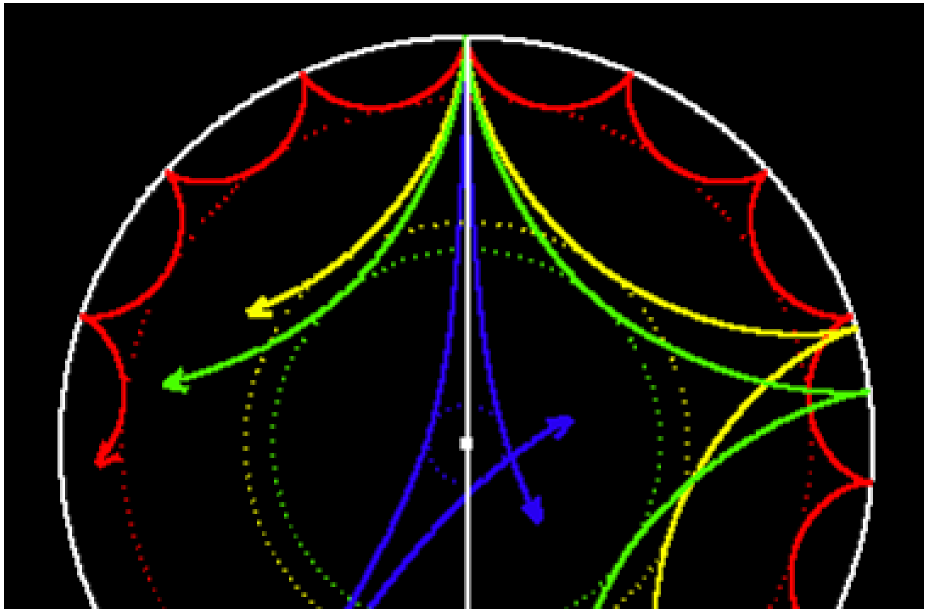
Lecture 12

- Solar 5 min oscillations
- Discrete frequencies
- Standing waves (Stix pp. 181-189)
- Helioseimology (Stix pp. 213, 214)

k_z^2



Recall lecture 11

Expand continuity eqn:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) = -\mathbf{u} \cdot \nabla \rho - \rho \nabla \cdot \mathbf{u}$$

Momntum eqn (isothermal):

$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\rho \mathbf{u} \cdot \nabla \mathbf{u} - \frac{\Re T}{\mu} \nabla \rho + \ldots$$

Linearized form

$$\frac{\partial \rho_1}{\partial t} = -\rho_0 \nabla \mathbf{u}_1$$

$$\rho_0 \frac{\partial \mathbf{u}_1}{\partial t} = -\frac{\Re T}{\mu} \nabla \rho_1$$

Trial solution

$$\rho_1(z,t) = \hat{\rho}_1 e^{ik_z z - i\omega t} + \text{c.c.}$$

$$u_{1z}(z,t) = \hat{u}_{1z} e^{ik_z z - i\omega t} + \text{c.c.}$$

$$\begin{pmatrix} i\omega & -ik_z \rho_0 \\ -ik_z \dfrac{\Re T}{\mu} & i\omega\rho_0 \end{pmatrix} \begin{pmatrix} \hat{\rho}_1 \\ \hat{u}_{1z} \end{pmatrix} = 0$$

Dispersion relation $\omega^2 = \dfrac{\Re T}{\mu} k_z^2$

$c_s = \sqrt{\Re T / \mu}$ Sound speed

Two solutions?

Real part $$u_{z1} = \hat{u}_{z1} \cos k_z \left(x \mp ct\right)$$

What if we superimpose the two?

$$u_{z1} = \hat{u}_{z1}\left[\cos k_z\left(x - ct\right) + \cos k_z\left(x + ct\right)\right]$$

A. Cancels to zero?
B. Oscillates only in space
C. Oscillates only in time
D. Oscillates both in space & time

Standing wave

2 nodes per wavelength

+ higher harmonics

sensitive to boundary conditions

→ music instruments

Sunspot & granulation

Size of the Earth 12 Mm

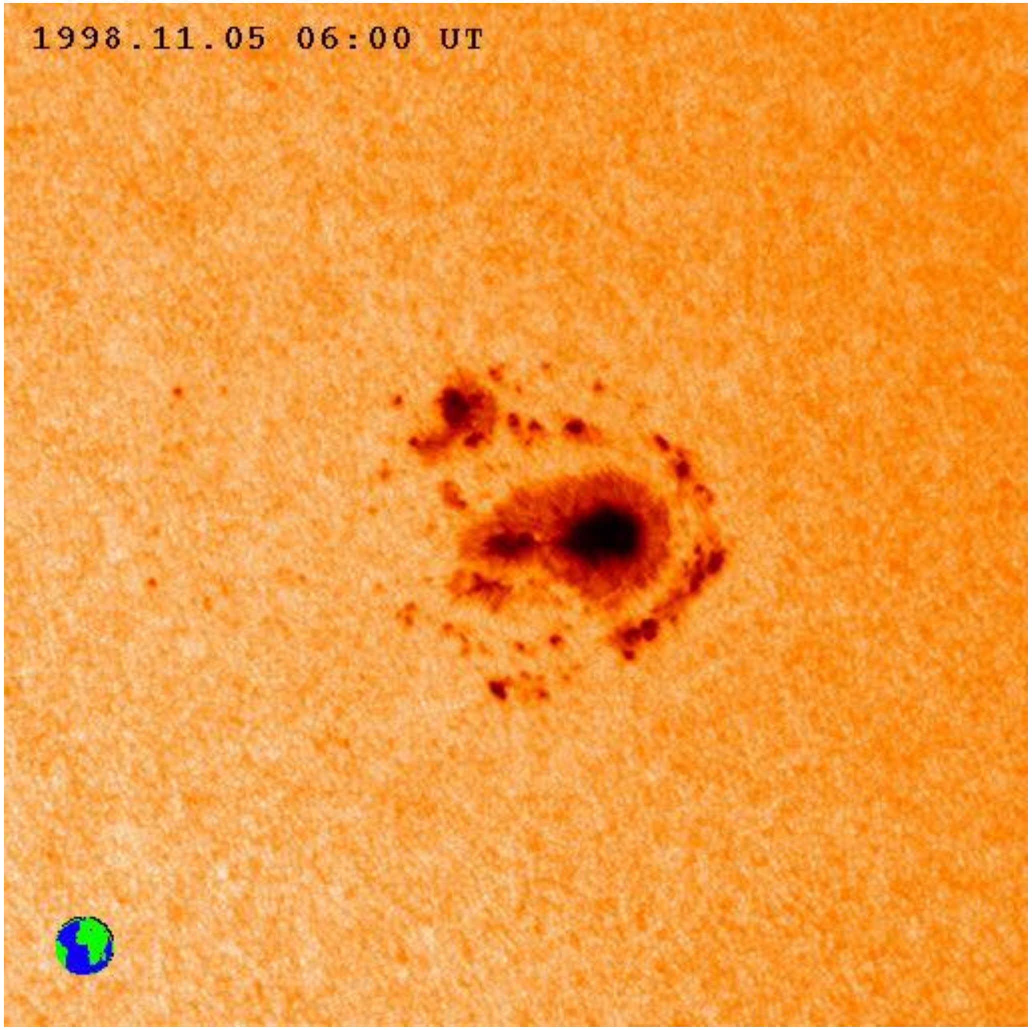
Size of Sunspots ~30 Mm

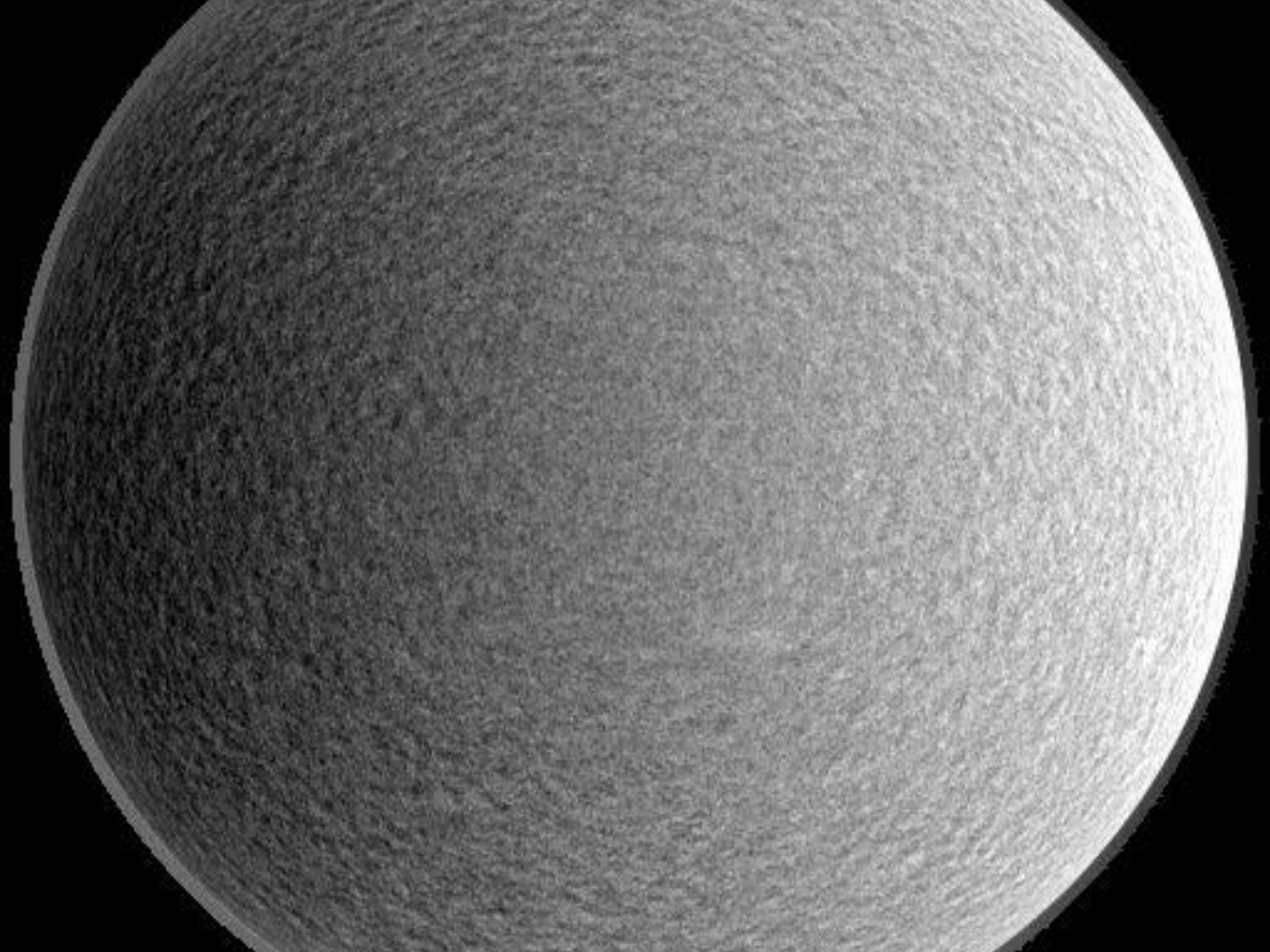
Life time ½ day – 3 months

Size of granules 1 Mm

Correlation time 5 min

Plus extra flickering!



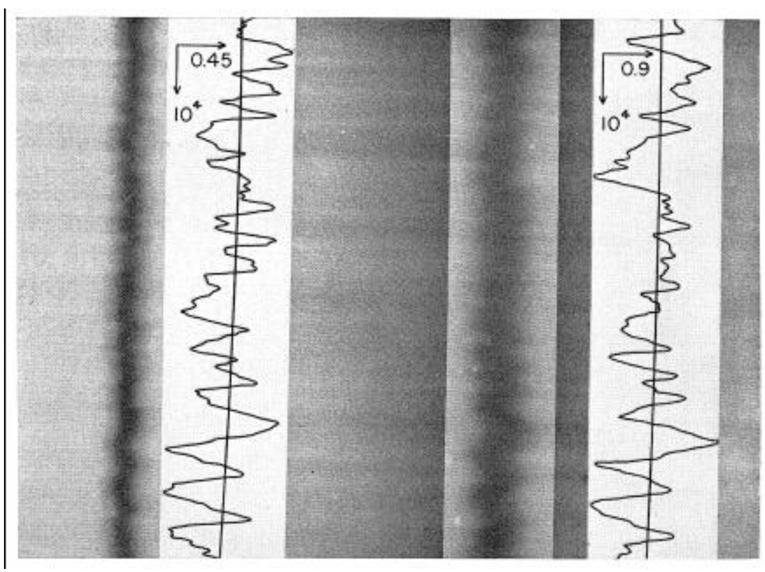


IAU meeting of 1960

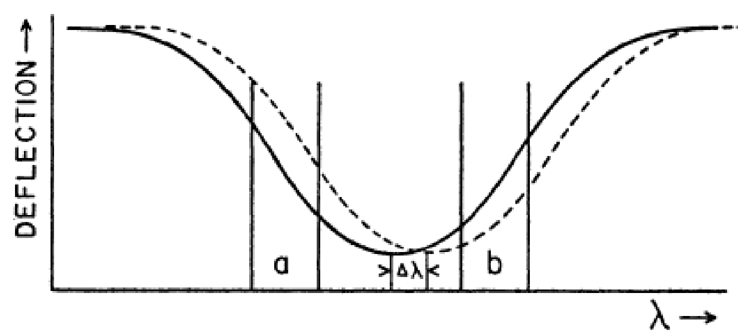
International Astronomical Union

— R. B. Leighton:

We have been spending about a week here discussing velocity fields, so I would like to take the liberty of showing you some as they appear on the surface of the sun. Let me first outline briefly the results which our observations have indicated to us. First, we have definite evidence for *horizontal motion* (*i.e.*, tangential to the solar surface) whose magnitude lies somewhere in the range 0.2 to 0.5 km/s, on a scale of about 30 000 km. This size is relatively large compared with the solar granulation. These motions represent



Discovered supergranulation (slow) and ?random vertical motion (fast)



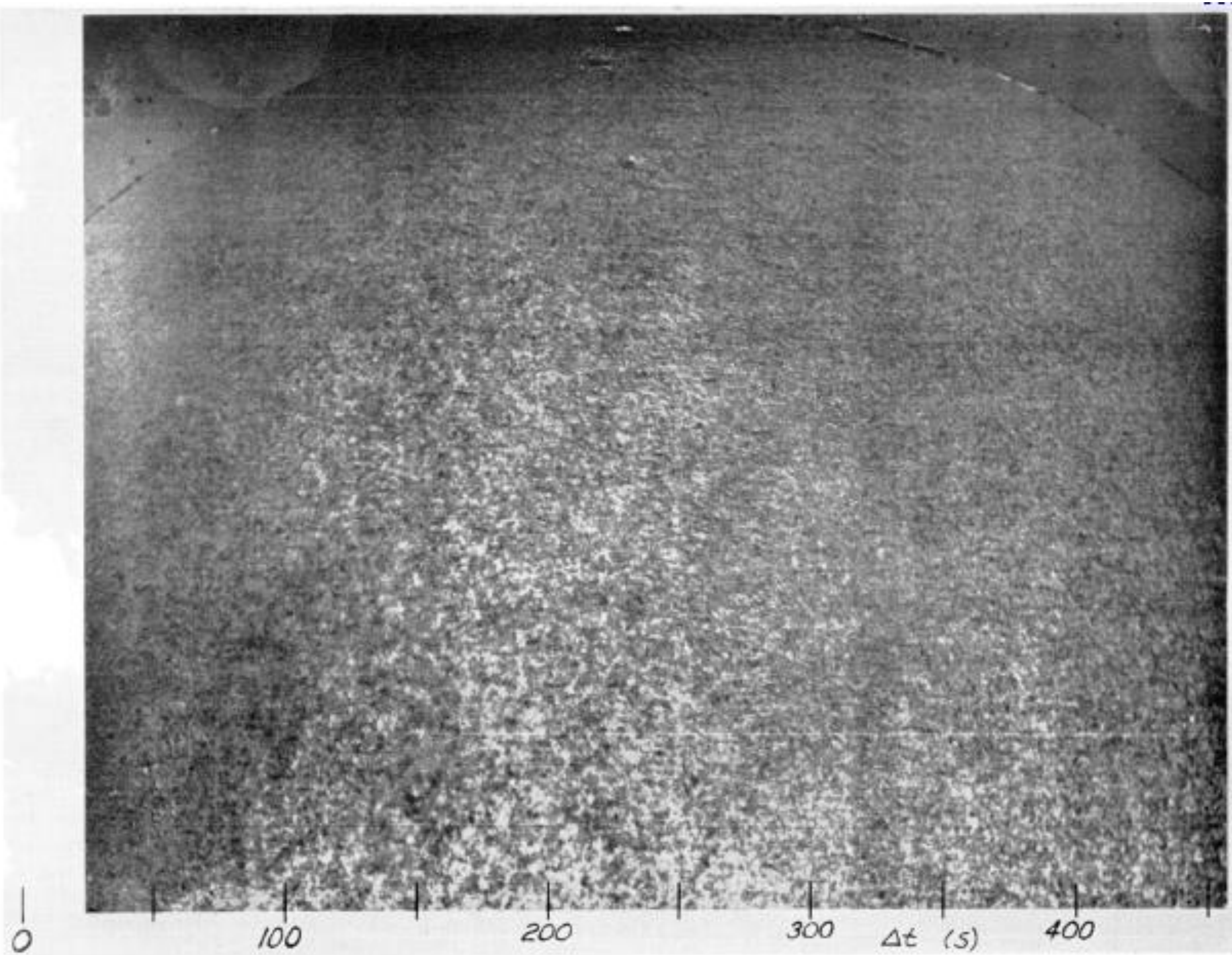
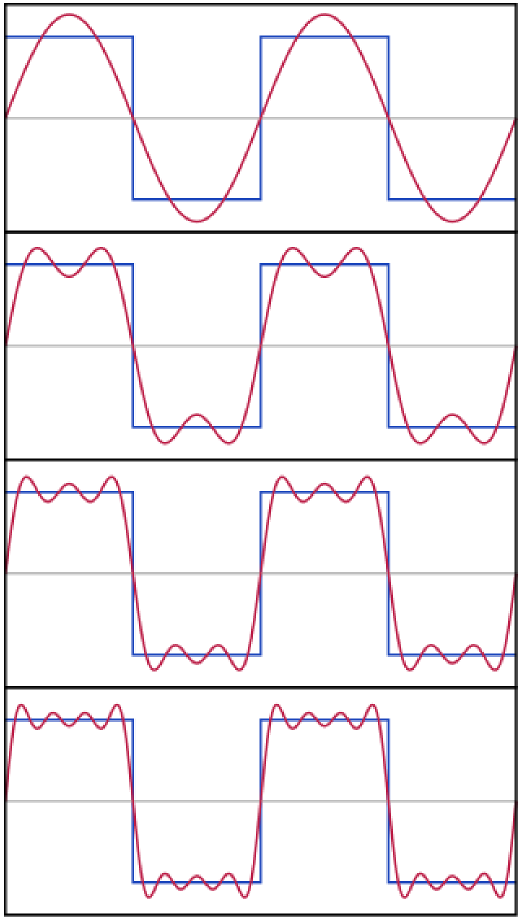


FIG. 14.—Doppler difference plate, showing the oscillatory time correlation of the small-scale velocity field. Ca 6103. June 10, 1960, 13^h40^m U.T.

Heard of Fourier series?

A. Yes
B. Maybe
C. Probably not
D. Certainly not

$$f(t) = \sum_{n=1}^{\infty} a_n \sin\underbrace{(2\pi n / P)}_{\omega_n} t$$



Fourier transform

Fourier sine series

$$f(t) = \sum_{n=1}^{\infty} a_n \sin \underbrace{(2\pi n / P)}_{\omega_n} t$$

Fourier cosine series

$$f(t) = \sum_{n=0}^{\infty} a_n \cos \underbrace{(2\pi n / P)}_{\omega_n} t$$

Continuous & complex version of Fourier series

$$f(t) = \int_{-\infty}^{\infty} \hat{f}(\omega)\, e^{-i\omega t} d\omega / 2\pi$$

Inverse transform

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{i\omega t} dt$$

Fourier transform: space & time

$$f(\mathbf{x},t) = \int_{-\infty}^{\infty} \hat{f}(\mathbf{k},\omega)\, e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t}\, \frac{d^2\mathbf{k}}{(2\pi)^2}\, \frac{d\omega}{2\pi}$$

Familiar from
Fourier ansatz
(=trial solution)

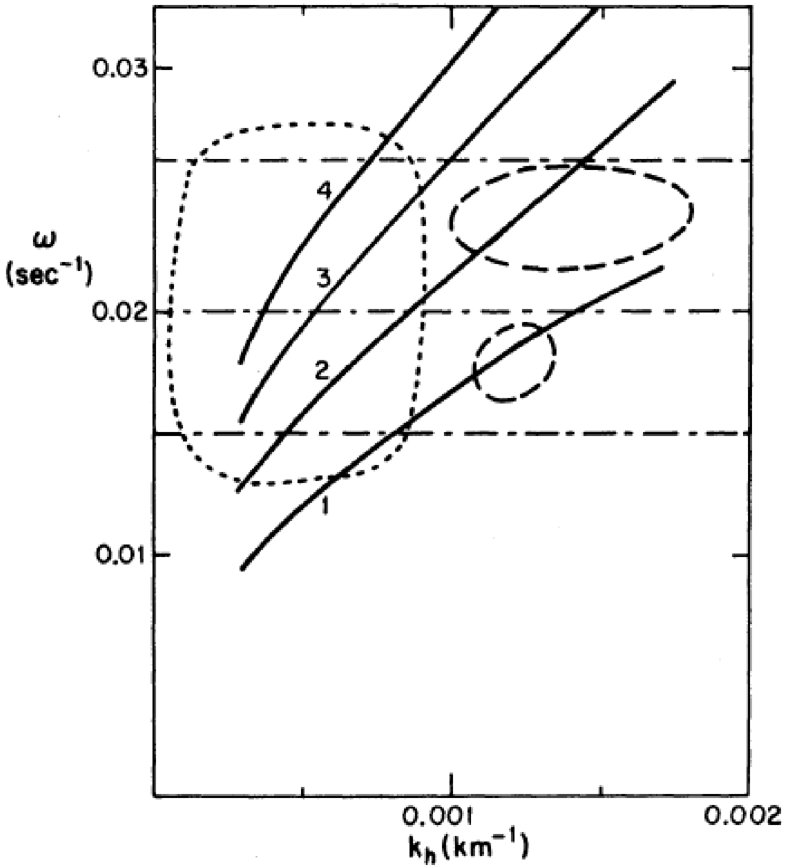
$$\rho_1(z,t) = \hat{\rho}_1 e^{ik_z z - i\omega t} + \text{c.c.}$$

$$u_{1z}(z,t) = \hat{u}_{1z} e^{ik_z z - i\omega t} + \text{c.c.}$$

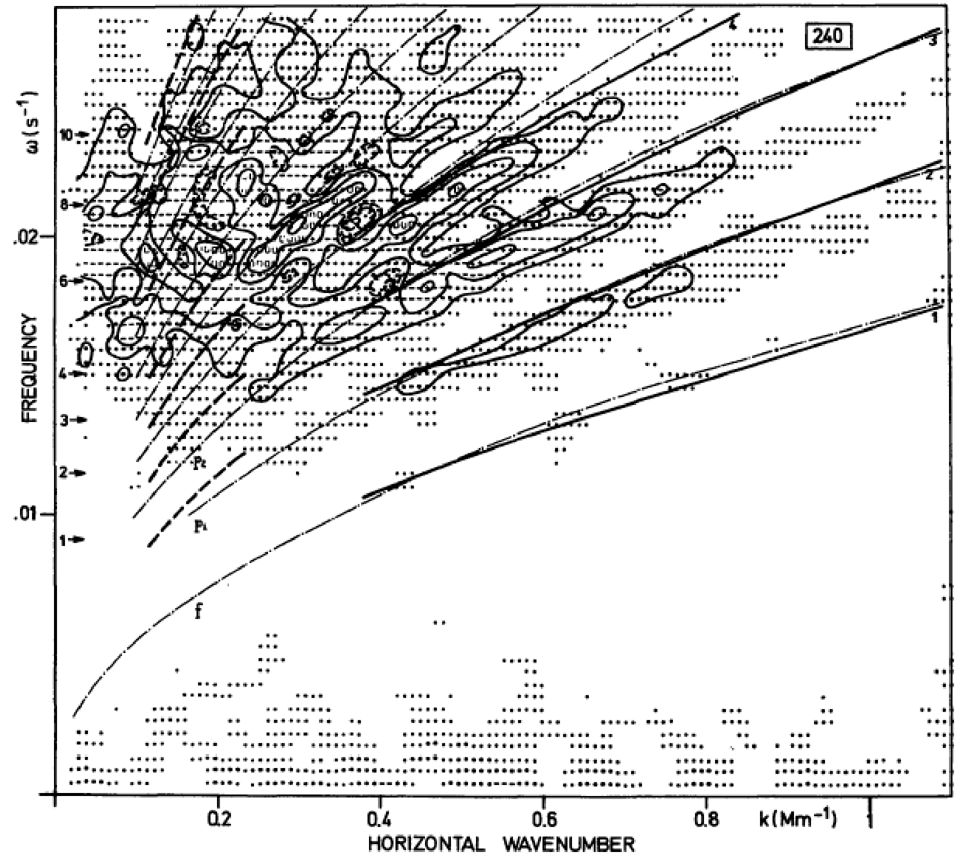
Computer routines: FFT (fast Fourier transform), forward & backward

$$\hat{f}(\mathbf{k},\omega) = \int_{-\infty}^{\infty} f(\mathbf{x},t)\, e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega t)}\, d^2\mathbf{x}\, dt$$

5 min osc are *global*



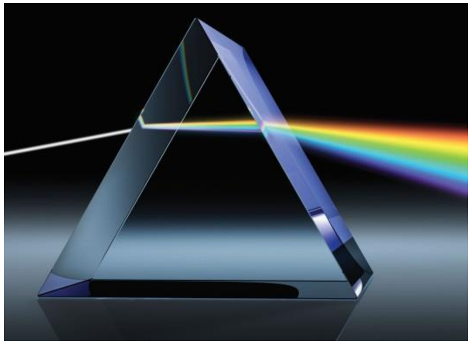
Roger Ulrich
(1970)



Franz-Ludwig Deubner
(1974)

Why should there be standing waves in the Sun?

- Is there a cavity?
- What forms it
- Think about this



and what is different?

Vertical wavenumber

Dispersion relation of Lecture 11

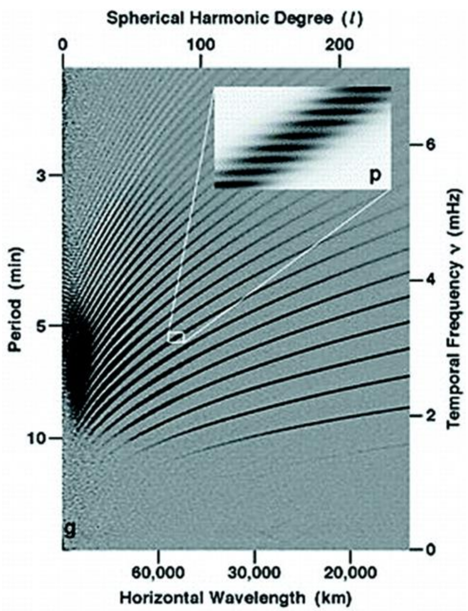
$$\omega^2 = \frac{\Re T}{\mu} k_z^2$$

In 3-D

$$\omega^2 = c_s^2 (\underbrace{k_x^2 + k_y^2}_{k_{\mathrm{hor}}^2} + \underbrace{k_z^2}_{k_{\mathrm{vert}}^2})$$

"Solve" for k_z

$$k_z^2 = \frac{\omega^2}{c_s^2} - k_{\mathrm{hor}}^2$$



Example

$$\omega = \frac{2\pi}{300\mathrm{s}} = 0.02\mathrm{s}^{-1}$$

$$k_{\mathrm{hor}} = \frac{\ell}{R} = \frac{100}{700\,\mathrm{Mm}}$$

$$c_s = \frac{\omega}{k_{hor}} = 0.02 \times 7\,\frac{\mathrm{Mm}}{\mathrm{s}} = 140\,\frac{\mathrm{km}}{\mathrm{s}}$$

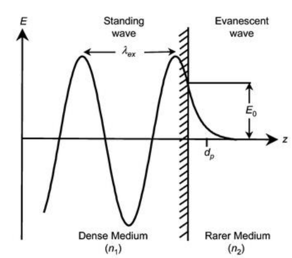
Consider $c_s = c_s(r)$ [oops?]

In quantum mechanics:
WKB approximation

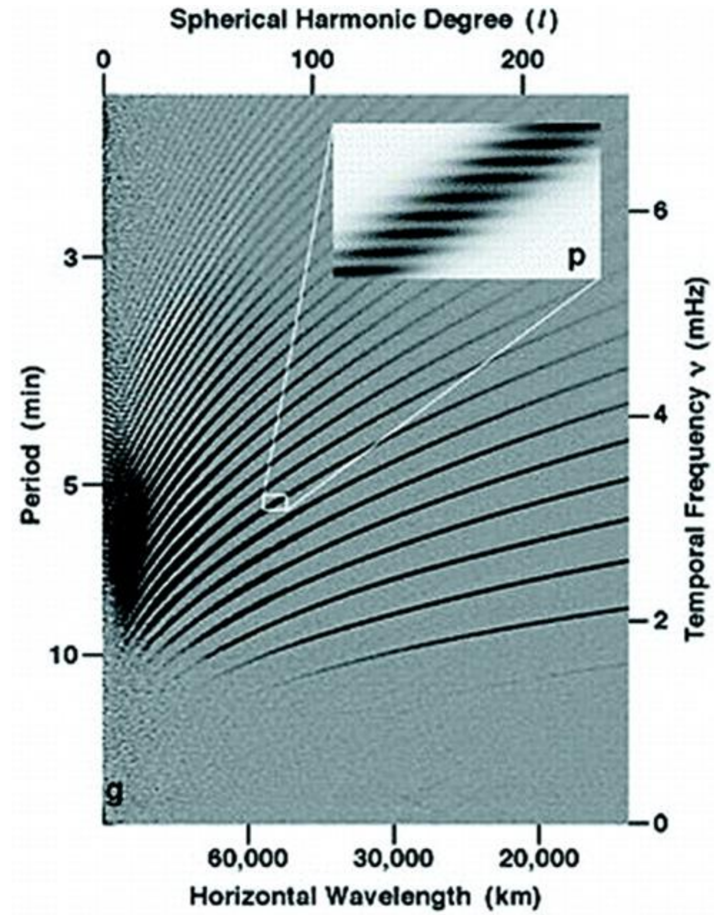
→ Jeffreys-Wrentzel-Kramers-Brilloin
→ Tunnel effect → Gamow!!

Deeper down: k_z imaginary!?

$$e^{ik_z z} = e^{\mp |k_z| z}$$



Which modes (k) needed to probe the core?



A. $kR>100$

B. $kR<100$

C. $kR<10$

D. $kR<1$ (i.e. impossible)

Number of nodes

$$n = \frac{L}{\lambda/2} = \frac{2k_z}{2\pi} L = k_z L/\pi$$

Continuous case

$$\pi n = \int_{z_{\mathrm{lower}}}^{z_{\mathrm{outer}}} k_z \, dz$$

$$k_z^2 = \frac{\omega^2}{c_s^2} - k_{\mathrm{hor}}^2$$

$$\pi n = \int_{z_{\mathrm{lower}}}^{z_{\mathrm{outer}}} \sqrt{\frac{\omega^2}{c_s^2} - k_{\mathrm{hor}}^2} \, dz$$

Just a function of *k/ω* …

$$\pi n/\omega = \int_{z_{\mathrm{lower}}}^{z_{\mathrm{outer}}} \sqrt{\frac{1}{c_s^2} - \frac{k_{\mathrm{hor}}^2}{\omega^2}} \, dz$$

Inversion: input/output

$$n\pi = \int_{r_0}^{R_0} k_r \,\mathrm{d}r$$

$$k_r = \sqrt{\frac{\omega^2}{c_s^2} - \frac{l(l+1)}{r^2}}$$

$$k_r = \frac{\omega}{r}\sqrt{\frac{r^2}{c_s^2} - \frac{l(l+1)}{\omega^2}}$$

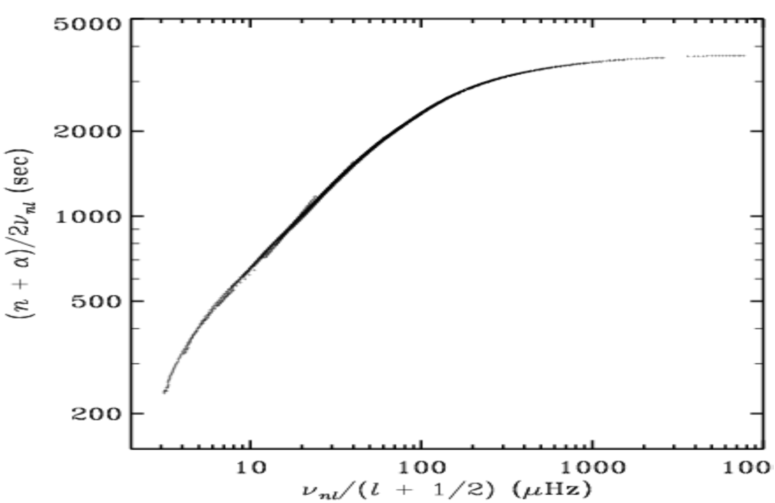
$$F(u) = \int_u^{\xi_0} \sqrt{\xi - u}\, G'(\xi)\,\mathrm{d}\xi$$

$$G(\xi) = \frac{2}{\pi}\int_{\xi}^{\xi_0} \frac{1}{\sqrt{\xi - u}} F'(u)\,\mathrm{d}u$$

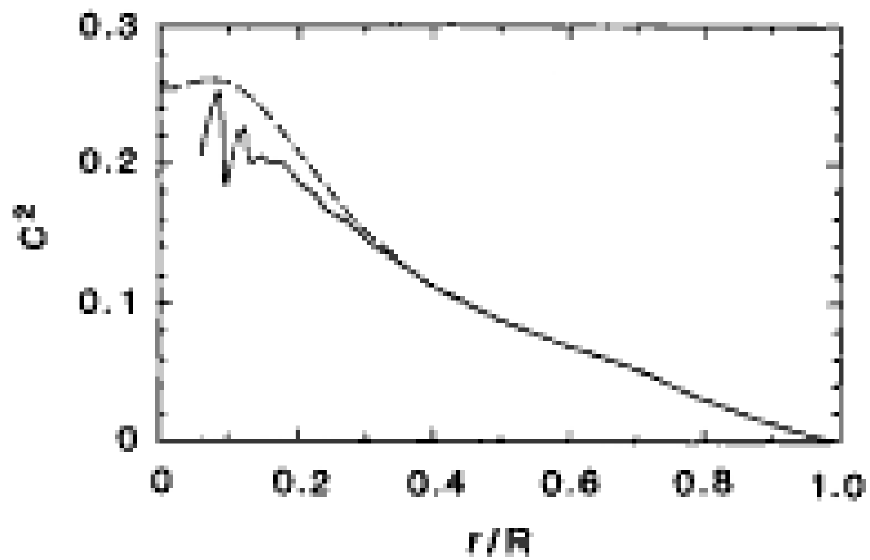
$$\xi \equiv \frac{r^2}{c_s^2}$$

$$u \equiv \frac{l(l+1)}{\omega^2}$$

$$G'(\xi) \equiv \frac{d \ln r}{d\xi}$$



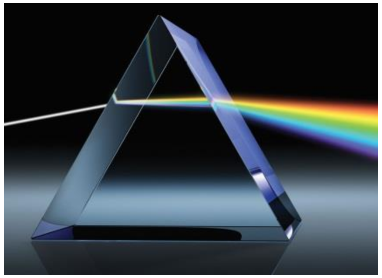
Duval law



Sound speed

What have we learnt today?

- Standing waves from 2 traveling ones
- granulation & oscillation different things
- Fourier transform: not so magic (perhaps)
- evanescent waves
- cavity in the Sun!



and what is different?