefore the question of the possible origin of a vertical net magnetic field cannot be addressed.

Another significant difference between BNST and the other simulations is that only BNST find that a large-scale toroidal field is generated by a dynamo. The boundary conditions of HGB and MT prohibit the generation of radial magnetic flux, which is a prerequisite for the shear to generate a toroidal net flux. BNST allow for this possibility by not using periodic upper and lower boundaries.

## ACKNOWLEDGEMENT

## REFERENCES

Balbus, S. A., Hawley, J. F., 1991, ApJ, 376, 214
Brandenburg, A., Nordlund, Å., Stein, R. F., Torkelsson, U., 1995, ApJ, 446, 741 (BNST)
Chandrasekhar, S., 1960, Proc. Nat. Acad. Sci., 46, 253
Foglizzo, T., Tagger, M., 1995, A&A, 287, 297 (also work presented at this meeting)
Hawley, J. F., Gammie, C. F., Balbus, S. A., 1995, ApJ, 440, 742 (HGB)
Matsumoto, R., et al., 1988, PASJ, 40, 171
Matsumoto, R., Tajima, T., 1995, ApJ, 445, 767 (MT)
Nordlund, Å., Stein, R. F., 1990, Comput. Phys. Commun., 59, 119
Safronov, V. S., 1972, Evolution of the protoplanetary cloud and formation of the Earth and the planets, Israel Program for Scientific Translation, Jerusalem
Shakura, N. I., Sunyaev, R. A., 1973, A&A, 24, 337
Velikhov, E. P., 1959, Sov. Phys. JETP, 9, 995
Wisdom, J., Tremaine, S., 1988, AJ, 95, 925