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Decay of trefoil and other magnetic knots

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Abstract. Two setups with interlocked magnetic flux tubes are used to study the evolution of magnetic energy and helicity on magnetohydrodynamical (MHD) systems like plasmas. In one setup the initial helicity is zero while in the other it is finite. To see if it is the actual linking or merely the helicity content that influences the dynamics of the system we also consider a setup with unlinked field lines as well as a field configuration in the shape of a trefoil knot. For helical systems the decay of magnetic energy is slowed down by the helicity which decays slowly. It turns out that it is the helicity content, rather than the actual linking, that is significant for the dynamics.

Keywords. Sun: magnetic fields

Magnetic helicity has been shown to play an important role in the dynamo process Brandenburg & Subramanian, 2005. For periodic systems where helicity is conserved simulations have shown that with increasing magnetic Reynolds number Re_M the saturation magnetic field strength decreases like $\mathrm{Re}_\mathrm{M}^{-1/2}$ (Brandenburg & Dobler, 2001). This is problematic for astrophysical bodies since for the Sun $\mathrm{Re}_\mathrm{M} = 10^9$ and galaxies $\mathrm{Re}_\mathrm{M} = 10^{14}$. In order to alleviate this quenching the magnetic helicity of the small scale fields needs to be shed (Brandenburg *et al.*, 2009).

In the active regions of the Sun twisted magnetic field lines have been observed (Pevstov *et al.*, 1995). Later it was shown (Leka *et al.*, 1996) that the magnetic field in sunspots gets twisted before it emerges out of the surface. Manoharan *et al.*(1996) and Canfield *et al.*, (1999) demonstrated that helical structures are more likely to erupt into coronal mass ejections. This suggests that the Sun sheds helicity.

The magnetic helicity is related to the mutual linking for two non-intersecting flux tubes via (Moffatt, 1969)

$$H = \int_V \boldsymbol{A} \cdot \boldsymbol{B} \ \mathrm{d}V = 2n\phi_1\phi_2,$$

where H is the magnetic helicity, $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ is the magnetic field in terms of the vector potential $\boldsymbol{A}$, ϕ_1 and ϕ_2 are the magnetic fluxes through the tubes and n is the linking number. The flux tubes may not have internal twist. In the limit of large Re_M H is a conserved quantity as well as the linking number.

In presence of magnetic helicity the magnetic energy decay is constrained via the realizability condition (Moffatt, 1969) which gives a lower bound for the spectral magnetic energy

$$M(k) \geqslant k|H(k)|/2\mu_0 \quad \text{with} \quad \int M(k) \ \mathrm{d}k = \langle \boldsymbol{B}^2 \rangle / 2\mu_0, \quad \int H(k) \ \mathrm{d}k = \langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle,$$

the magnetic permeability μ_0, where $\langle . \rangle$ denotes volume integrals.

In this work we extend earlier work (Del Sordo *et al.*, 2010) where the dynamics of interlocked flux rings, with and without helicity, was studied as well as a non-interlocked configuration. Here we also study a self-interlocked flux tube in the form of a trefoil knot.

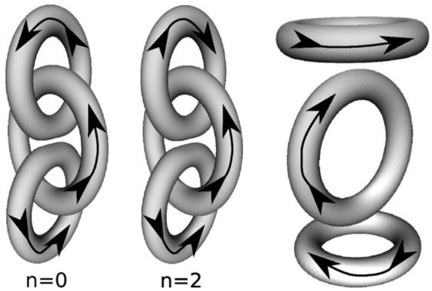

Figure 1. The three triple ring configurations for the initial time. From left to right: interlocked rings with no helicity, interlocked rings with finite helicity and non-interlocked rings without helicity. The arrows indicate the direction of the magnetic field. Adapted from Del Sordo *et al.*, 2010.

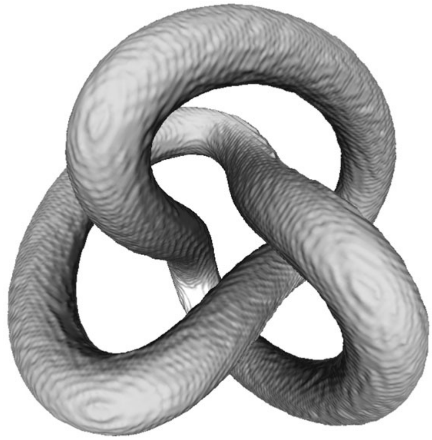

Figure 2. The initial magnetic field configuration for the trefoil knot.

The three-rings setups consist of three magnetic flux tubes. In two configurations they are interlocked where in one the helicity is zero and in the other one it has a finite value, as shown in Fig. 1. In the third setup we instead consider unlocked rings. Since the rings do not have internal twist, the helicity of this last configuration is zero. We also study the evolution of a self-interlocked flux tube having the form of a trefoil knot, with finite helicity (Fig. 2). In this case we have $H = 3\phi^2$, so the linking number is $n = 3/2$. All of these setups evolve according to the full resistive equations of MHD for an isothermal compressible medium. The Alfvén time is used as time unit.

As a consequence of the realizability condition the magnetic energy cannot decay faster than the helicity. The setups with finite H show a slower decay than the setups with no helicity (Fig. 3). The decay of the trefoil knot follows approximately the same decay law as the other configuration consisting of three rings with finite H. Within the simulation time H decays only to about one half of the initial value conserving then the topology. During later times field lines reconnect and the helicity seems to go into internal twist, which is topologically equivalent to linking; see Fig. 4.

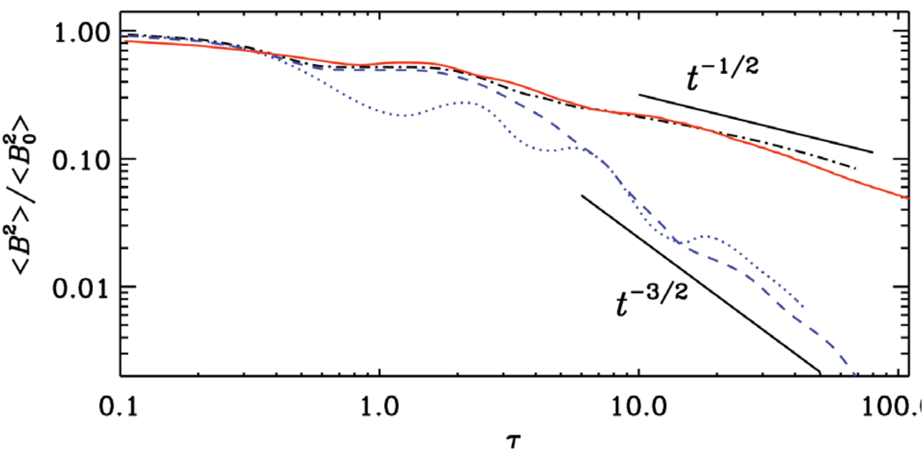

Figure 3. Evolution of the normalized magnetic energy for the trefoil know (solid/red line) compared with various three-ring configurations with $n = 2$ (dash-dotted line), $n = 0$ (dashed/blue line), and the non-interlocked case (dotted/blue line).

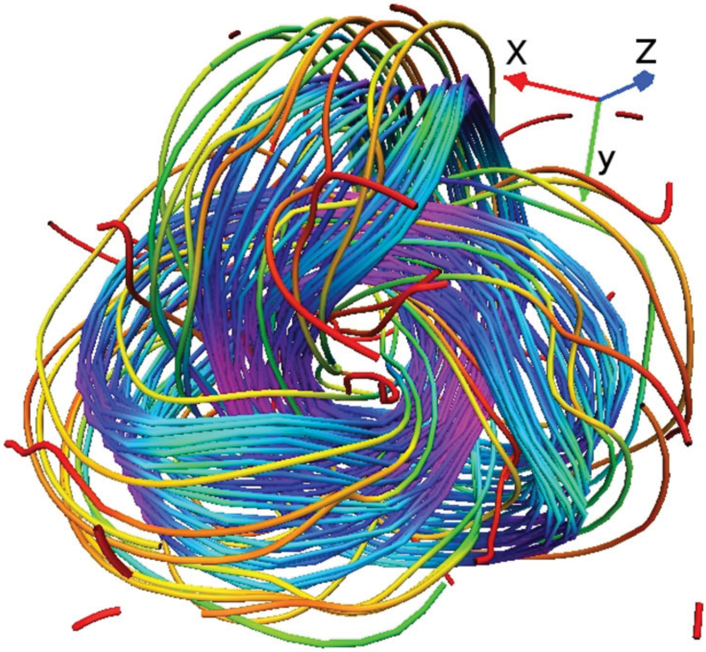

Figure 4. Magnetic field lines at 5 Alfvén times for the trefoil knot. The colors represent the magnitude of the magnetic field. Note that internal twist generation is weak.

The slow decay of H conserves the topology of the system. The linking is then eventually transformed into internal twisting during magnetic reconnection. Since both non-helical setups evolve similarly we conclude that it is mainly the magnetic helicity and not the actual linking which influences the dynamics. The helical trefoil knot evolves in a similar manner. This confirms the hypothesis that the decay of interlinked flux structures is governed by magnetic helicity and that higher-order invariants, advocated for example by Yeates *et al.*, 2010, may not be essential for describing this process.

In conclusion, we can say that magnetic helicity is decisive in controlling the decay of interlocked magnetic flux structures. If the magnetic helicity is zero, resistive decay will be fast while with finite magnetic helicity the decay will be slow and the speed of decay of magnetic energy depends on the speed at which magnetic helicity decays. This is likely an important aspect also in magnetic reconnection problems that has not yet received sufficient attention.

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