Please try to be neat when writing up answers. In cases where calculations are called for, please show all of the intermediate steps, including any approximations you choose to make and any sketches you may need to illustrate what's what. Be careful to properly evaluate units and significant figures. Calculations given without 'showing the work' will receive zero credit, even if the final answer is correct.

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1. Your favorite space weather story.

(a) Describe in a few words your favorite space weather story.

(b) Try to complement the story with some simple numerical estimates to check whether it makes sense.

(c) How frequent is an event of the type you described?

2. Eddington approximation.

In class we used several approximations to derive the temperature structure $T(\tau)$ of the solar photosphere. If one uses this to solve for the source function S, and then plug S back into the equation of radiative transfer, we get a differential equation for $I(\mu,\tau)$. The solution to that equation is:

$$I(\mu,\tau) = \begin{cases} 3H_\odot\,(\tau+\mu+q) & \text{for } \mu > 0 \text{ (i.e., upward propagating rays),} \\ 3H_\odot\left[(\tau+\mu+q) - (\mu+q)\,e^{\tau/\mu}\right] & \text{for } \mu < 0 \text{ (i.e., downward propagating rays).} \end{cases}$$

where $q = 2/3$ and $H_\odot$ is proportional to the constant energy flux of radiation through the atmosphere. Also, let's define

$$I_{\rm up}(\tau) = I(+1,\tau) \qquad \text{and} \qquad I_{\rm down}(\tau) = I(-1,\tau)$$

(a) For the limiting case of $\tau \gg 1$, write simpler approximations for $I_{\rm up}$ and $I_{\rm down}$. Compute the ratio $I_{\rm up}/I_{\rm down}$. Does it make sense for this limit of the "deep interior?"

(b) Write the angle dependence $I(\mu)$ for the limiting case of $\tau \to 0$. Does it agree with what was discussed in class about the phenomenon of "limb darkening" at the solar surface?

(c) At the surface ($\tau = 0$), compute the angle moments

$$J = \frac{1}{2}\int_{-1}^{+1} d\mu\; I(\mu) \qquad H = \frac{1}{2}\int_{-1}^{+1} d\mu\; \mu\; I(\mu) \qquad K = \frac{1}{2}\int_{-1}^{+1} d\mu\; \mu^2\; I(\mu)$$

as functions of $H_\odot$. *Note:* You can set aside (for now) the fact that the moment quantity H is supposed to be the same thing as $H_\odot$. We're trying to see how "self-consistent" this model is.

(d) From your answer to part (d), evaluate whether Eddington's approximations ($J = 3K$ and $J = 2H$) are good approximations at the solar surface. In other words, if these approximations are not exactly true, then compute how "bad" they are (i.e., percentage error). You can also compare H to $H_\odot$ to get another estimate of the error.

3. Need for solar wind.

In class, we will derive that a hot corona (with $T \sim 10^6$ K) will be dominated by heat conduction in the regions far above the surface. In these regions, the temperature drops off very slowly with increasing distance,

$$T(r) = T_0 \left(\frac{r_0}{r} \right)^{2/7}$$

where r_0 is a base radius in the corona, often assumed to be about 2 $R_\odot$, and T_0 is the temperature at the base radius.

(a) Assuming a corona with no fluid flow (i.e., $\mathbf{u} = 0$ everywhere) in spherical symmetry, write the equation of hydrostatic equilibrium and show that it can be simplified into the form

$$\frac{d}{dr} \left(\frac{\rho}{r^{2/7}} \right) = -C_1 \frac{\rho}{r^2}$$

and give an expression for the constant C_1 in terms of the solar mass, the properties at r_0, and other physical constants.

(b) Show that the following solution satisfies the above differential equation,

$$\rho(r) = \rho_0 \left(\frac{r}{r_0} \right)^{2/7} \exp \left\{ C_2 \left[\left(\frac{r_0}{r} \right)^{5/7} - 1 \right] \right\} \qquad \text{where} \qquad C_2 = \frac{7\, C_1}{5\, r_0^{5/7}}$$

and ρ_0 is the density at r_0.

(c) If $r_0 = 2\, R_\odot$, $T_0 = 2 \times 10^6$ K, and $\rho_0 = 2 \times 10^{-15}$ kg m^{-3}, then solve the above expression for the *hydrogen number density* $n_{\rm H}$ at a distance of 1 AU. Feel free to assume that the corona is all hydrogen (i.e., just protons and electrons).

(d) The observed number density at 1 AU is usually between 1 and 10 protons per cm^3. Is the hydrostatic model a good one?

(e) Write an expression for the gas pressure P using the above hydrostatic model. Show that as $r \rightarrow \infty$, P approaches a constant value (call it P_∞) and derive an expression for P_∞.

(f) Compute a value for P_∞ given the constants from part (c).

(g) Astronomers find that the gas pressure in the interstellar medium (far outside the influence of the Sun) is about 10^{-14} to 10^{-13} pascals (i.e., N m^{-2}). Again, do you think this hydrostatic model is realistic? What do you think is *really* happening?