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Dynamo-generated Turbulence in Disks: Value and Variability of Alpha

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Abstract

Dynamo-generated turbulence seems to be a universal mechanism for angular momentum transport in accretion disks. We discuss the resulting value of the viscosity parameter alpha and emphasize that this value is in general not constant. Alpha varies with the magnetic field strength which, in turn, can vary in an approximately cyclic manner. We also show that the stress does not vary significantly with depth, even though the density drops by a factor of about 30.

1. Introduction

Differentially rotating disks with angular velocity decreasing outwards are susceptible to a magnetic shear (or Balbus-Hawley) instability (Velikhov 1959; Chandrasekhar 1960, 1961; Balbus & Hawley 1991). This occurs not only in accretion disks, but also in major parts of galactic disks. Balbus & Hawley (1992) suggested that the growth rate of the instability is given by the Oort A-value, $-\frac{1}{2}\partial\Omega/\partial\ln r \sim \Omega$. The magnetic field grows until the energy density becomes a tenth (or more) of the thermal energy density. In this way a weak magnetic field produces turbulent fluid motions (Hawley et al. 1995, Matsumoto & Tajima 1995). These motions are practically always strong

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enough (i.e. the magnetic Reynolds number is large enough) that dynamo action becomes possible so that an initial seed magnetic field is amplified further (Brandenburg et al. 1995a, Hawley et al. 1996, Stone et al. 1996). It is difficult to imagine how this process can be avoided (except maybe in protoplanetary disks where there is a lack of charge carriers). In other words, non-magnetic disks are probably rare.

The standard accretion disk model is nonmagnetic, but Shakura & Sunyaev (1973) did anticipate the importance of magnetic fields for transporting angular momentum in disks. Campbell (1992) produced a thin disk model that is inherently magnetic, with field strengths comparable to the thermal equipartion value. However, the value of α is not constant, but depends on the magnetic field strength. Now, such information can be extracted from three-dimensional simulations of dynamo-generated turbulence.

2. The model

The Balbus-Hawley instability is local and can be simulated in the framework of the shearing box approximation (Lynden-Bell & Ostriker 1967), where the radial boundary conditions are periodic in a Lagrangian system following the azimuthal shear flow. The radial dependence of gravity is expanded around $r=R$,

$$\frac{GM}{r^2}=\frac{GM}{R^2}\left[1-2\frac{x}{R}+3\left(\frac{x}{R}\right)^2-...\right], \tag{1}$$

where $x=r-R$. Normally only the linear term is retained. In a local frame of reference rotating with angular velocity $\Omega_0=(GM/R^3)^{1/2}$ the centrifugal force and the radial components of gravity and Coriolis force balance,

$$-\frac{GM}{R^2}\left(1-2\frac{x}{R}\right)+\Omega_0^2R\left(1+\frac{x}{R}\right)+2u_y^{(0)}\Omega_0=0, \tag{2}$$

which means that $u_y^{(0)}=-\frac{3}{2}\Omega_0 x$. In practice we solve the basic hydromagnetic equations for the deviations from the linear shear flow $u_y^{(0)}$.

The shearing box approximation is now frequently being used in local disk simulations. The disadvantage is that curvature terms such as B_ϕ^2/r and B_rB_ϕ/r are ignored. These terms can be important not only for the stability of the flow (Knobloch 1992), but also for breaking the symmetry between inflow and outflow. Brandenburg et al (1996) restored such curvature terms and included also the quadratic terms in Eq. (1). They found that the mean accretion rate $\dot{M}$ is consistent with the standard formula (Frank et al 1992)

$$\dot{M}=3\pi\alpha c_s H\Sigma,\quad\text{where}\quad\Sigma=\int\rho dz. \tag{3}$$

Furthermore, it turns out that both $\dot{M}$ and α vary cyclically over a timescale of about 30 orbits (for further details see Brandenburg et al 1996). In the following we discuss the average of α and its variations.

3. The average value of the viscosity parameter α

The horizontal components of the Reynolds and Maxwell stress, that are crucial in accretion disk theory, are parametrized in terms of a turbulent viscosity ν_t,

$$\tau_{xy} \equiv \langle \rho u_x u_y - B_x B_y/\mu_0 \rangle = -\nu_t \langle \rho \rangle r \frac{\partial \Omega}{\partial r}. \tag{4}$$

When ν_t is expressed in terms of the natural units for disks, the sound speed c_s and the vertical disk scale height H, the proportionality constant is the viscosity parameter α, that is, $nu_t = \alpha c_s H$. Since $c_s = \Omega H/\sqrt{2}$ we have $\nu_t = \alpha \Omega H^2/\sqrt{2}$.

There are certain properties of local simulations that may affect the value of α: stratification, field strength, and distance from the central object. First, in the absence of stratification, the vertical scale height is formally infinite and $\alpha = \sqrt{2}\nu_t/(\Omega H^2)$ goes to zero, even though the stress remains finite. The reason is that the Balbus-Hawley instability operates on scales smaller than $\lambda_{\rm BH} = v_A/\Omega$, where v_A is the Alfvén speed. In real disks, $\lambda_{\rm BH} < H$, but in a simulation with no (or very weak) stratification $\lambda_{\rm BH}$ is limited by the size of the box. In that case, a better parametrization of the stress would be to normalize by the height of the box L_z, i.e. $\alpha_L = \sqrt{2}\nu_t/(\Omega L_z^2)$. In Fig. 1 we compare two runs of Torkelsson et al (1996) with stratification ($L_z/H \approx 2$) and with almost no stratification ($L_z/H = 0.2$). The average viscosity parameters are $\alpha \approx 0.005$ and $\alpha_L \approx 0.017$, so $\alpha_L \approx 3.2\alpha$. In other words, given the "conversion factor" α_L/α, one could (in principle) estimate α even if there is no stratification in the simulation.

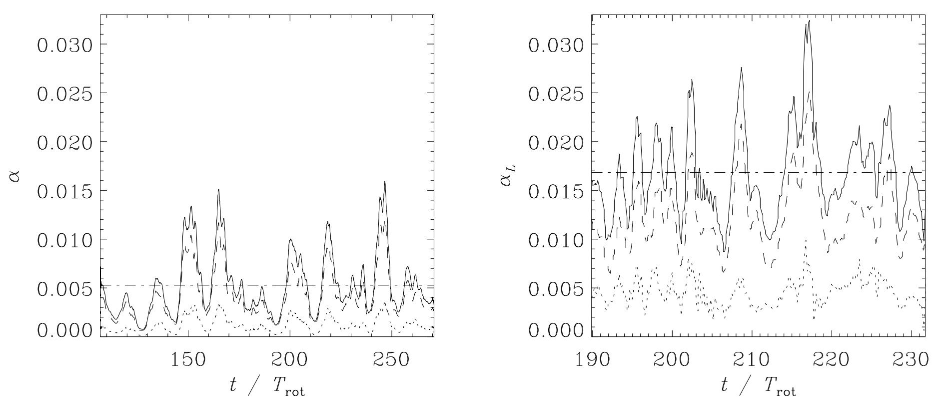

Fig. 1. Evolution of α and α_L. The dotted (dashed) line refers to the contribution of the Reynolds (Maxwell) stress.

Obviously, the value of α depends on the definition of H. For isothermal stratification, the density in the equilibrium state goes like $\rho = \rho_0 \exp(-z^2/H^2)$, but it is not uncommon to define a scale height by writing $\rho = \rho_0 \exp(-z^2/2\tilde{H}^2)$, where $\tilde{H} = H/\sqrt{2}$ (e.g. Frank et al 1992). Note that now $c_s = \Omega\tilde{H}$ and therefore $\tilde{\alpha} = \nu_t/(\Omega\tilde{H}^2)$. With this definition, the two values $\tilde{\alpha}_L$ and $\tilde{\alpha}$ are closer together ($\tilde{\alpha} = 0.008$ and $\tilde{\alpha} = 0.012$), suggesting perhaps some advantage in using the second definition, because then $\tilde{H}$ is closer to the "effective" disk height than H. However, in order to be consistent with previous work, we continue to use the old definition.

Note that with the new definition $\tilde{\alpha}$ would be larger than α by a factor of $\sqrt{2}$. However, there is yet another definition of alpha: the ratio of stress to pressure, which yields a value that is again 3/2 times larger, so altogether 2.1 times larger than our definition. Thus, it is important to keep this in mind when comparing work of different authors. We also note that the values obtained by Hawley et al (1996) and Stone et al (1996) agree with those of Brandenburg et al (1995a, 1996) within a factor of two or less.

4. Variability of α

It is important to realize that α is not constant. Variability of α is caused not only by turbulent motions which are inherently time-dependent, but especially by the slow changes of the large-scale magnetic field. In our model a large scale magnetic field is generated by some kind of (large scale) dynamo process. In Fig. 2 we compare contours of the total stress and the azimuthal field $\langle B_y \rangle$ in a $z-t$ diagram.

The long-term evolution of the magnetic field on a time scale of 30 orbits is perhaps somewhat reminiscent of the solar cycle. In the case of an accretion disk around a compact object of $1M_\odot$ and at a distance of 10^6 km (typical of dwarf novae systems) 30 orbits would correspond to about 20 min. However, the cyclic variations are the result of a local simulation. Applied at different radii, we would have interference of different frequencies from different radii. In the absence of simulations in global geometry we cannot say what kind of variability (if any) can be expected in reality. What is important, however, is that α and $\langle \mathbf{B} \rangle$ are closely connected. Torkelsson et al (1996) and Brandenburg et al (1996) proposed the following fit formula

$$\alpha(\langle \mathbf{B} \rangle) = \alpha^{(0)} + \alpha^{(B)} \langle \mathbf{B} \rangle^2 / B_0^2, \tag{5}$$

where $B_0 = \langle \mu_0 \rho c_s^2 \rangle^{1/2}$ is the equipartition field strength based on the thermal energy, and $\alpha^{(B)}$ is around 0.5, which is much larger than the quiescent value $\alpha^{(0)}$, which is around 0.001.

It is conceivable that an improved α-disk model would yield a self-consistent description of the disk structure and the large scale magnetic field $\langle \mathbf{B} \rangle$. Eq. (5) could be an important ingredient of such a model.

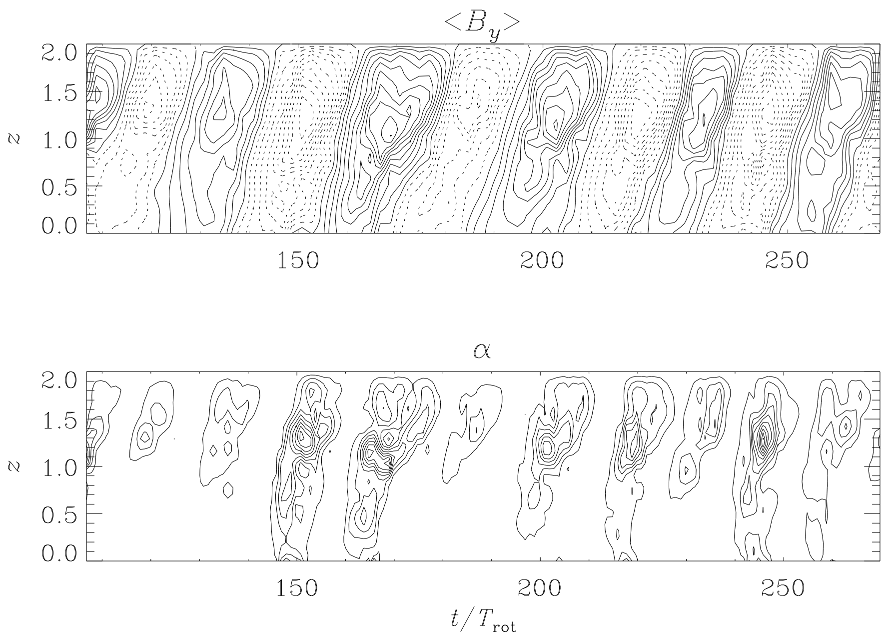

Fig. 2. Comparison of the azimuthal field $\langle B_y \rangle$ and the horizontal components of the total magnetic stress in a $z-t$ diagram. Dotted contours refer to negative values. Note that maxima of α coincide with those of $\langle B_y \rangle$.

5. The vertical dependence of α

Originally the concept of α-viscosity was employed within the framework of vertically integrated models. However, α has also been used to model the "subgrid-scale" viscosity of models that resolve the vertical dependence explicitly (e.g. Kley et al 1993). If one assumes a constant α, and if c_s is approximately constant (isothermal disk), then Eq. (4) might suggest that the stress τ_{xy} is proportional to the density and would fall off towards the surface. This is however not confirmed by the simulations; see Fig. 3. We find that τ_{xy} is *not* proportional to $\rho(z)$ or $c_s(z)$. Indeed, a better approximation might be to use the vertically averaged stress

$$\tau_{xy} \sim \alpha c_s \Sigma, \tag{6}$$

This new description is likely to affect the vertical disk structure of α-disks and in that way some conclusions regarding instabilities associated with the ionization state of the disk, like dwarf nova instabilities.

The models presented here lack radiation transport and are therefore nearly isothermal. Preliminary steps toward models with surface cooling have been presented by Brandenburg et al (1995b). Their models show narrow convection zones right beneath the cooled surface.

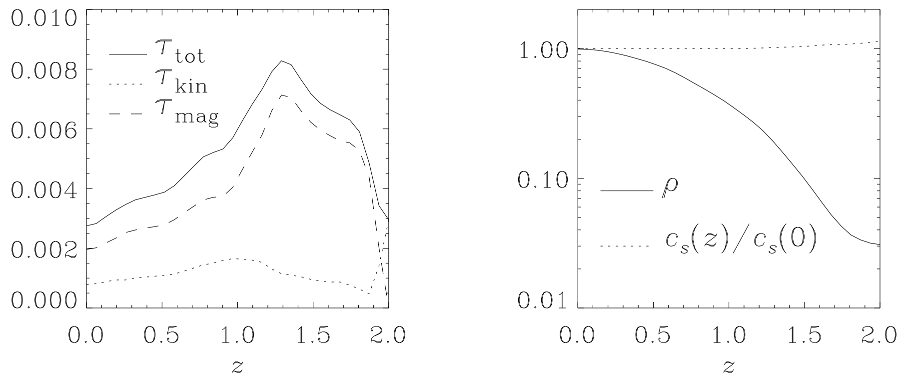

Fig. 3. Dependence of $\tau_{xy}(z)$, normalized by $\langle\rho\rangle c_s H$ (left). Note that τ_{xy} is not proportional to $\rho(z)$ or $c_s(z)$ (right).

References

Balbus, S. A. & Hawley, J. F. 1991, ApJ 376, 214

Balbus, S. A. & Hawley, J. F. 1992, ApJ 400, 610

Brandenburg, A., Nordlund, Å., Stein, R. F., & Torkelsson, U. 1995a, ApJ 446, 741

Brandenburg, A., Nordlund, Å., Stein, R. F., & Torkelsson, U. 1995b, Lecture Notes in Physics 462, 385

Brandenburg, A., Nordlund, Å., Stein, R. F., & Torkelsson, U. 1996, ApJL (in press)

Campbell, C. G. 1992, Geophys. Astrophys. Fluid Dyn. 63, 179

Chandrasekhar, S. 1960, Proc. Nat. Acad. Sci. 46, 253

Chandrasekhar, S. 1961, Hydrodynamic and Hydromagnetic Stability (Dover Publications, Inc., New York)

Frank, J., King, A. R., & Raine, D. J. 1992, Accretion power in astrophysics (Cambridge: Cambridge Univ. Press)

Hawley, J. F., Gammie, C. F., & Balbus, S. A. 1995, ApJ 440, 742

Hawley, J. F., Gammie, C. F., & Balbus, S. A. 1996, ApJ (in press)

Kley, W., Papaloizou, J. C. B., Lin, D. N. C. 1993, ApJ 416, 679

Knobloch, E. 1992, MNRAS 255, 25p

Lynden-Bell, D. & Ostriker, J. P. 1967, MNRAS 136, 293

Matsumoto, R. & Tajima, T. 1995, ApJ 445, 767

Shakura, N. I., & Sunyaev, R. A. 1973, A&A 24, 337

Stone, J. M., Hawley, J. F., Gammie, C. F., & Balbus, S. A. 1996, ApJ (in press)

Torkelsson, U., Brandenburg, A., Nordlund, Å., Stein, R. F. 1996, Astrophys. Letter & Comm. (in press)

Velikhov, E. P. 1959, Sov. Phys. JETP 36, 1398