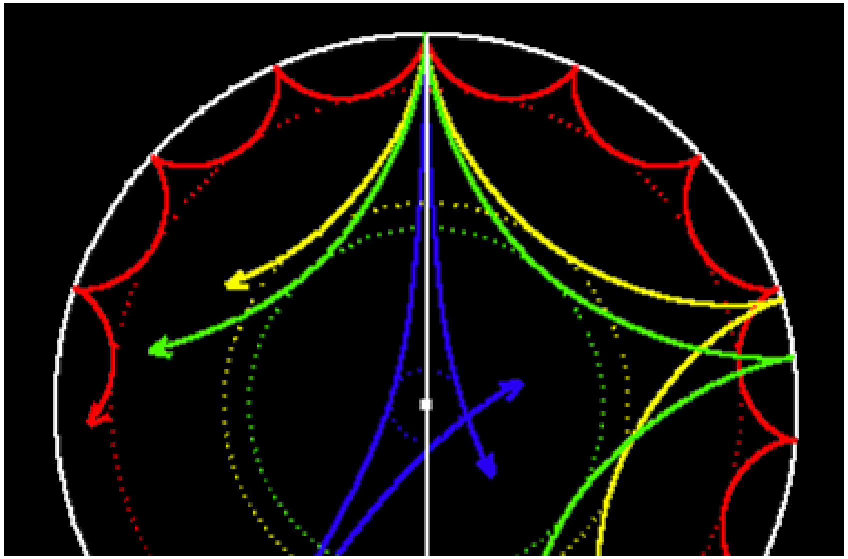
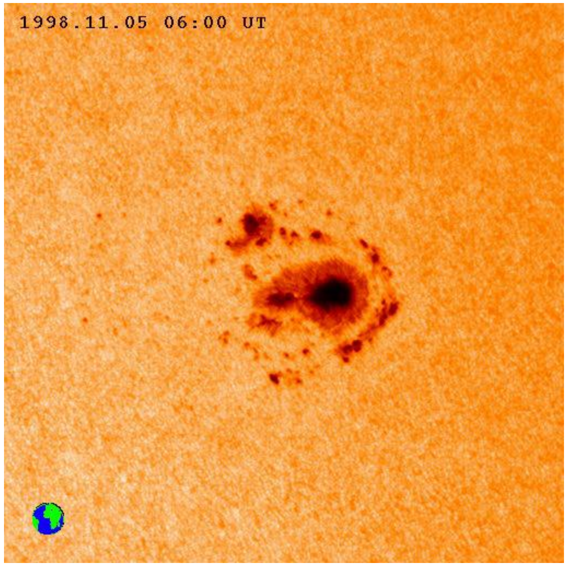
Lecture 13

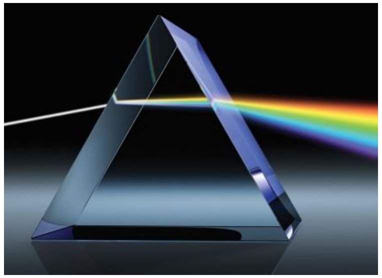
- FFT (Fast Fourier Transform): resolution
- Solar abundance (Stix pp. 215-219)
- Doppler shift (Stix pp. 226-228)





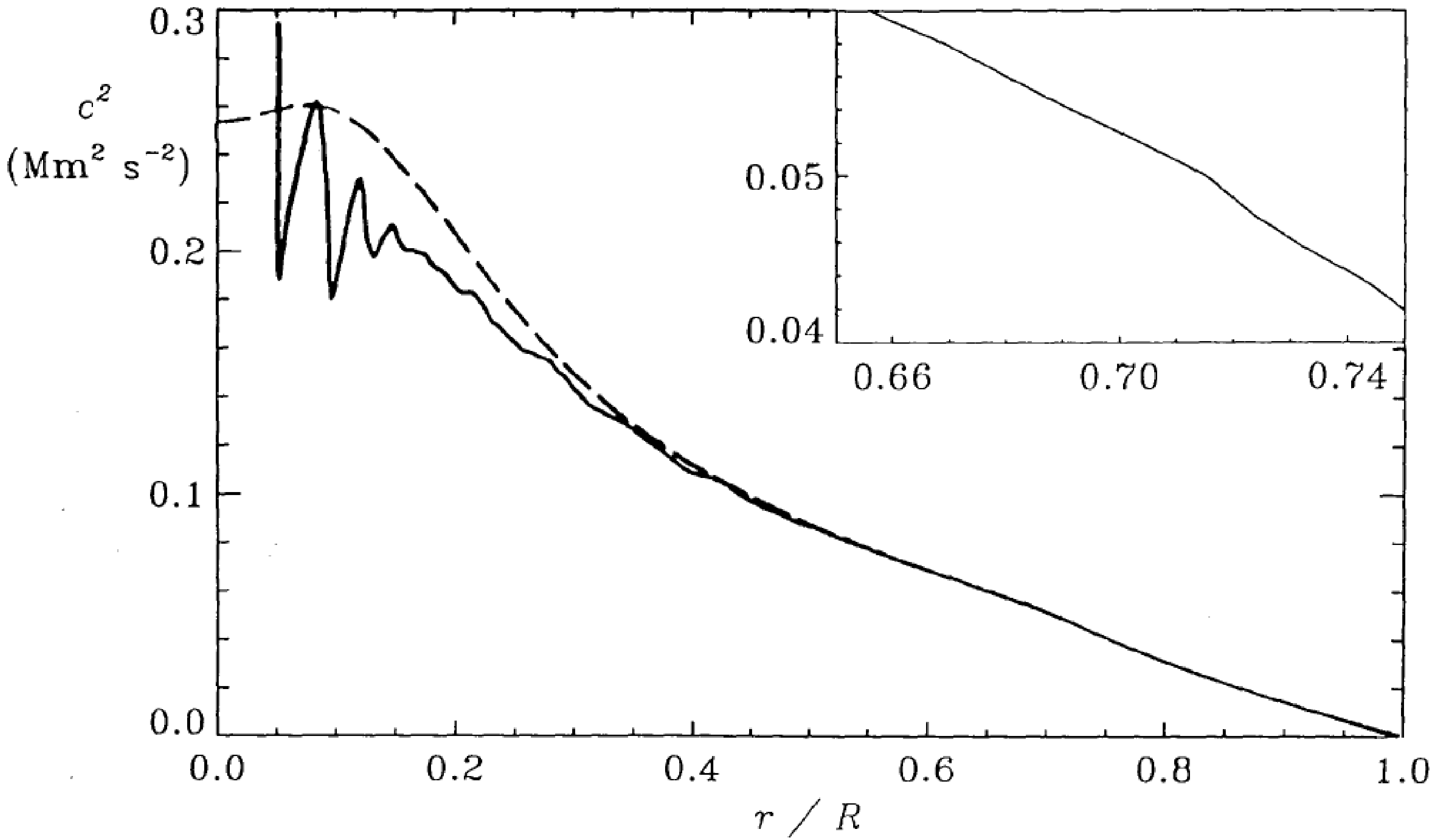
Last time

- Standing waves from 2 traveling ones
- granulation & oscillation different things
- Fourier transform ← quantum mechanics
- evanescent waves → tunneling
- cavity in the Sun!



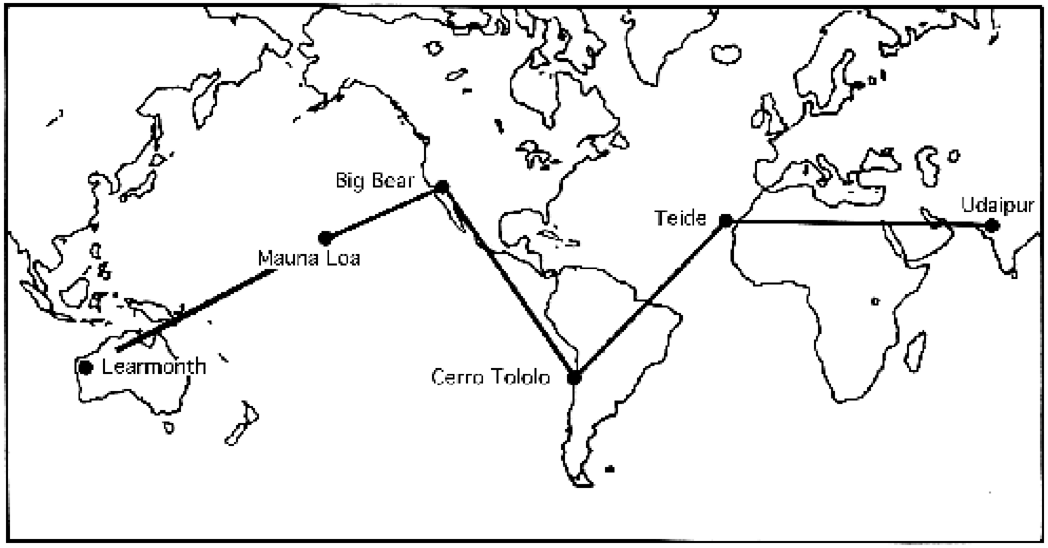
and what is different?

Helioseismology: change at 0.7R



GONG

global oscillation network group



Since late 1980ties

Continuous coverage: why?

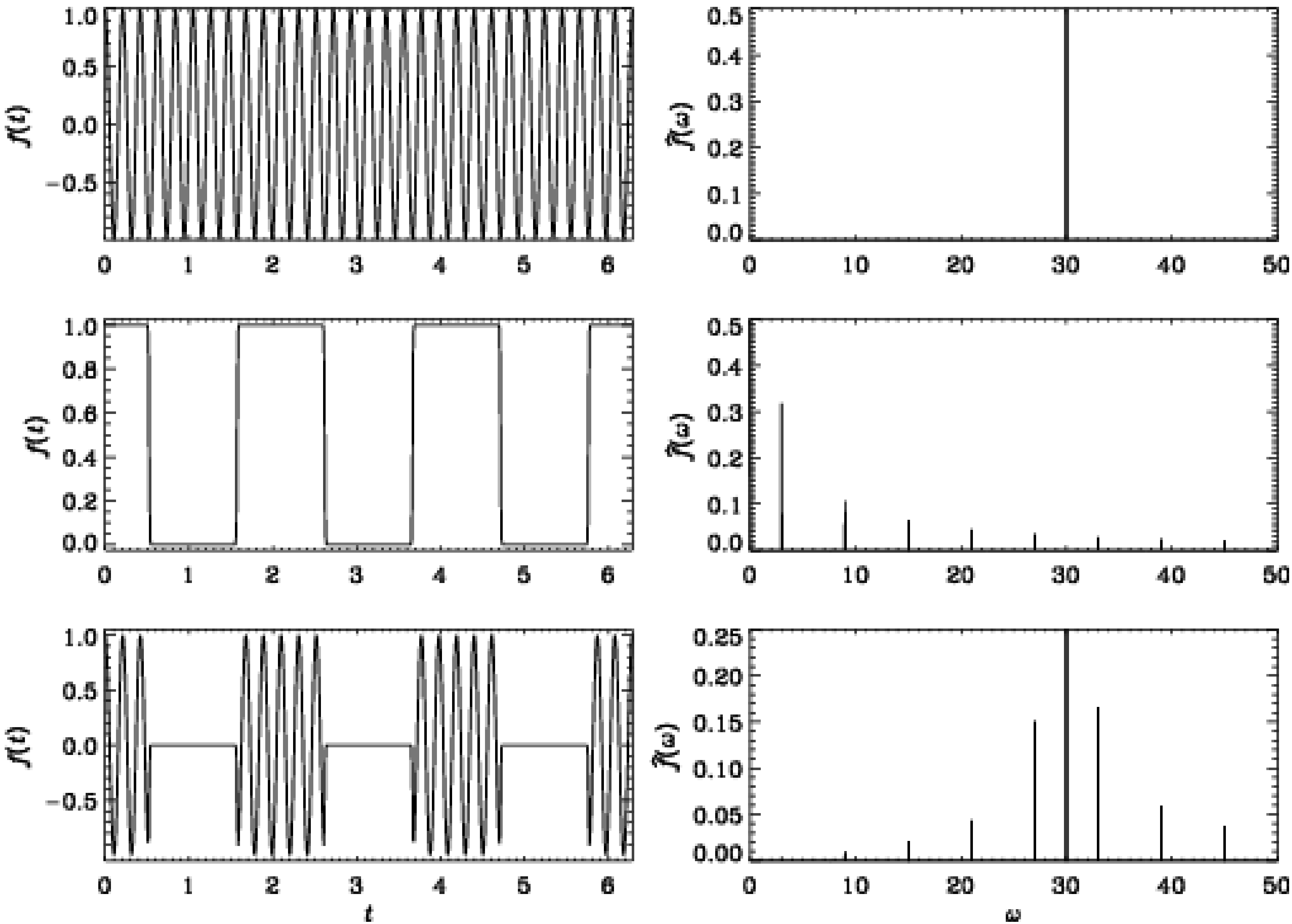
A. Avoid missing rare events
B. To get sharper lines
C. To include larger frequencies
D. To include smaller frequencies
E. To avoid artifacts like side lobes

$$\Delta\omega = \frac{2\pi}{t_{\max}} \qquad \omega_{\max} \overset{?}{=} \frac{2\pi}{\Delta t} \qquad \omega_{\text{Nyquist}} = \frac{\pi}{\Delta t}$$

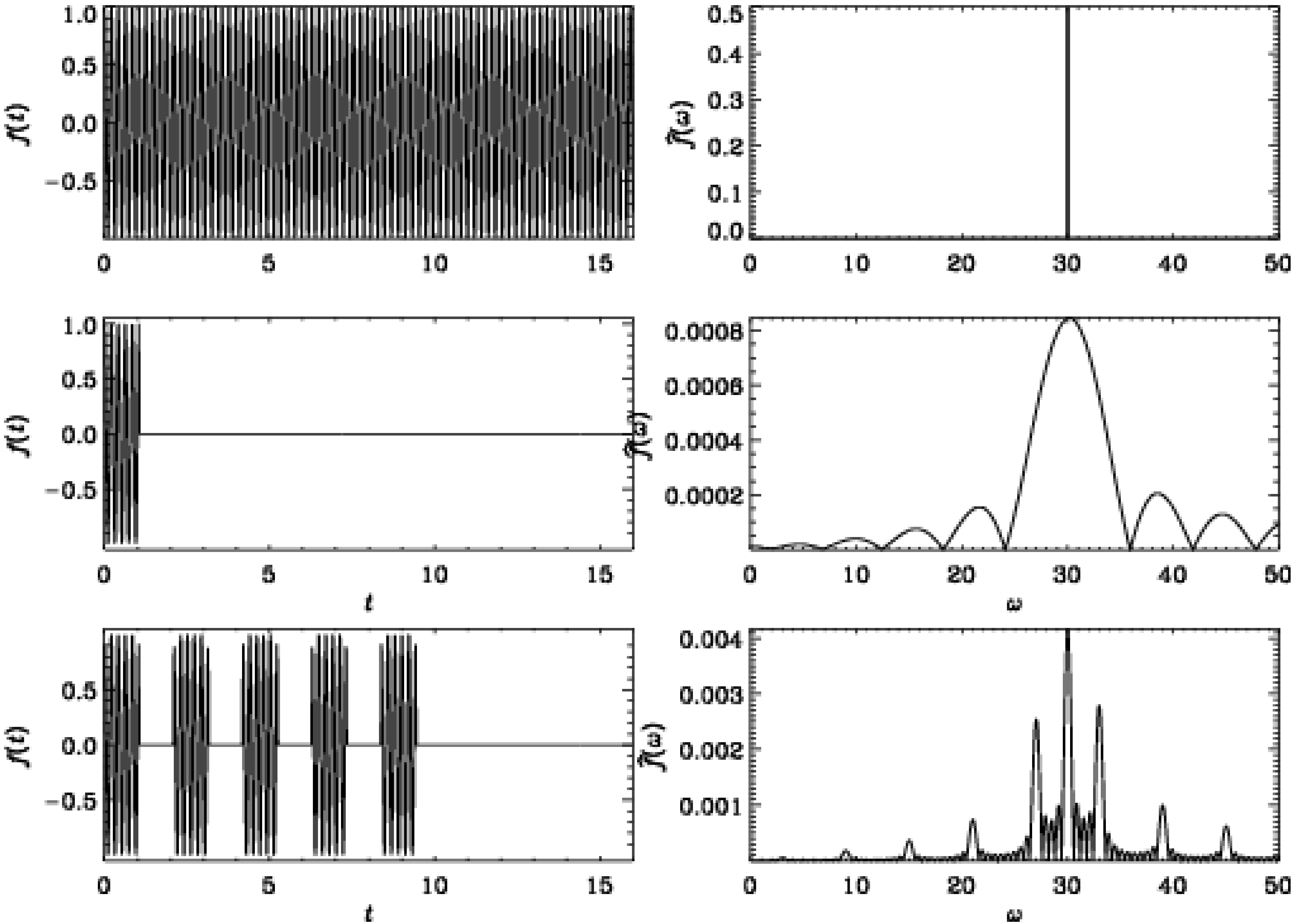
Because there are also negative frequencies

$$-\omega_{\text{Nyquist}} \le \omega \le \omega_{\text{Nyquist}} - \Delta\omega$$

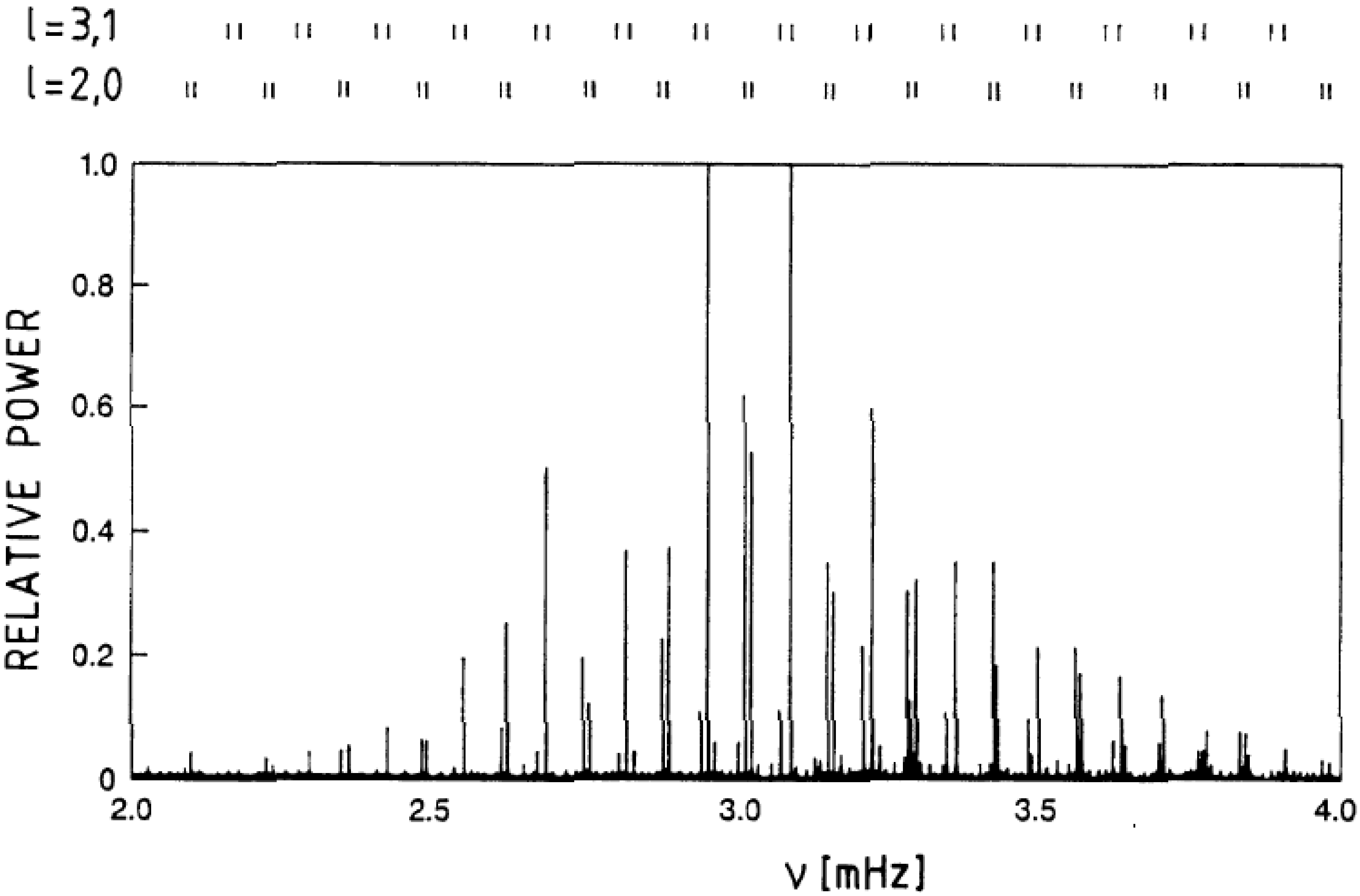
Numerical FFT experiments



Advantage of accumulating nights



3 months observation

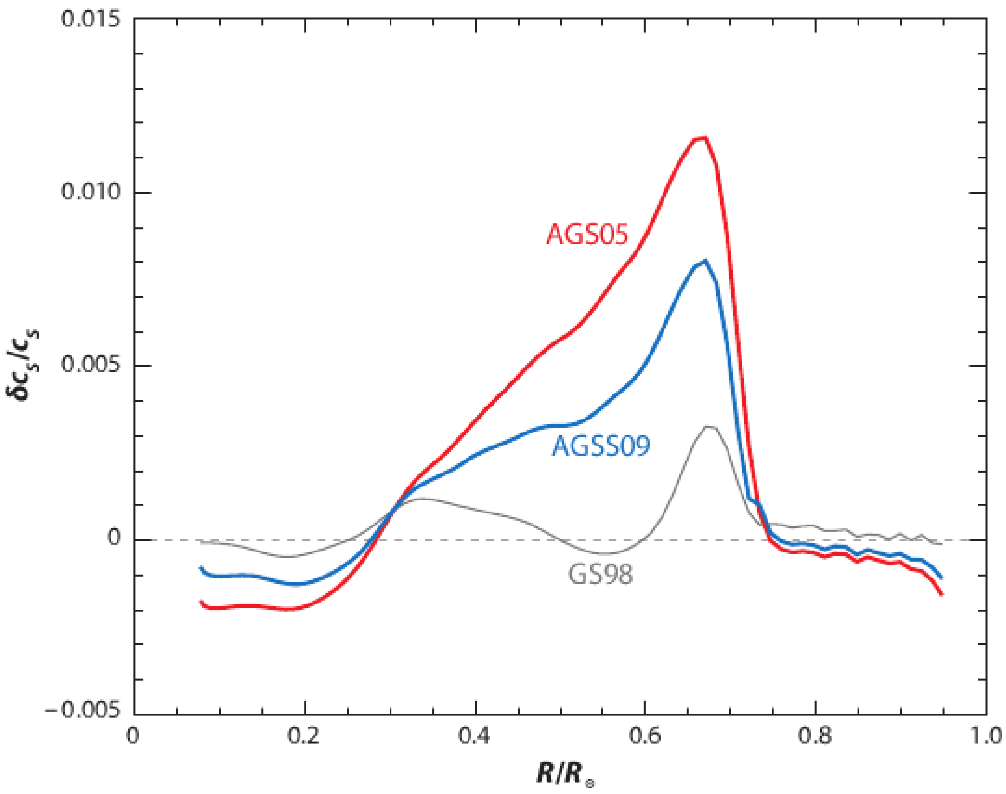


Open problems

Abundance of heavier elements (Z)

	X	Y	Z
GS98	.735	.249	.0231
AGS05	.739	.249	.0165

Opacity sensitive to Z
Theory of convection
Convective overshoot



Doppler shift: linearize about u_0=const

Expand continuity eqn: $$\frac{\partial \rho}{\partial t} = -\mathbf{u}\cdot\nabla\rho - \rho\nabla\cdot\mathbf{u}$$

Momntum eqn (isothermal): $$\rho\frac{\partial \mathbf{u}}{\partial t} = -\rho\mathbf{u}\cdot\nabla\mathbf{u} - \frac{\Re T}{\mu}\nabla\rho + \ldots$$

Linearized form

$$\frac{\partial \rho_1}{\partial t} = -\mathbf{u}_0\cdot\nabla\rho_1 - \rho_0\nabla\cdot\mathbf{u}_1$$

$$\rho_0\frac{\partial \mathbf{u}_1}{\partial t} = -\mathbf{u}_0\cdot\nabla\mathbf{u}_1 - \frac{\Re T}{\mu}\nabla\rho_1$$

Trial solution ="ansatz"

$$\rho_1(z,t) = \hat{\rho}_1 e^{ik_z z - i\omega t} + \text{c.c.}$$

$$u_{1z}(z,t) = \hat{u}_{1z} e^{ik_z z - i\omega t} + \text{c.c.}$$

$$\begin{pmatrix} \mathrm{i}\omega - u_{0z}ik_z & -ik_z\rho_0 \\ -ik_z\dfrac{\Re T}{\mu} & \mathrm{i}\omega\rho_0 - u_{0z}ik_z \end{pmatrix}\begin{pmatrix}\hat{\rho}_1 \\ \hat{u}_{1z}\end{pmatrix} = 0$$

Dispersion relation

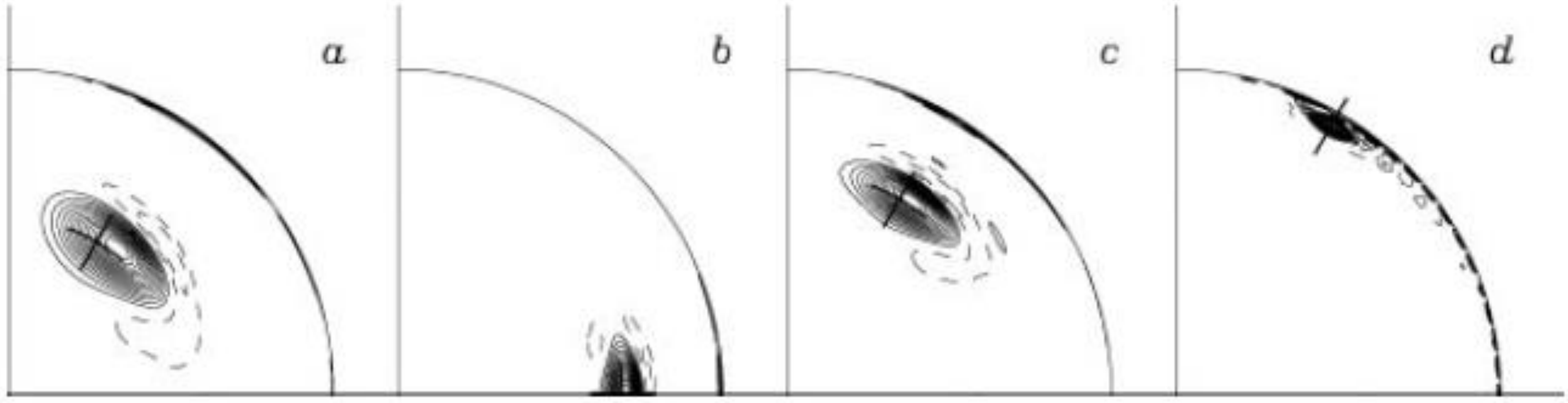
$$(\omega - u_{0z}k_z)^2 = \frac{\Re T}{\mu}k_z^2$$

$$c_s = \sqrt{\Re T/\mu}$$ Sound speed

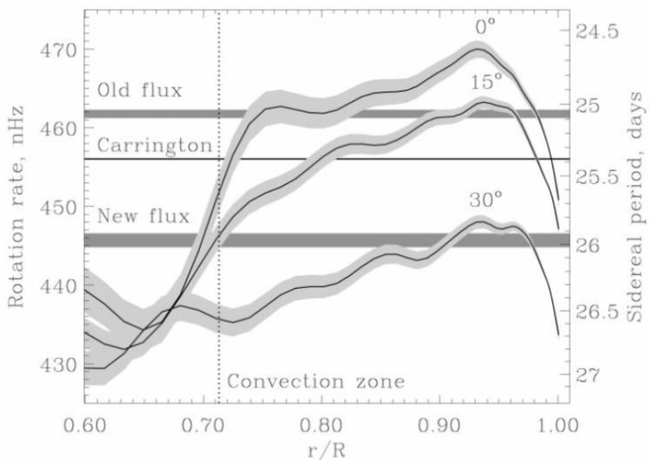
Internal angular velocity

Rotational splittings

$$\omega_{nlm} - \omega_{nl0} = m \int_0^{\pi} \int_0^{R} K(r,\theta)\,\Omega(r,\theta)\, r\mathrm{d}r\, \mathrm{d}\theta$$

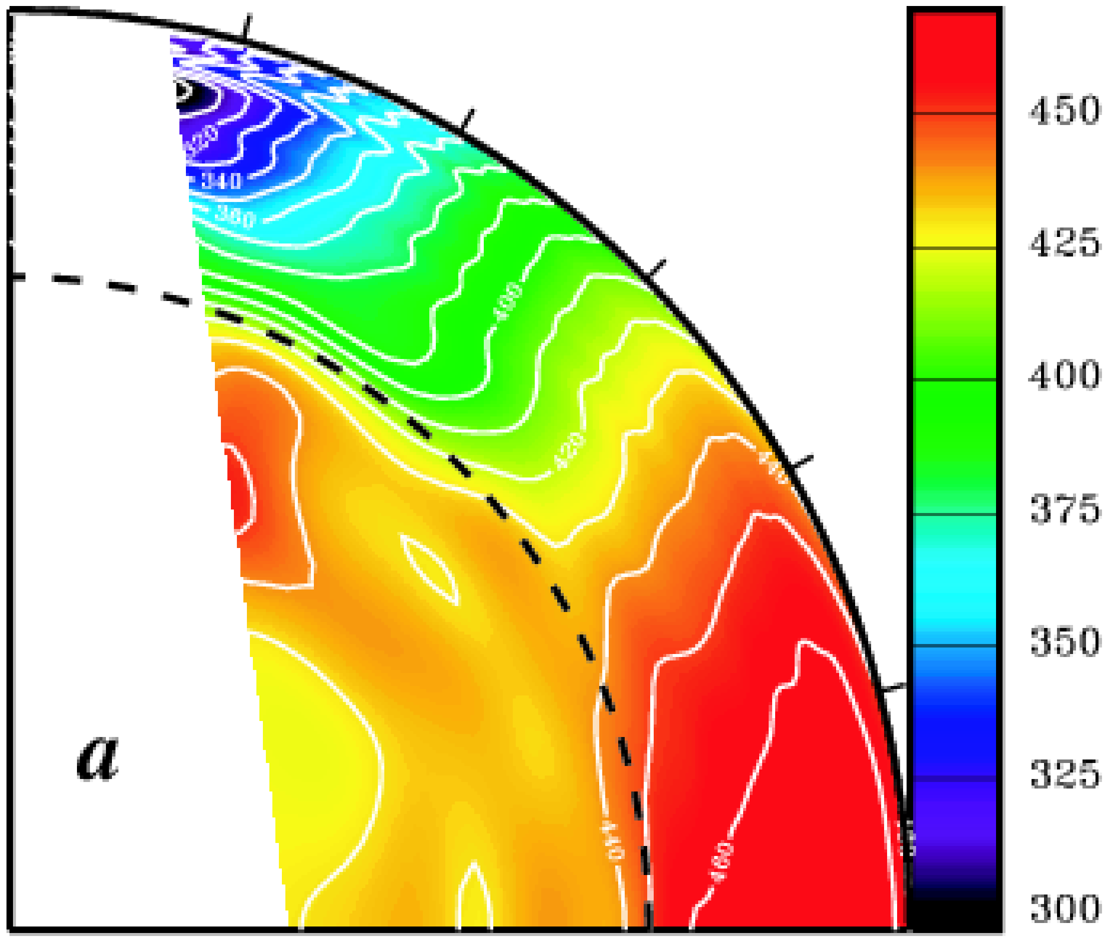


Internal angular velocity from helioseismology

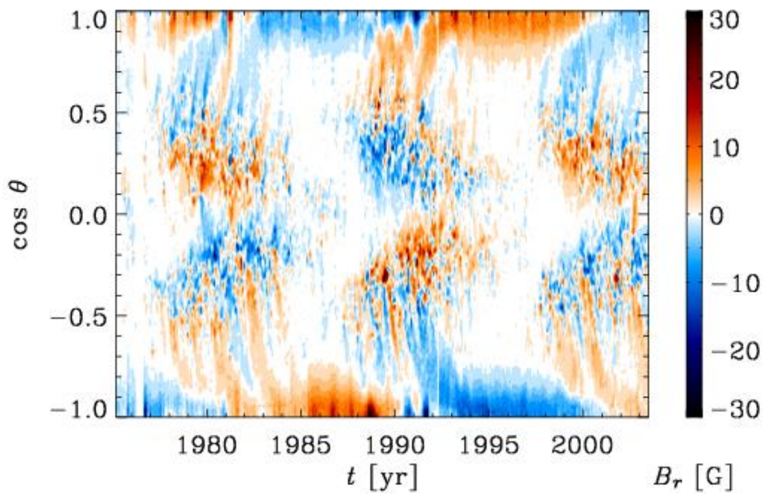


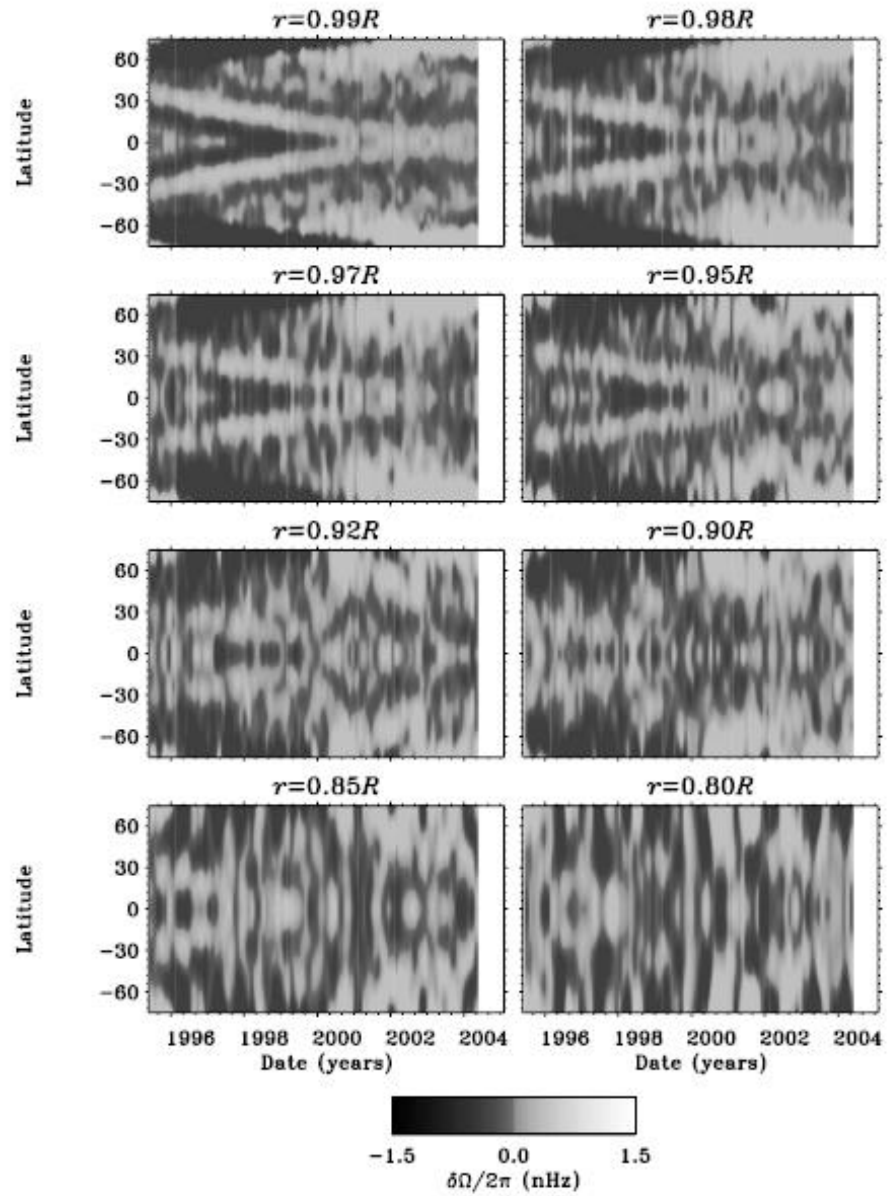
spoke-like at equ.
dΩ/dr>0 at bottom

? dΩ/dr<0 at top

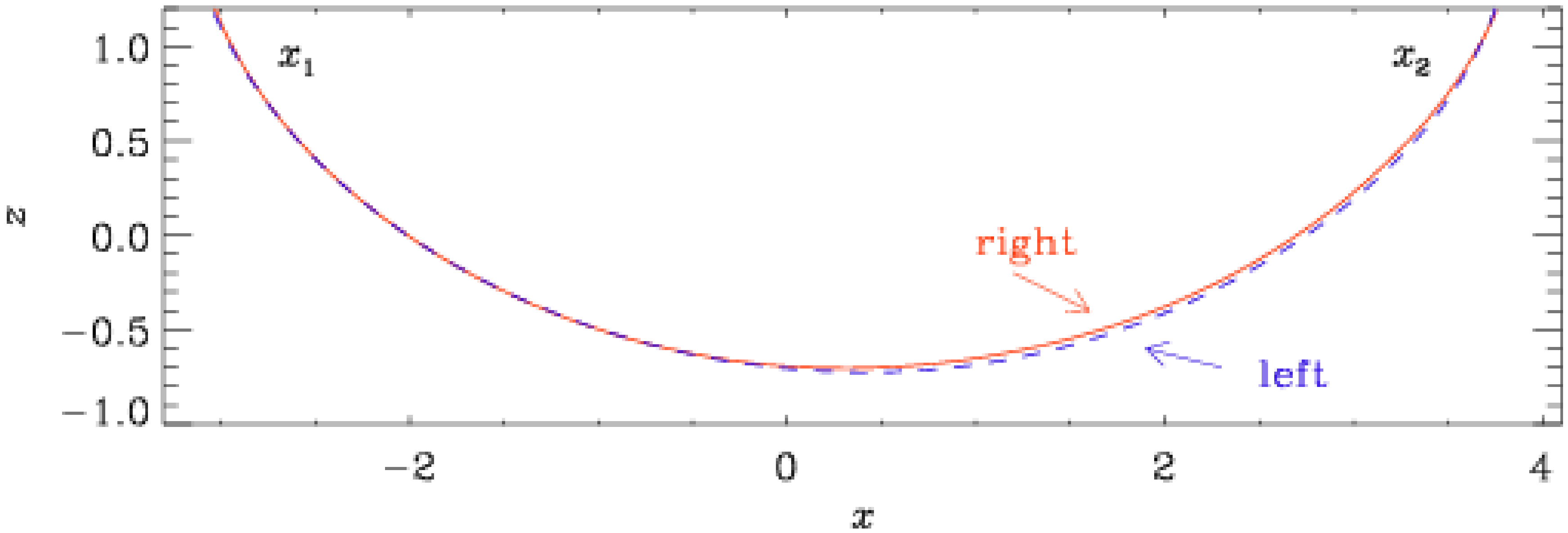


Cycle dependence of Ω(r,θ)



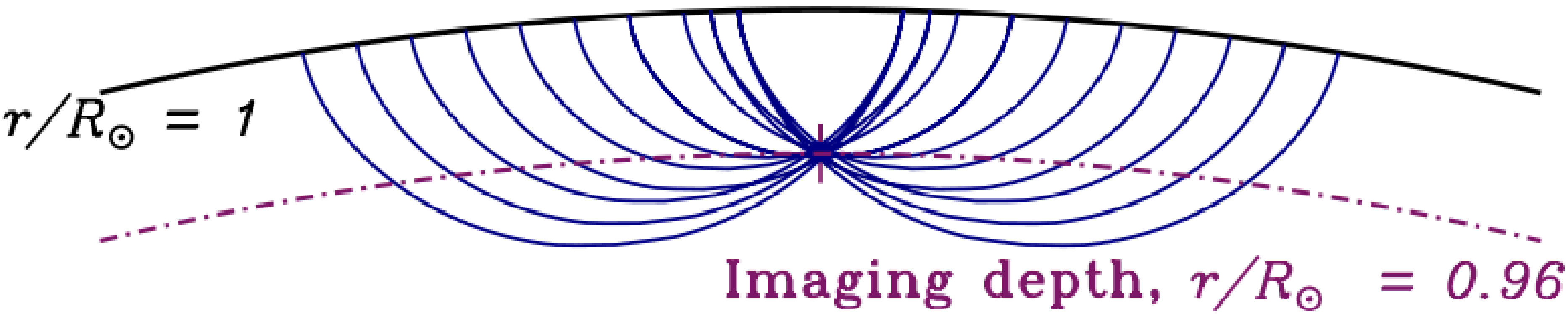


Travel time differences

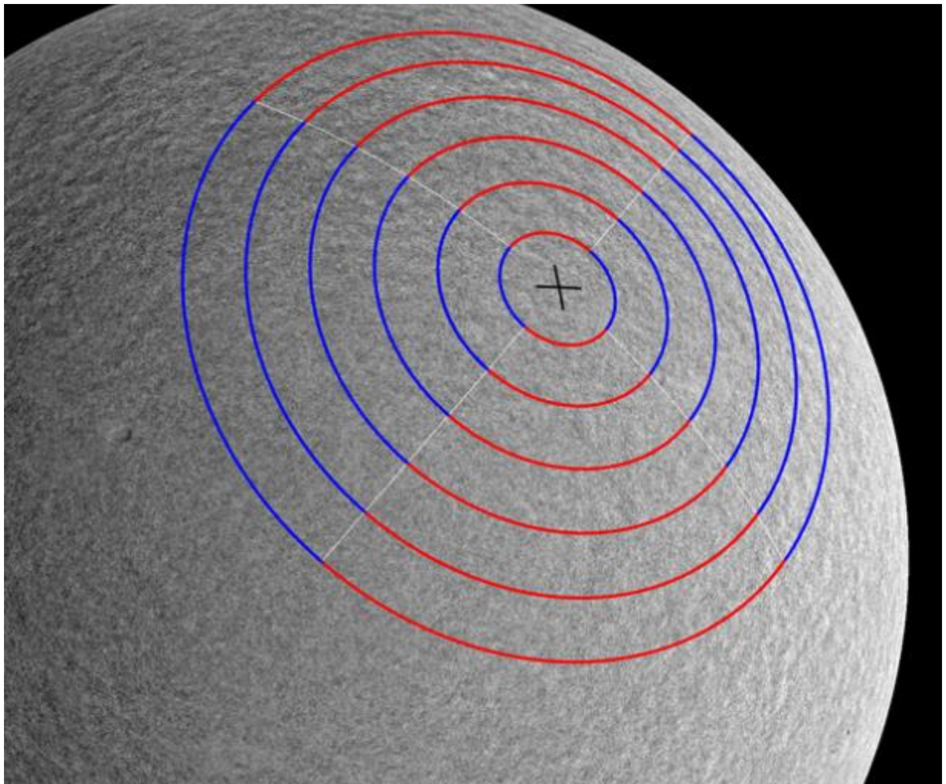


- Contrib. from whole path $$\tau_+ - \tau_- = 2\int \frac{\mathbf{u}\cdot\mathbf{ds}}{c_s^2}$$
- Esp. top layers (c_s small)
- → averaging over rays through same point

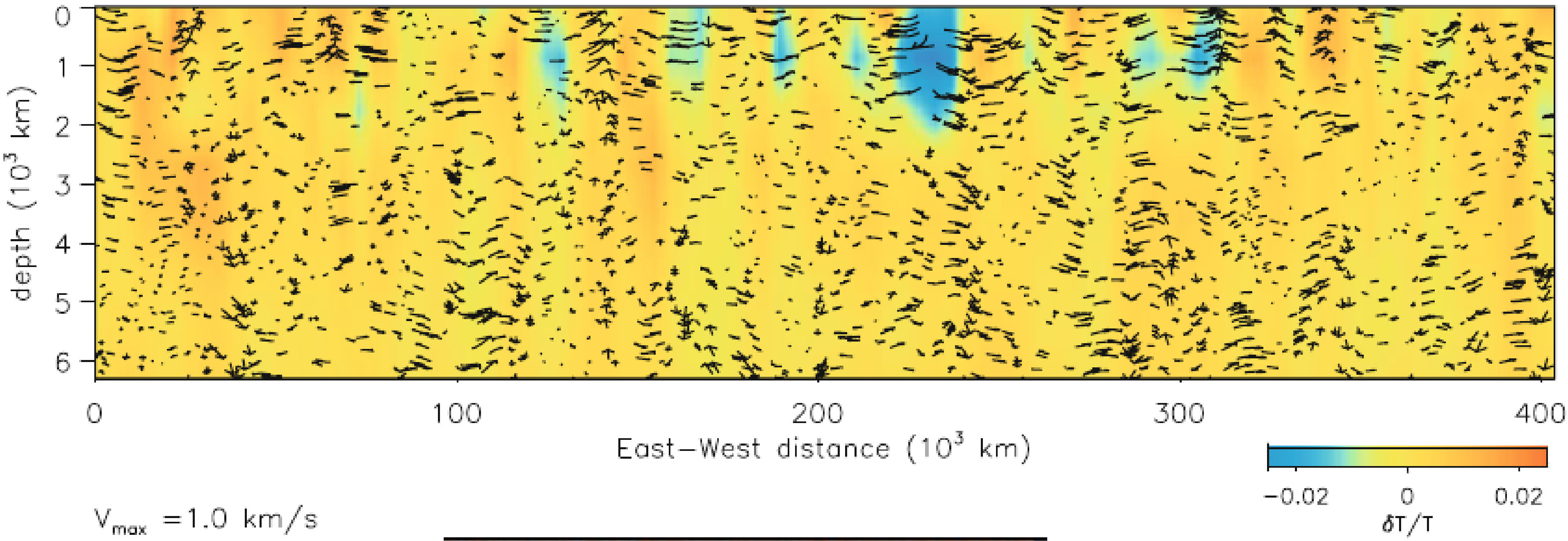
Deep-focusing geometry



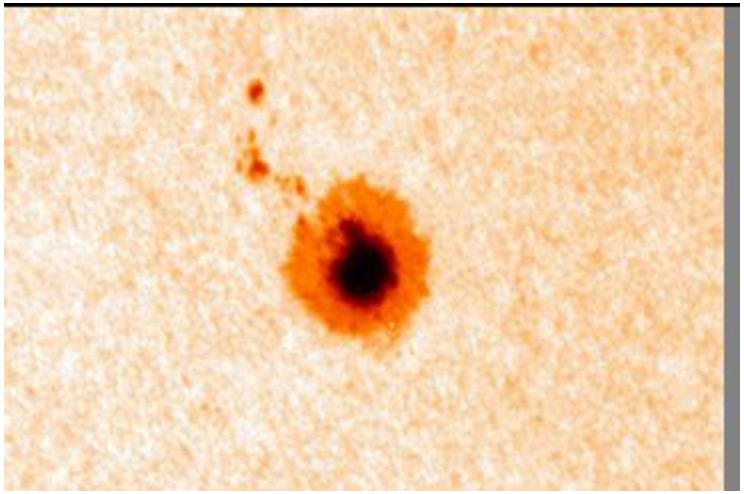
- Removes strong contributions from top layers
- Could they be right?

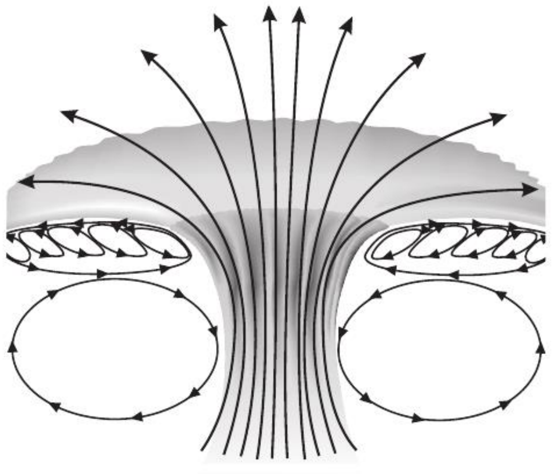


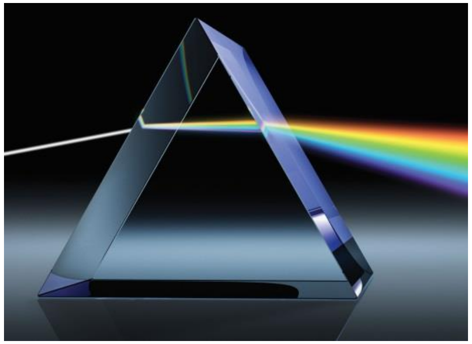
Turbulence imaging



Sunspot imaging
…but uncertain!







By contrast, in the Sun:

A. Waves at high *k* travel faster
B. Wave speed independent of *k*
C. Only near surface high *k* slower
D. Only for long waves high *k* slower

What have we learnt today?

- FFT: line width, side lobes, …
- Doppler shift: from dispersion relation
- Monday: convection & mixing length theory