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Spontaneous chiral symmetry breaking by hydromagnetic buoyancy

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Evidence for the parity-breaking nature of the magnetic buoyancy instability in a stably stratified gas is reported. In the absence of rotation, no helicity is produced, but the nonhelical state is found to be unstable to small helical perturbations during the development of the instability. The parity-breaking nature of this magnetohydrodynamic instability appears to be the first of its kind and has properties similar to those in chiral symmetry breaking in biochemistry. Applications to the production of mean fields in galaxy clusters are discussed.

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The phenomenon of spontaneous breakdown of chiral symmetry, i.e., the bifurcation of an achiral state into two states with opposite chirality (handedness), has attracted attention in many different fields of physics, even at the macroscopic level. A well studied example from hydrodynamics is the Taylor-Couette flow with counterrotating cylinders. In this case, linear stability analysis reveals the presence of two oppositely helical states (spiral vortices) with identical growth rates. Experiments and weakly nonlinear analysis show that, depending on the initial conditions, one of these two helical states is selected [1]. One might suppose that rotation (which is an axial vector) is essential to enable the occurrence of such symmetry-degenerate unstable eigenmodes and hence the breaking of chiral symmetry. In other words, the conjecture is that, in the absence of such a vector, helical states will never show up. Then the question arises naturally whether another global axial vector can replace rotation in this respect. Here the magnetic field is a suggestive candidate.

However, more puzzlingly, helical flows are possible even in nonrotating, nonmagnetic setups as demonstrated in Ref. [2] for one of the asymmetric square patterns of Bénard convection. There, the sign of helicity realized in a given cell is expected to depend only on the initial condition. Yet, when considering that this type of pattern emerges not by a bifurcation from the trivial (nonconvective) solution, but from convective rolls instead, we can again identify an axial vector in the unstable reference state, namely the vorticity of the rolls. Note that this vector can, at least with respect to a single roll, be considered global.

As helical flows are known to be crucial in producing large-scale magnetic fields by the so-called α effect [3], the strongest interest in such flows is likely to be found in those branches of astrophysics, geophysics, and planetology which deal with the origin of galactic, stellar, and planetary magnetic fields. However, helicity in astrophysical flows is normally not supposed to be caused by a spontaneous parity breaking. Indeed, bodies such as the Sun tend to produce helicity with opposite signs in their two hemispheres. This is not surprising because the Sun is stratified in the radial direction by its own central gravity $\boldsymbol{g}$ and rotates with finite angular velocity $\boldsymbol{\Omega}$. These two vectors form a pseudoscalar $\boldsymbol{g}\cdot\boldsymbol{\Omega}$ that changes sign about the equatorial plane and is, by virtue of the Coriolis force, directly related to the kinetic helicity.

The question now emerges whether finite kinetic and magnetic helicity can also be produced in the absence of ingredients that are *a priori* given as, for example, $\boldsymbol{g}$ or $\boldsymbol{\Omega}$. In search of such a process we have to look for an instability which shows a preference for amplifying helical velocity (and possibly magnetic) perturbations in comparison with nonhelical perturbations, but showing, of course, no preference for one specific sign of helicity. One particular example is the magnetic buoyancy instability. It is well known that this instability can produce helical magnetic and velocity fields owing to the presence of rotation and stratification [4]. Nevertheless, it has been explicitly stated [4,5] that, in the absence of rotation or at the equator, the α effect must vanish because the eigenmodes of the instability are then degenerate and have opposite helicities. By contrast, recent work [6] delivered indications that finite helicity can emerge even at the equator. In this paper we elaborate on the possibility that helicity may be finite, even in the absence of rotation. We present new simulations and argue that our findings provide strong evidence for spontaneous chiral symmetry breaking during the nonlinear phase of the magnetic buoyancy instability. In addition to our example, there is now also that of the Tayler instability which leads to spontaneous chiral symmetry breaking [7]. In both cases there is no rotation and helicity emerges in the absence of quantities that are *a priori* given such as $\boldsymbol{\Omega}$ and either $\boldsymbol{g}$ or at least boundary effects.

There are quite a number of cases where spontaneous chiral symmetry breaking has been discussed. A somewhat hypothetical example is the production of magnetic helicity during electroweak baryogenesis [8] and may be connected with the emergence of baryon asymmetry [9]. Chiral symmetry breaking is also known in biochemistry where it refers to the consequent selection of one of two possible forms of biomolecules (mainly sugars and amino acids) that are mirror images of each other. This selection may have taken place during the emergence of life on Earth [10]. It requires the preferred replication of molecules of the same handedness, known as autocatalysis [11,12]. Equally important is the effect of mutual antagonism. This has been identified in the context of DNA polymerization, where it characterizes the inability to continue polymerization with monomers of opposite handedness [13]. Thus, an initial imbalance in chirality is strongly amplified and the final selection of one chirality over the other consequently is a result of random fluctuations in the equipartition of right- and left-handed monomers in a prebiotic mixture [14].

The equations governing the agent concentrations in biochemistry [e.g., Ref. [10] on the one hand and the velocity mode amplitudes in hydrodynamics [e.g., Ref. [15] on the other show remarkable similarities. Hence, it could be fruitful to keep the mentioned biochemical processes in mind when trying to understand bifurcations into helical states in hydrodynamics. In particular, it deserves interest if something like "mutual antagonism" can be identified in the nonlinear evolution of two competing eigenmodes with opposite helicity.

We consider a Cartesian slab of ideal gas with a stable stratification in the z direction, which is altered by the presence of a narrow horizontal magnetic layer in magnetostatic equilibrium. That is, in the layer a fraction of the gas pressure is substituted by magnetic pressure. Equilibrium is achieved by reducing the temperature correspondingly, but keeping the density unchanged, so the magnetic layer is initially not buoyant. However, the local temperature change in the layer perturbs the thermodynamic equilibrium. We solve the compressible hydromagnetic equations in a setup described in detail in Ref. [6], but focus here on the case without rotation, $\mathbf{\Omega} = \mathbf{0}$, which shows no *a priori* preference of positive or negative helicity.

Our computational domain has an aspect ratio $L_x : L_y : L_z = 1{:}3{:}1$. The ratio of the thickness of the magnetic layer, H_B, to pressure scale height H_p and L_z is $H_B : H_p : L_z = 0.1{:}0.3{:}1$. Our fluid and magnetic Prandtl numbers are equal to 4. The Lundquist number based on the thickness of the magnetic layer is $v_{A0} H_B/\eta = 1000$, where $v_{A0} = B_0/\sqrt{\rho_0 \mu_0}$ is the Alfvén speed associated with the initial magnetic field of strength B_0 in the y direction, ρ_0 is the initial density at the bottom of the domain, μ_0 the vacuum permeability, and η is the magnetic diffusivity. The initial stratification is a polytrope with index 3, so density scales to temperature like $\rho \sim T^3$. We use the fully compressible PENCIL CODE [16] for all our calculations and the number of mesh points is 64^3.

Considering reflectional symmetries of the basic equations together with the unperturbed initial state and the boundary conditions, we observe that reflections at planes $z = \text{const}$ clearly change the system already due to its stratification. In contrast, reflections at planes $y = \text{const}$ leave it unchanged while those at planes $x = \text{const}$ are equivalent with merely changing the sign of the initial field of the magnetic layer. Hence there is no essential difference in the behavior of a solution and a corresponding counterpart obtained by one of the two latter reflections. In particular, if there are helical eigenmodes of the linearized system they should occur in pairs with opposite helicities, but equal growth (or decay) rates. Further, if there were stable quasistationary helical solutions of the nonlinear system they should again occur in such pairs.

Our results depend decisively on the details of the initial velocity perturbations, which possess a slight imbalance of random sign in net kinetic helicity. Depending on this sign we find exponential amplification of positive or negative kinetic and current helicities during the initial stage of the instability. An important consequence of this parity breaking is the occurrence of what is known in dynamo theory as the α effect. This effect is crucial for the amplification and sustainment of a mean magnetic field $\overline{\boldsymbol{B}}$. Its evolution is essentially governed by the mean electromotive force, $\overline{\boldsymbol{\mathcal{E}}}$, which is the correlation $\overline{\boldsymbol{u} \times \boldsymbol{b}}$ of velocity and magnetic field fluctuations, $\boldsymbol{u} = \boldsymbol{U} - \overline{\boldsymbol{U}}$ and $\boldsymbol{b} = \boldsymbol{B} - \overline{\boldsymbol{B}}$, respectively, where the overbars indicate horizontal (xy) averaging. If u lacks reflectional symmetry, and in particular, if it exhibits handedness, $\overline{\boldsymbol{u} \times \boldsymbol{b}}$ can have a constituent proportional to the mean field

$$(\overline{\boldsymbol{u} \times \boldsymbol{b}})_i = \alpha_{ij} \circ \overline{B}_j - \eta_{ij} \circ \overline{J}_j, \qquad (1)$$

where the operator "$\circ$" denotes convolution in space, and in general, also in time, but this will be ignored here. Note that α_{ij} and η_{ij} are tensorial kernels, the former a pseudo and the latter a true tensor. The symmetric part of α_{ij} results in the aforementioned α effect, while that of η_{ij} describes what is known as turbulent diffusion. Given that we are working with horizontal averages, $\overline{\boldsymbol{B}} = \overline{\boldsymbol{B}}(z,t)$, we have $\overline{B}_z = 0$ in the absence of an imposed field. Therefore, the evolution of $\overline{\boldsymbol{B}}$ is governed by only 2×2 components of both α_{ij} and η_{ij} with α_{xx} and α_{yy} being of particular interest.

In Fourier space (with respect to z) the convolutions in Eq. (1) turn into multiplications with tensors depending on the z wavenumber k. However, due to the intrinsic inhomogeneity of our system they depend at the same time also explicitly on z. The functions $\hat{\alpha}_{ij}(z,k)$ and $\hat{\eta}_{ij}(z,k)$ can be directly determined by the so-called test-field method [17,18]; for specific details see [6]. For the purpose of the present paper it suffices to consider the Fourier constituents with the smallest wave number, $k = \pi/L_z$ and we omit the argument k in what follows.

In this paper we discuss two pairs of runs, A$\pm$ and B$\pm$. For each pair, the initial velocity of one run is an exact mirror image of that of the other one with respect to a vertical plane. Consequently, the two runs have opposite initial kinetic helicity. Depending on its sign, the runs are labeled by $+$ or $-$. In both pairs the initial velocity contains a random pattern of horizontal eddies with Mach numbers of the order of 10^{-5}. In pair A$\pm$, there are additional random vertical motions with the same Mach number, whereas in pair B$\pm$ these are set to zero. Thus, the pair A$\pm$ differs from pair B$\pm$ in the magnitude of the initial kinetic helicity, whose normalized value $\langle \boldsymbol{w} \cdot \boldsymbol{u} \rangle / w_{\rm rms} u_{\rm rms}$ is, with the vorticity $\boldsymbol{w} = \text{curl}\, \boldsymbol{u}$, $\approx 2 \times 10^{-5}$ for the former and, due to numerical noise only, $\approx 4 \times 10^{-10}$ for the latter. In the linear phase of the instability the two runs of each pair yield exactly the same growth rate. Likewise, the magnetic and kinetic energies in the saturated stage are nearly the same. The evolution of the normalized kinetic helicity is shown in Fig. 1 for all four runs. Here, angular brackets denote the average over the full computational domain. Note that parity breaking becomes obvious after about one Alfvén time $t_{A0} = L_y/v_{A0}$. In the lower panel of Fig. 1 we show the moduli of the kinetic helicity together with the energy of the components $B_{x,z}$ generated from the initial B_y by the instability. Evidently, the evolution of the magnetic energy is nearly the same in all cases. Note that, along with the kinetic helicity, the current helicity also is generated with the same growth rate, again with positive or negative sign, depending on the initial condition. Furthermore, the final sign of the helicity does not need to agree with the initial one. For example, A+ had positive helicity initially, but ends up with negative helicity, undergoing several sign changes during the kinematic stage. Given that the helicity is a volume integral comprising patches with opposite signs of

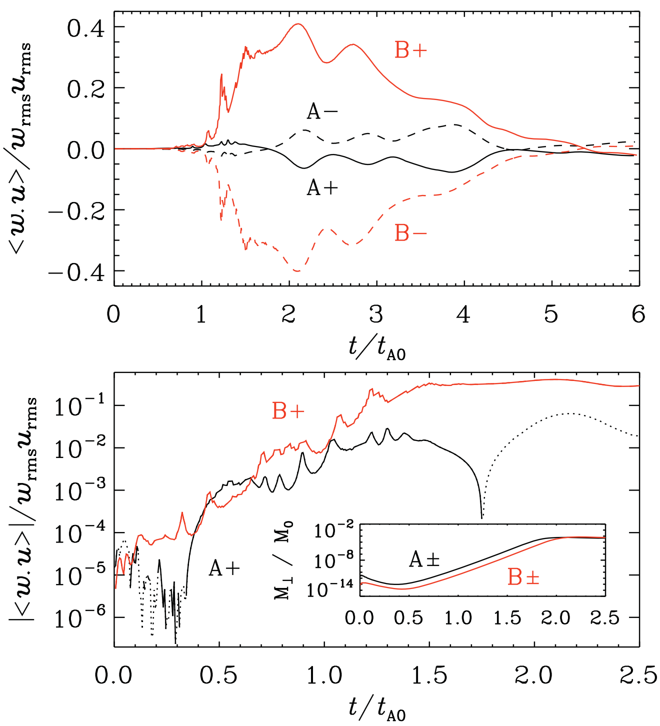

FIG. 1. (Color online) Upper panel: Evolution of the normalized kinetic helicity for runs A± and B±. Lower panel: Same as above but for the absolute value (dotted) overplotted with a solid line to indicate the intervals of positive kinetic helicity. Inset: $M_\perp = \langle B_x^2 + B_z^2 \rangle$, normalized by the averaged initial magnetic energy density, $M_0 = \langle B_{y0}^2 \rangle$ for the same set of runs.

$\boldsymbol{w} \cdot \boldsymbol{u}$, such sign changes can be understood in terms of local volume changes of these patches. The sign change for the A+ solution at $t/t_{A0} = 1.7$ could then be explained by a gradual shrinking of a patch with positive helicity that occupied a much larger fraction at earlier times. We speculate that this scenario might be analogous to the case of chiral molecules, where patches of one handedness would shrink in the direction of the curvature vector [19].

In Fig. 2 we plot the dependence of $\hat{\alpha}_{xx}$ and $\hat{\alpha}_{yy}$ on z and t for the runs A±.

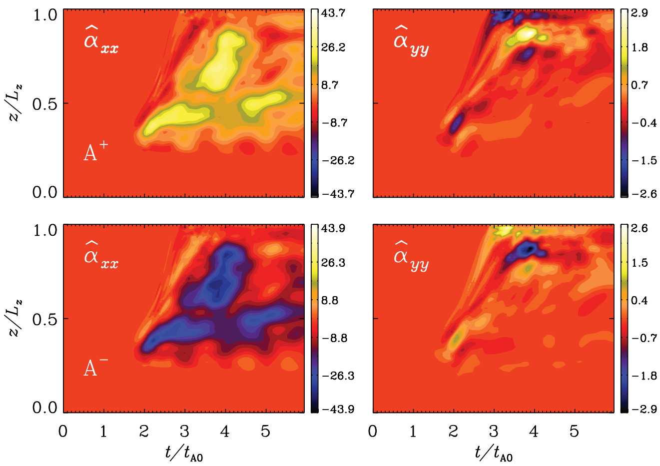

FIG. 2. (Color online) $\hat{\alpha}_{xx}$ and $\hat{\alpha}_{yy}$ as functions of time and height for runs A+ (top) and A− (bottom). Values scaled by $10^3/v_{A0}$.

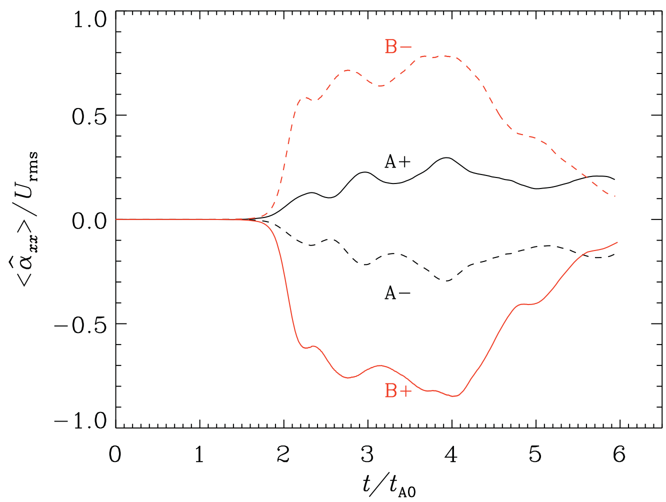

FIG. 3. (Color online) $\langle \hat{\alpha}_{xx} \rangle$ as a function of time for runs A± and B±.

Note that their patterns reflect the rise of the magnetic layer from its original position at $z = 0.3\,L_z$ to the top of the domain at $z = L_z$. Obviously, within each pair of runs the $\hat{\alpha}_{yy}$ map of the run with negative initial helicity labeled "−" can be obtained from the one of the run with positive initial helicity, labeled "+" to reasonable accuracy by a simple sign inversion.

The component $\hat{\alpha}_{xx}$ has been found to have the same sign throughout the entire domain at any instant of time. Hence it is sufficient to consider the evolution of the average $\langle \hat{\alpha}_{xx} \rangle$ which is shown in Fig. 3 for all four runs. Its behavior is similar to that of $\langle \boldsymbol{w} \cdot \boldsymbol{u} \rangle$ in Fig. 1 which, however, shows a sign change at $t/t_{A0} \approx 2$, whereas the sign of $\langle \hat{\alpha}_{xx} \rangle$ does never change.

The phenomenon of spontaneous parity breaking has been observed earlier for hydrodynamic instabilities, in particular those of the Taylor-Couette flow [20]. For such instabilities, systems of amplitude equations of weakly nonlinear, Ginzburg-Landau type have already been described in Refs. [15,20]. In all these cases, the helical states, consisting of traveling waves, emerge due to the presence of rotation which is an axial vector. In our case this role is played by the magnetic field. Deriving nonlinear amplitude equations for the present problem is nontrivial and will be described elsewhere. Here it suffices to say that the amplitude equations in Ref. [15] show a striking similarity to the equations that describe homochirality in the biological context [11]. In particular, both sets of equations contain terms of mutual antagonism, which allows one sign of helicity to grow at the expense of the other. The analogies between homochirality and the present problem extend even further. (a) In both cases one can start with an initial state with small excess of one sign of helicity or chirality, but the final state can have the opposite sign [19] (here, e.g., the set of runs A). (b) In a closed system, which in the biological context implies that the resources available to the evolving species are finite, both problems fail to reach a statistically stationary state [12,21].

As eluded to in the beginning, we are not aware of any similar spontaneous parity-breaking instability in magnetohydrodynamics. A related example is the selection of one type of nonmirror symmetric crystals during their growth in the presence of grinding [22,23]. In the case of the hydromagnetic buoyancy instability presented here, we can only speculate about possible applications. In any case we must be thinking of a stably stratified layer with negligible rotation. A prime example fitting this description is clusters of galaxies. Indeed, magnetic and thermal buoyancy effects are invoked to explain what looks like cold bubbles in such clusters [24].

However, the magnetic buoyancy instability might be just one example showing the parity-breaking property described here. The magnetorotational instability (MRI) could be another one. In that case there is rotation, but in the absence of stratification and current density there would still be no pseudo-scalar constructible and hence no net helicity, unless the MRI were capable of spontaneous parity breaking.

Large-scale magnetic fields have been found in simulations without net helicity including cases with shear, in particular the MRI [25] and the magnetic buoyancy instability with a sinusoidal shear profile [26]. Magnetic field generation occurs also in the case of a linear shear flow with isotropic nonhelically forced turbulence Although it is not clear whether in this case an instability (like the magnetic buoyancy instability) is really required for parity breaking, it should be noted that there is generally a production of mean vorticity [27,28] whose possible importance has been discussed in connection with dynamos from linear shear flows [29]. Such a vorticity production would indeed be the result of a (hydrodynamic) instability, supporting the idea that an instability might be needed for parity breaking. An alternative explanation for the origin of large-scale fields in this setup is the incoherent α-shear dynamo [30], where kinetic helicity is expected to fluctuate about zero. Revisiting this assertion in the light of the present results could be a worthwhile effort.

In conclusion, we have found a mechanism that is able to amplify magnetic and velocity fields with either sign of helicity, but the selection of one of them occurs during the nonlinear stage where it gives rise to an α effect under otherwise parity-invariant conditions. This mechanism might be applicable to clusters of galaxies. Moreover, it is very likely that the magnetic buoyancy instability is just a first example in a class of spontaneous parity-breaking instabilities in magnetohydrodynamics.

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