THE DYNAMO EFFECT IN STARS

Axel Brandenburg
NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark; and
Department of Mathematics, University of Newcastle upon Tyne, NE1 7RU, UK
brandenb@nordita.dk

Abstract Stars with outer convection zones are all magnetically active. They possess magnetic fields that have a strong large scale component, which can sometimes show cyclic reversals, like in the sun. Over the past thirty years mean-field dynamo theory has been used to explain structure and evolution of those large scale fields. The main ingredients of this theory are the alpha-effect and turbulent diffusion, but the physical nature of these effects has shifted from a purely hydrodynamical origin to more magnetically controlled scenarios, where thermal buoyancy, for example, is replaced by magnetic buoyancy and other magnetically driven instabilities.

Keywords: dynamos, stellar cycles, magnetohydrodynamics

1. INTRODUCTION

Stellar chromospheric and coronal activity is usually explained by some kind of dynamo process, which converts kinetic energy into magnetic energy. For example, for turbulent convection at sufficiently high magnetic Reynolds numbers (small enough magnetic diffusivity) small scale magnetic fields are produced (Meneguzzi & Pouquet 1989, Nordlund et al. 1992, Brandenburg et al. 1996). However, many stars show cyclic behavior. Explaining such behavior requires some extra ingredients, such as rotation, shear, and vertical density stratification (e.g. Moffatt 1978). Those extra ingredients tend to give the flow some swirl and make it helical–just like cyclones. The shear from *differential* rotation tends to align the field with the toroidal direction, converting poloidal magnetic field into toroidal. To close the cycle, poloidal magnetic field is generated from toroidal via cyclonic convection. This effect is usually referred to as the alpha-effect.

K.S. Cheng et al. (eds.), Stellar Astrophysics, 1–8.

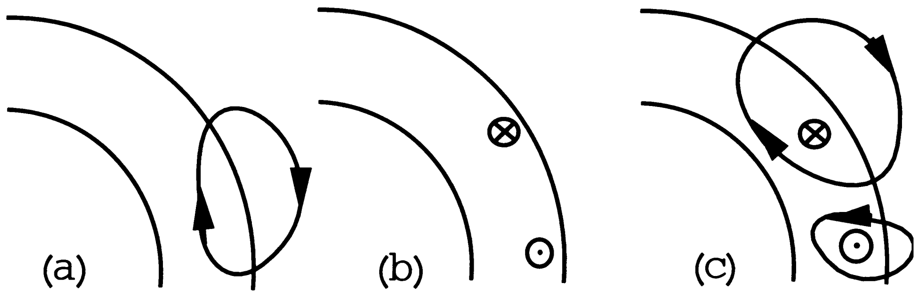

Figure 1 A poloidal magnetic field loop is being sheared out by differential rotation causing toroidal fields. The α-effect produces new poloidal field loops, but shifted polewards.

In the standard picture, rotation twists a rising flux loop such that an extra *turbulent* electric field is induced (the turbulent electromotive force $\boldsymbol{\mathcal{E}}$), which has a component perpendicular to the mean magnetic field, i.e. $\boldsymbol{\mathcal{E}} = \alpha\langle \boldsymbol{B} \rangle$.

The α parameter gives rise to exponentially growing solutions of the induction equation provided the magnitude of α is large enough (large enough dynamo number). If this α-effect is supplemented by differential rotation (the Ω-effect), one talks about an $\alpha - \Omega$ dynamo (e.g., Parker 1979, Krause & Rädler 1980). Figure 1 gives a qualitative explanation of why this α-effect can lead to dynamo waves propagating in the toroidal direction. Consider first some pre-existing poloidal magnetic field loop (Fig. 1a). In lower latitudes the deeper regions of the sun spin slower, giving rise to a toroidal field as shown in Fig. 1b. This toroidal field induces a current and the α-effect produces a new magnetic field parallel to it (Fig. 1c).

Comparing Figs. 1a and 1c we notice the emergence of a new loop near the equator with *opposite* orientation relative to the original loop in Fig. 1a. The larger loop in 1c, however, has the same orientation as the loop in Fig. 1a, which therefore appears to have migrated away from the equator. (Three further applications of shear and α-effect bring the situation back full circle to the configuration in Fig. 1a.) The field migration seen in this model is of course in the "wrong" direction, because in the sun the sunspot belts migrate equatorward. This is known as the dynamo dilemma (Parker 1987). In the early days of dynamo theory, before helioseismology told us otherwise, one used to believe that the sun spun faster in deeper layers than at the surface. In that case the dynamo waves would go in the right direction. There are some indications supporting this possibility in the case of accretion discs (Brandenburg &

Donner 1997), but it is not clear that similar circumstances apply to the case of stars. An opposite angular velocity gradient would indeed be consistent with the observation that very young sunspots have faster proper motion than older spots. (There is at present no good explanation for this property of sunspot proper motions. However, helioseismology is now beginning to resolve a negative radial gradient of the angular velocity in low latitudes and near the surface, see also Chan's article in these proceedings.)

If the sign of α was for some reason reversed (negative in the northern hemisphere) then this would turn the dynamo wave into the right direction. Another possibility that has been discussed already by Parker (1987), and more recently by Durney (1996) and Choudhuri, Schüssler, & Dikpati (1995) is to invoke meridional circulation to turn the dynamo wave around. The most recent model of that type is by Dikpati & Charbonneau (1999), where a more realistic profile of differential rotation has been adopted.

2. MAGNETICALLY DRIVEN α-EFFECTS

Over the past few years there have been several suggestions that the dynamo effect should actually increase with field strength. In the paper of Brandenburg, Saar & Turpin (1998) this was just a hypothesis that appeared plausible in view of other simulated dynamos that share the property of becoming more and more effective as the magnetic field strength increases. One possible and straightforward explanation would be that α may *not* be driven by thermal buoyancy, but by *magnetic* buoyancy. This idea goes back to Schmitt (1985), who was the first to derive in detail the α-effect resulting from such a system. Recent simulations have been presented by Brandenburg & Schmitt (1998), and model calculations have been carried out by Moss, Shukurov & Sokoloff (1999). The stronger the magnetic field, the more the flux tubes are evacuated (total pressure = magnetic pressure + gas pressure) and the more buoyant they are. It may therefore not be so implausible that α could indeed increase with increasing field strength.

If α really does increase with field strength we need some other mechanism for saturation of the dynamo. This could be again magnetic buoyancy: once the magnetic buoyancy effect exceeds a certain value it would no longer lead to field growth, because the generated flux would be removed too quickly from the dynamo-active region. In the case of the magnetorotational instability, which is primarily relevant to *accretion discs*, the growth would cease once the Alfvén speed becomes so large that the travel distance of an Alfvén wave within one orbit becomes comparable to some relevant scale of the disc (e.g. the disc height in the case of a vertical field). This would effectively limit the mean field strength. This system provides an important example of a magnetically driven α-effect (Brandenburg et al. 1995, Brandenburg & Donner 1997). Here the turbulence is driven by the magnetorotational or Balbus-Hawley (1991) instability.

3. SIMULATIONS

Brandenburg, Nordlund & Stein (1999) have simulated a convective dynamo with imposed shear trying to capture both the effects of latitudinal differential rotation in the convection zone proper and vertical shear at the bottom of the convection zone. In that simulation the total magnetic energy, $\langle \boldsymbol{B}^2 \rangle$, as well as the energy in the mean magnetic field, $\langle \boldsymbol{B} \rangle^2$, increase exponentially until saturation is reached. The mean field shows unsteady behavior without real cycles and field reversals. However this is strongly related to geometrical effects and boundary conditions, because the large scale field extends over the scale of the box making global effects important. Furthermore, the energy of the mean field to the total magnetic energy, $f = \langle \boldsymbol{B} \rangle^2 / \langle \boldsymbol{B}^2 \rangle$, which is a measure of the filling factor, also increase with time. Thus, again, the large scale field becomes better defined (relative to the fluctuations) once it reaches appreciable field strength. Those results are encouraging in that they confirm the observational finding that the solar magnetic field shows a great deal of coherence even though it is basically of turbulent origin.

In the case of local turbulence simulations of accretion disc dynamos (Brandenburg et al. 1995) we found that the mean magnetic field (averaged azimuthally and over some radius interval) shows spatio-temporal coherence as evidenced by a "butterfly-type" diagram of the mean toroidal field as a function of time and height above and below the midplane of the disc. This result is however markedly dependent on boundary conditions. If one adopts perfectly conducting boundary conditions instead of vacuum boundary conditions one finds steady dipole-type solutions instead of oscillatory quadrupole-type solutions (Brandenburg 1998). It is remarkable, however, that the same change of behavior is reproduced by an $\alpha - \Omega$ dynamo model. In that sense simulations and $\alpha - \Omega$ model show an important similarity.

There is another point that needs to be emphasized. While simulations such as the accretion disc simulations show fairly well-defined large scale fields, they also display an extremely "noisy" behavior for the turbulent electromotive force and hence the α-effect. Although it has been shown that in the presence of shear and turbulent diffusion,

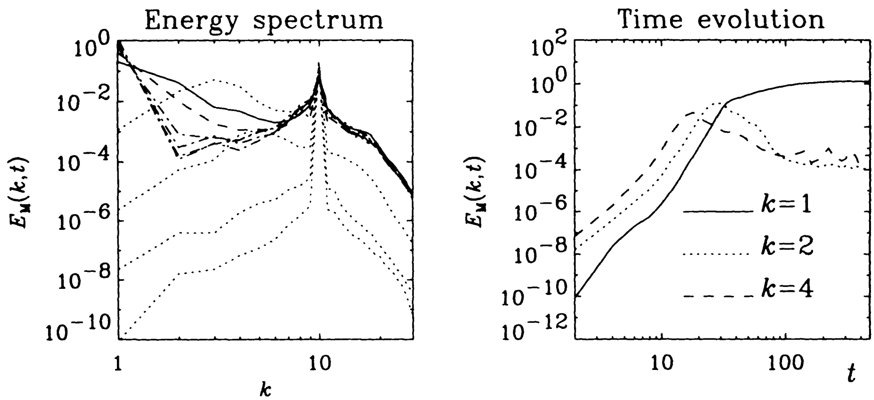

Figure 2 Spectral magnetic energy, $E_{\rm M}(k,t)$, as a function of wavenumber k for different times: dotted lines are for early times ($t = 2, 4, 10, 20$), the solid and dashed lines are for $t = 40$ and 60, respectively, and the dotted-dashed lines are for later times ($t = 80, 100, 200, 400$).

noisy α-effects are quite capable of producing mean fields that are not very noisy (Vishniac & Brandenburg 1997), it remains still somewhat of a mystery as to how such a noisy α-effect can have anything to do with a fairly well-behaved large scale magnetic field as seen in the simulations.

4. THE INVERSE MAGNETIC CASCADE

It is actually very difficult to verify that it is really the α-effect that is responsible for the large scale field generation. From the seminal papers of Frisch et al. (1975) and Pouquet et al. (1976) it is clear that the amplification of large scale fields can be explained by an inverse cascade of magnetic helicity. This effect too is rather difficult to isolate in simulations of astrophysical turbulence. However, under somewhat more idealized conditions, for example when magnetic energy is injected at high wave numbers, one clearly sees how the magnetic energy increases at large scales; see Fig. 2. (For more details of those calculations see Brandenburg 1999.)

A somewhat different situation is encountered in the absence of any forcing where some initial magnetic field can only decay. However, if initially most of the magnetic energy is in the small scales, there is the possibility that magnetic helicity and thereby also magnetic energy is transferred to large scales. This is exactly what happens (Fig. 3), provided there is initially some net helicity. The inset of Fig. 3 shows that in the absence of initial net helicity the field at large scales remains

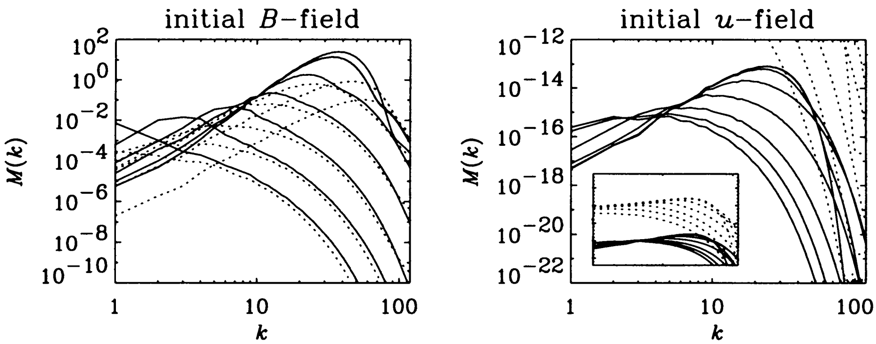

Figure 3 Power spectra of magnetic energy (solid lines) and kinetic energy (dotted lines) in a decay run. The left hand panel is for a case where the flow is only driven by an initial helical magnetic field. In the right hand panel the field is weak and governed by strong decaying fluid turbulence. The inset shows both velocity and magnetic spectra in the same plot.

unchanged, until diffusion kicks in and destroys the remaining field at very late times.

If the magnetic field has the possibility to tap energy also from the large scale velocity the situation is somewhat different and a large scale magnetic field can also be driven without net helicity. In that case even without any helicity (kinetic or magnetic helicity) the large scale field can increase. In astrophysical settings there is usually large scale shear from which energy can be tapped. This was indeed the case in the simulations discussed in the previous section.

5. CONCLUSIONS

While dynamo theory in its present form is in principle able to reproduce basic behavior of solar and stellar magnetic fields and cycles, there are a number of problems of theoretical and practical nature, as well as a number of new hypotheses that could resolve some of these problems.

The main theoretical problem is related to the functional dependence between the electromotive force and the mean magnetic field. Comparison with simulations suggests tentatively that α may work preferentially at the largest possible scale. If that is true one could solve the (practical) problem of explaining the increase of stellar cycle frequencies with increasing inverse Rossby number by assuming that the α-effect increases with field strength (anti-quenching).

Another rather practical problem concerns the shape of the solar butterfly diagram. Theoretically one would expect that the dynamo wave should migrate poleward; see the pioneering simulations of Gilman (1983) and Glatzmaier (1985). In order to explain the observed equatorward migration one would either need to have a negative α in the northern hemisphere (some simulations do predict this, but it is not clear that this applies really to the solar regime), or one might be able to explain the migration directly by invoking a suitable meridional circulation. Recent work by G. Rüdiger and collaborators (private communication) suggests that this is indeed a viable possibility (see also Dikpati & Charbonneau 1999). This was first suggested by Parker (1987) and Durney (1996), and confirmed by a model calculation by Choudhuri et al. (1995), but it seemed to be a rather special case given that meridional circulation usually tends to make oscillatory models stationary (Rädler 1986).

In any case, the theoretical foundations of $\alpha - \Omega$ are sufficiently shaky that one may consider a realistic high-resolution simulation of stellar dynamos as absolutely crucial before one can try to use $\alpha - \Omega$ type dynamos with real predictive power.

Acknowledgements

This work has been supported in part by the British Council (JRS 98/39).

References

Balbus, S. A., & Hawley, J. F. 1991, ApJ 376, 214

Brandenburg, A. 1998, in Theory of Black Hole Accretion Discs, ed. M. A. Abramowicz et al. (Cambridge University Press), p.61

Brandenburg, A. 1999, in Magnetic Helicity, ed. A. A. Pevtsov (American Geophysical Union, Florida), (in press)

Brandenburg, A., & Donner, K. J. 1997, MNRAS 288, L29

Brandenburg, A., & Schmitt, D. 1998, A&A 338, L55

Brandenburg, A., Nordlund, Å., & Stein, R. F. 1999, A&A (to be submitted)

Brandenburg, A., Saar, S. H., & Turpin, C. R. 1998, ApJ (Letters) 498, L51

Brandenburg, A., Nordlund, Å., Stein, R. F., & Torkelsson, U. 1995, ApJ 446, 741

Brandenburg, A., Jennings, R. L., Nordlund, Å., Rieutord, M., Stein, R. F., & Tuominen, I. 1996, J. Fluid Mech. 306, 325

Choudhuri, A. R., Schüssler, M., & Dikpati, M. 1995, A&A 303, L29

Dikpati, M. & Charbonneau, P. 1999, ApJ 518, 508

Durney, B.R. 1996, Solar Physics 166, 231

Frisch, U., Pouquet, A., Léorat, J., Mazure, A. 1975, J. Fluid Mech. 68, 769

Gilman, P. A. 1983, ApJ Suppl. 53, 243

Glatzmaier, G. A. 1985, ApJ 291, 300

Krause, F., & Rädler, K.-H. 1980, Mean-Field Magnetohydrodynamics and Dynamo Theory (Akademie-Verlag, Berlin; also Pergamon Press, Oxford)

Meneguzzi, M., Pouquet, A. 1989, J. Fluid Mech. 205, 297

Moffatt, H. K. 1978, Magnetic Field Generation in Electrically Conducting Fluids (Cambridge University Press, Cambridge)

Moss D., Shukurov A., Sokoloff D. 1999, A&A 343, 120

Nordlund, Å., Brandenburg, A., Jennings, R. L., Rieutord, M., Ruokolainen, J., Stein, R. F., & Tuominen, I. 1992, ApJ 392, 647

Parker, E. N. 1979, Cosmical Magnetic Fields (Clarendon Press, Oxford)

Parker, E. N. 1987, Solar Phys. 110, 11

Pouquet, A., Frisch, U., Léorat, J. 1976, J. Fluid Mech. 77, 321

Rädler, K.-H. 1986, Astr. Nachr. 307, 89

Schmitt, D. 1985, Dynamowirkung magnetostrophischer Wellen (PhD thesis, University of Göttingen)

Vishniac, E. T., & Brandenburg, A. 1997, ApJ 475, 263