ASTR/GEOL-2040-001: Search for Life in the Universe

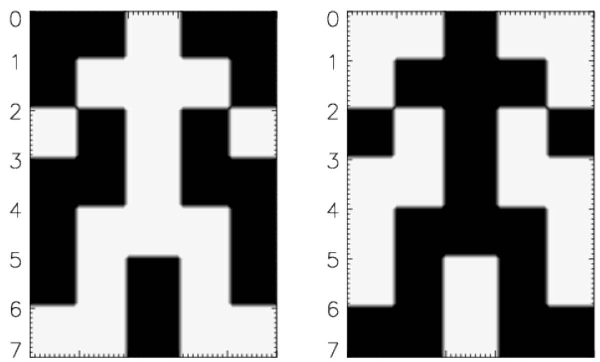
Homework #6 <u>Due: Friday December 8, 2017</u>

1. An extraterrestrial is sending you the following message.

   1 1 0 1 1 0 1 0 1 0 0 1 1 1 0 0 0 1 0 0 1 0 1 0 1 0 1 1 1 0 0 0 1 0 0

   Look at Figure 9.9 of the Rothery et al. book and assume that this extraterrestrial is sending you a picture just like in this figure. Draw this picture and write what you see.

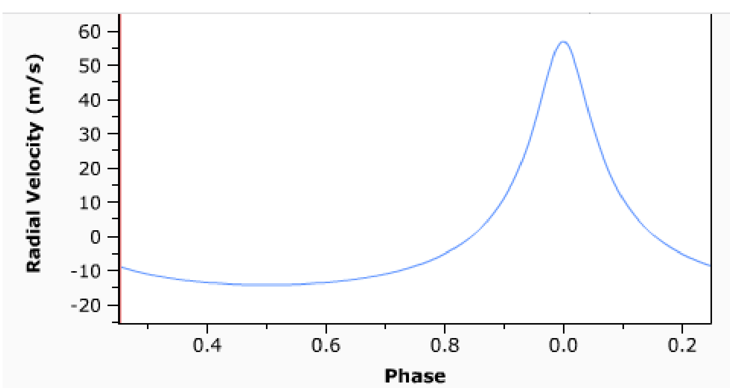
   The total number of transmitted digits is 35. This is the product of the two prime numbers 5 and 7. We arrange the pictures as 7 rows, each with 5 pixels. (Alternatively, we could have chosen 5 rows, each with 7 pixels.) The image is shown as a "positive" and a "negative". [2pts]

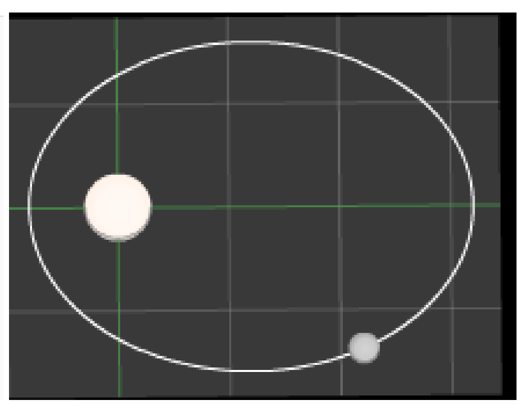


[3pts]

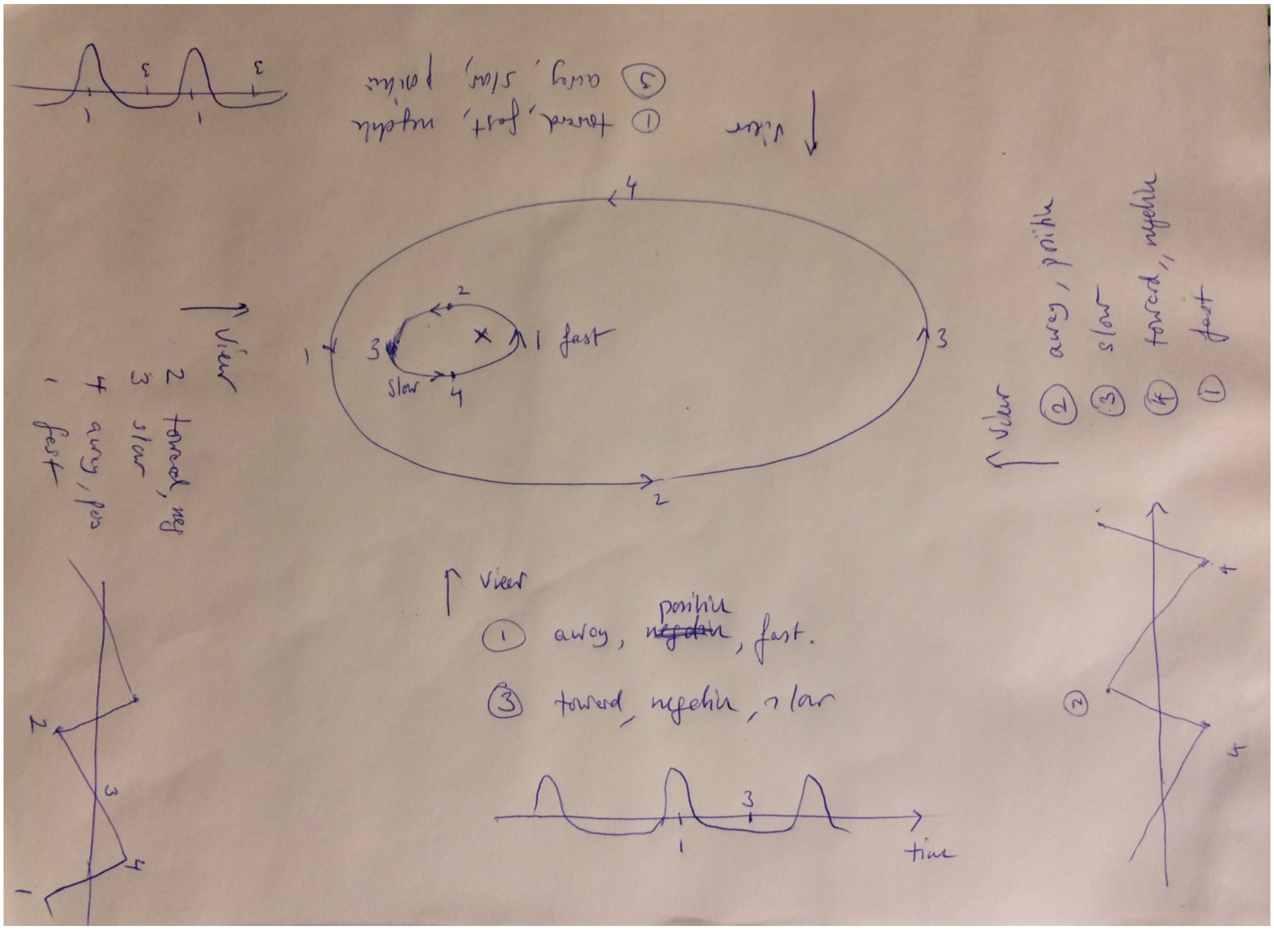
   This picture depicts a humanoid. [1pts]

2. Below you see a curve of the radial velocity versus time (or phase). The curve is rather asymmetric. In the other figure below you see an elliptic orbit. Positive velocities corresponds to a *star* moving away from the observer.





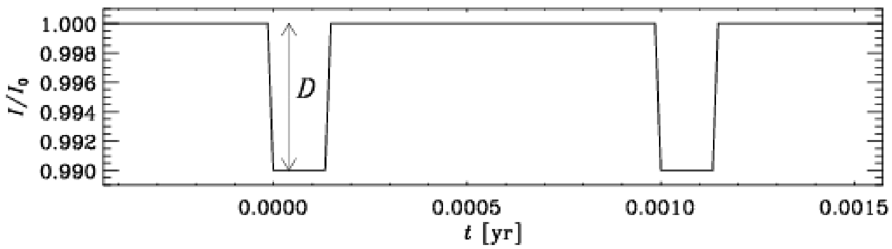
(a) Reproduce the sketch with the elliptic orbit of planet and star orbiting around a common center of mass. Indicate the direction in which the bodies orbit with arrows.
See the sketch below. [2pts]



(b) The observer measures a temporal change of the frequency of spectral lines. Explain how a velocity can be measured in this way and of which of the two bodies.
When the frequency is high (low), the star, which is the most luminous body, moves toward (away from) you, so the radial velocity of the star is negative (positive), and the planet moves away from (toward) you. [2pts]

(c) In which direction does the observer lie? Indicate the position in your sketch. Sketch the time dependence of the radial velocity from other viewing angles to make sure your answer is correct.
In the sketch above, four different viewing angles are shown. The bottom one matches the in the exercise: in phase 1, planet and star move fastest, the star moves away from you, so the velocity is positive, while in phase 3, planet and star move slowest, the star moves toward you, so the velocity is negative. Therefore, the curve have short positive peaks and extended negative valleys. [2pts]

3. Below is a light curve of a solar "twin" with periodic dips. Intensity I is normalized by the maximum value, I_0. This solar twin has a radius just like the Sun ($R = 700,000\,\text{km}$) and the same mass as our Sun. Thus, you can write Kepler's law as $P^2 = r^3$ if the orbital period P is measured in yr and the orbital radius r is measured in AU.



(a) How long is the orbital period P of the planet (in yr)?
The light curve is in the same phase at $t = 0$ and then next time at $t = 0.001$ yr, so the period is $P = 0.001\,\text{yr} = 10^{-3}\,\text{yr}$. [3pts]

(b) What is the radius r of the planet's orbit (in AU)?
Since $r^3 = P^2$, we have $r = (P^2)^{1/3} = 10^{-3\times2/3}\,\text{AU} = 10^{-2}\,\text{AU}$. [3pts]

4. Figure 1 shows a $400\,\text{km} \times 400\,\text{km} = (400\,\text{km})^2$ patch with "synthetic" lunar craters. Determine the age of the surface hosting these craters using the left panel of Figure 1, where isochrones are shown in a graph showing the number count $N(D)$ versus diameter D per $10^6\,\text{km}^2$ for constant logarithmic mass bins (corresponding roughly to the intervals between 32–48 km, 48–64 km, 64–96 km, etc).

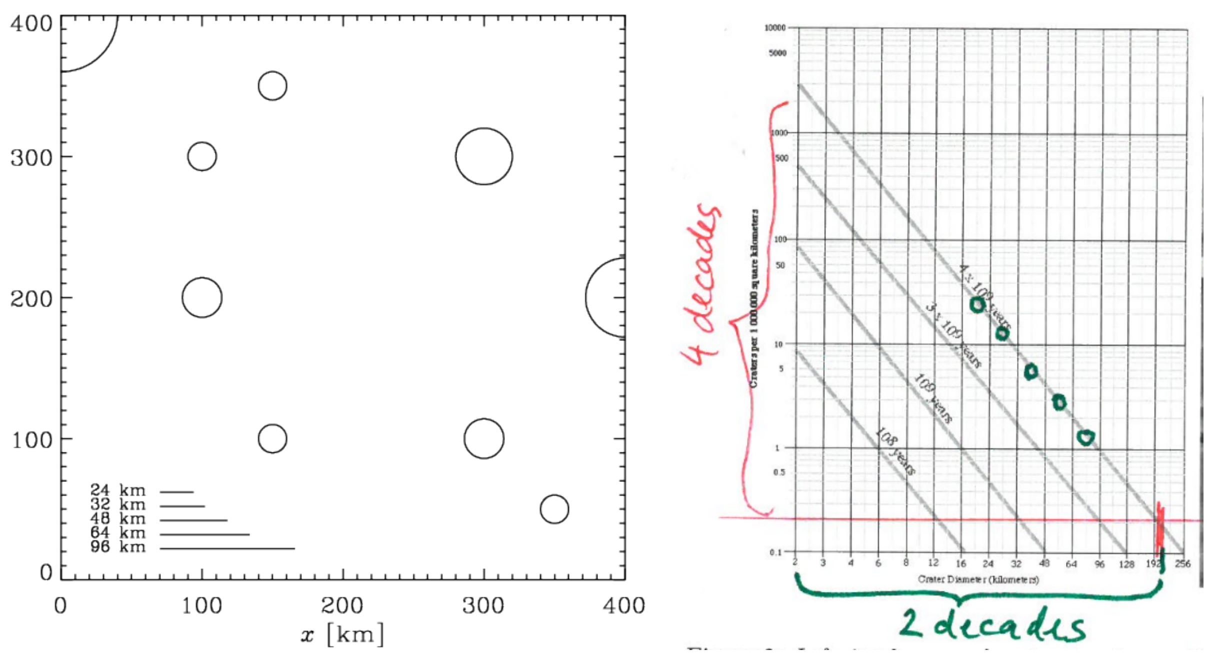


Figure 1: *Left*: $400\,\text{km} \times 400\,\text{km} = (400\,\text{km})^2$ patch with "synthetic" lunar craters. *Right*: the 5 datapoints from the table overplotted on the isochrones.
The slope is approximately $-4/2 = -2$.

(a) Count the number of craters for each of the 5 size bins given in the table below and enter that number in the 2nd column, which is the number count per $(400\,\text{km})^2$. Count 1/2 craters as 1/2, etc.
The counts are 1/4, 1/2, 1, 2, and 4; see the table. [2pts]

(b) To use the left panel of Figure 1, convert your number count to one per $10^6\,\text{km}^2$ and enter those values in the 3rd column.
Since $(400\,\text{km})^2 = (0.4 \times 1000\,\text{km})^2 = 0.16 \times (1000\,\text{km})^2$, we have to divide by $0.16 \times (1000\,\text{km})^2$

and multiply by $(1000\,\mathrm{km})^2$, so just divide by 0.16. The rescaled counts are then 1.56, 3.1, 6.2, 12.5, and 25. [2pts]

(c) Check whether you did this right: should the value in column 3 be larger or smaller than the value in column 2?
A surface area of $(1000\,\mathrm{km})^2$ is larger than that of $(400\,\mathrm{km})^2$, so the crater count per $(1000\,\mathrm{km})^2$ should also be larger. Comparing columns 2 and 3, this is indeed the case. [2pts]

(d) Overplot these values in Fig. 1 and, thus,
See the dots in the figure. [2pts]

(e) estimate the age of the surface.
The points lie on the line corresponding to 4 Gyr of age.
The slope is approximately $-4/2 = -2$. [2pts]

(f) The isochrones in the figure obey $N(D) \propto D^{-q}$, where N is the number count and D the radius. What is the value of the exponent q? Indicate your working in the figure and on this paper (e.g., mark approximate decades on the axes.)
As seen from the annotations on the plot, it covers a little below 2 orders of magnitude in the x direction, and a little over 4 orders of magnitude in the y direction. so the slope is 2. The number $N(D)$ is decreasing with increasing D, but because q is defined with a minus sign, q should be positive, i.e., $q \approx 2$. [2pts]

(A more accurate calculation is given by the intersections of the 4 Gyr isochrone, which intersects the x axis at $D = 256$ and $N = 0.1$ and the y axis at $D = 2$ and $N = 300$, so the slope can be obtained by

$$q = \frac{\ln 3000 - \ln 0.1}{\ln 256 - \ln 2} = \frac{5.70 + 2.30}{5.55 - 0.69} \approx 2.12.$$

size range	crater count per $(400\,\mathrm{km})^2$	crater count per $(1000\,\mathrm{km})^2$
16–24 km	4	25
24–32 km	2	12.5
32–48 km	1	6.2
48–64 km	1/2	3.1
64–96 km	1/4	1.56