ASTR/GEOL-2040-001: Search for Life in the Universe

Homework #6 Monday November 6, 2017 model solutions

Remember to show all details of your work and staple your pages! Don't use this homework page to squeeze your answers on the margins or between the lines.

1. **How big a rocky core does Titan have?** Titan has a mass of $M = 1.346 \times 10^{23}\,\mathrm{kg}$ and a radius of $R = 2,576\,\mathrm{km}$.

(i) Compute Titan's volume, $V = \frac{4\pi}{3}R^3$. What is Titan's mean density, $\overline{\rho} = M/V$? [Show the details of your work!]
Density = Mass/Volume, and the volume is $V = 4\pi R^3/3 = 4\pi(2.576 \times 10^6\,\mathrm{m})^3/3 = 7.160 \times 10^{19}\,\mathrm{m}^3$ and so the mean density is $\overline{\rho} = 1.346 \times 10^{23}\,\mathrm{kg}/7.160 \times 10^{19}\,\mathrm{m}^3 = 1.88 \times 10^3\,\mathrm{kg\,m}^{-3} = 1880\,\mathrm{kg\,m}^{-3}$. [2pts]
Rounded values are also acceptable, e.g., $V \approx 10 \times (3 \times 10^6\,\mathrm{m})^3/3 = 9 \times 10^{19}\,\mathrm{m}^3$, so the mean density is $\overline{\rho} \approx 1.5 \times 10^{23}\,\mathrm{kg}/9 \times 10^{19}\,\mathrm{m}^3 \approx 1.7 \times 10^3\,\mathrm{kg\,m}^{-3}$.

(ii) Your result above should lie somewhere between the mean densities of rock, $\rho_{\rm rock} = 3100\,\mathrm{kg\,m}^{-3}$, and ice, $\rho_{\rm ice} = 950\,\mathrm{kg\,m}^{-3}$. Explain why?
Common constituents of the bodies of the solar system include water and water ice, silicates or rocks, and iron. Water ice is the lighted of them. Thus, if Titan consisted to 100% if water ice, its density would be $950\,\mathrm{kg\,m}^{-3}$. Titan's density is higher, so there must also be heavier constituents. If the mean density were larger than that of rock, there would have been a significant contribution of iron, but this is not the case. However, we can never exclude an additional contribution from iron. [2pts]

(iii) Assume that Titan is composed of 50% rock and 50% ice, what would the mean density be? Compare this with the mean density of Titan.
We add 50% of the density of rock (i.e., a fraction of 0.5) and 50% of the density of water ice (again a fraction of 0.5) to get the mean density, i.e.,

$$\overline{\rho} = 0.5 \times 3100\,\mathrm{kg\,m}^{-3} + 0.5 \times 950\,\mathrm{kg\,m}^{-3}.$$

Thus, we have $\overline{\rho} = 1550\,\mathrm{kg\,m}^{-3} + 475\,\mathrm{kg\,m}^{-3} = 2025\,\mathrm{kg\,m}^{-3}$. [2pts]
Also acceptable are rounded values, i.e., $\overline{\rho} = 0.5\times3000\,\mathrm{kg\,m}^{-3}+0.5\times1000\,\mathrm{kg\,m}^{-3} = 2000\,\mathrm{kg\,m}^{-3}$.

(iv) Assume that half of Titan's volume consists of rock, what would the radius $R_{\rm rock}$ of such a rocky core be? Check that its volume $V_{\rm rock} = \frac{4\pi}{3}R_{\rm rock}^3$ is indeed half of Titan's value that you computed in (i).
The volume of a spherical rocky core is proportional to $R_{\rm rock}^3$, so $R_{\rm rock}$ must be by a fraction of $(1/2)^{1/3}$ smaller than R. Using a calculator, we find $(1/2)^{1/3} = 0.794$, and therefore

$$R_{\rm rock} = 0.794 \times R = 0.794 \times 2,576\,\mathrm{km} = 2045\,\mathrm{km}.$$

[2pts]
Also acceptable is to approximate

$$(1/2)^{1/3} = \left(1 - \tfrac{1}{2}\right)^{1/3} \approx 1 - \tfrac{1}{3} \times \tfrac{1}{2} = \tfrac{5}{6} = 0.83,$$

and so $R_{\rm rock} \approx \frac{5}{6} \times 2000\,\mathrm{km} = 1667\,\mathrm{km}$.

(v) Now assume that Titan is composed of 40% rock and 60% ice, what would the mean density be? Again, compare this with the mean density of Titan.
We add 40% of the density of rock (i.e., a fraction of 0.4) and 60% of the density of water ice (again a fraction of 0.6) to get the mean density, i.e.,

$$\overline{\rho} = 0.4 \times 3100\,\mathrm{kg\,m^{-3}} + 0.6 \times 950\,\mathrm{kg\,m^{-3}}.$$

Thus, we have $\overline{\rho} = 1240\,\mathrm{kg\,m^{-3}} + 570\,\mathrm{kg\,m^{-3}} = 1810\,\mathrm{kg\,m^{-3}}$. [2pts]
Also acceptable are rounded values, i.e., $\overline{\rho} = 0.4\times3000\,\mathrm{kg\,m^{-3}}+0.6\times1000\,\mathrm{kg\,m^{-3}} = 1800\,\mathrm{kg\,m^{-3}}$.

(vi) Do you expect the fraction of rock in Titan to be more or less than 40%?
The latter estimate of 40% for the rocky core resulted in a mean density of $1810\,\mathrm{kg\,m^{-3}}$, which is closer to the actual mean density of Titan ($1880\,\mathrm{kg\,m^{-3}}$) than the 50% estimate, but it slightly overshoots the goal. There, the correct fraction must be between 40% and 50%, but closer to 40%. [2pts]
In the book by Rothery et al., there is a related question, Question 4.1 on page 132, with an answer on page 309, where a formula is derived for calculating the fraction x directly, i.e.,

$$x = \frac{\overline{\rho} - \rho_{\mathrm{ice}}}{\rho_{\mathrm{rock}} - \rho_{\mathrm{ice}}}.$$

Inserting our values gives $(1880 - 950)/(3100 - 950) = 0.43$.
Incidently, the mass of the Earth is 6×10^{24} kg, and with a radius of 6400 km, the mean density is $5460\,\mathrm{kg\,m^{-3}}$. This is more than the density of just rock, so we must have an iron core. Using our formula above together with the density of $8000\,\mathrm{kg\,m^{-3}}$ for iron, we find its fraction to be $(5460 - 3100)/(8000 - 3100) = 0.48$. From seismology, we know that the radius of the core is 2900 km, so $x = 2900/6400 = 0.45$ is not so far away from the truth.

2. **Planetary protection categories.** Place each of the following five space missions in the appropriate planetary protection category (I to V, with V being the most restrictive), giving the reason for each case. There is room for debate here. In the following, the answers of the Rothery et al. book are given; see page 308.

(i) Cometary nucleus lander.
Comets are of interest for understanding the origins of life and contamination could jeopardize future experiments, **Category IV** seems appropriate. However, since there are many comets, it might be argued that we can afford to spoil a few, so a category below the top one can be justified. [2pts]

(ii) Mercury orbiter mission.
Mercury is not of direct interest for understanding the process of chemical evolution relevant to the origin of life, so **Category I** is appropriate. [2pts]

(iii) Jupiter orbiter with Mars fly-by en route.
Since only a fly-by of Mars is planned, the concern here is primarily over unintentional impact which place is it in **Category II**, unless this seems any likelihood of eventual impact into Europa, which would require Category IV. [2pts]

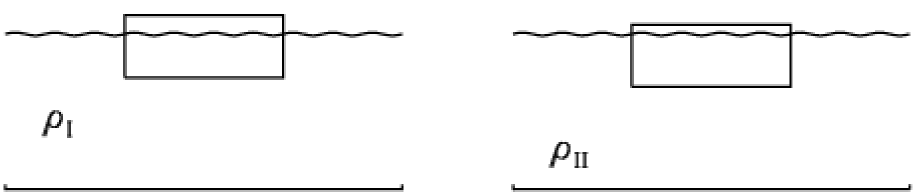
(iv) Mars orbiter and lander.
Category IV is appropriate for missions to targets of interest for understanding the origins of life and for which contamination could jeopardize future experiments. Mars is in this category. [2pts]

(v) Mission to return cometary dust to Earth.
Always sample-return missions are in **Category V**, because there is a concern of the terrestrial system. [2pts]

This question was taken from the RGS book (Question 3.7), page 122.

3. **Is the water in Lake Vostok salty or fresh?** Lake Vostok beneath the ice of Antarctica is believed to be a good analog of Europa's ocean. However, Lake Vostok has fresh water and Europa's ocean is salty. This makes the density of Europa's ocean bigger than that of Lake Vostok.

Think of the ice as floating rafts. Read Box 4.7 of the text book by Rothery et al. on page 157. In the sketch below you see two identical ice rafts, but they are floating in media of different density, $\rho_{\rm I}$ and $\rho_{\rm II}$. Which one corresponds to Lake Vostok and which one to Europa's ocean? Explain your answer.



The buoyancy force of a body is equal to the weight of the displaced water. The higher the density of this displaced water, the larger the buoyancy, and the more it will stick out of the water, or the less it will need too lie in the water to attain the buoyancy needed to balance the weight of the floating body (the ice raft). This is the case in the left sketch, so $\rho_{\rm I} = \rho_{\rm Europa}$. In fresh water, the ice raft will sink more. This is the case in the right sketch, so $\rho_{\rm II} = \rho_{\rm Vostok}$. [6pts]

The following was not asked for, and is only given for completeness. Box 4.7 on page 157 of the Rothery et al. book explains that the weight of the icy raft is equal to its density ($\rho_{\rm ice}$) times its volume, say $L_x \times L_y \times L_z$. Here, L_x and L_y are the side lengths, and $L_z = h + w$ is the height of the raft, being the sum of the part underneath the water (w) and the part above the water surface (h). The weight of the raft is thus $\rho_{\rm ice} \times L_x \times L_y \times L_z$. The buoyancy, on the other hand, is $\rho_{\rm water} \times L_x \times L_y \times w$. Setting them equal to each other gives

$$\rho_{\rm ice} \times L_x \times L_y \times L_z = \rho_{\rm water} \times L_x \times L_y \times w.$$

Evidently, the factor $L_x \times L_y$ on both sides cancels, so we have

$$\rho_{\rm ice} \times L_z = \rho_{\rm water} \times w.$$

Eliminating for w, we rearrange and have

$$w = \frac{\rho_{\rm ice}}{\rho_{\rm water}} \times L_z$$

, so we see that if the density of water increases (salty water instead of fresh), w decreases, so it lies less deep in thee water.