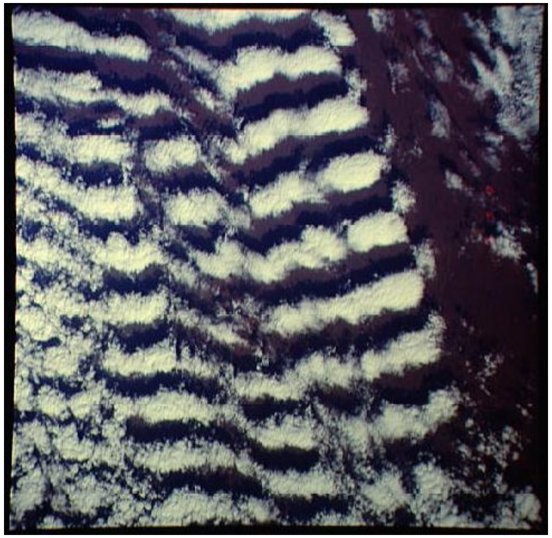
Lecture 16

- Talk about homework 2
- Buoyancy oscillations (Stix pp. 237)


Last time

- Changes in entropy
- Energy equation
- Talked about Homework 3
- ?activity at SBO & Fiske

Instructor's evaluation from

A. Electronically?
B. On paper?

RE HW2.1: from lecture 5

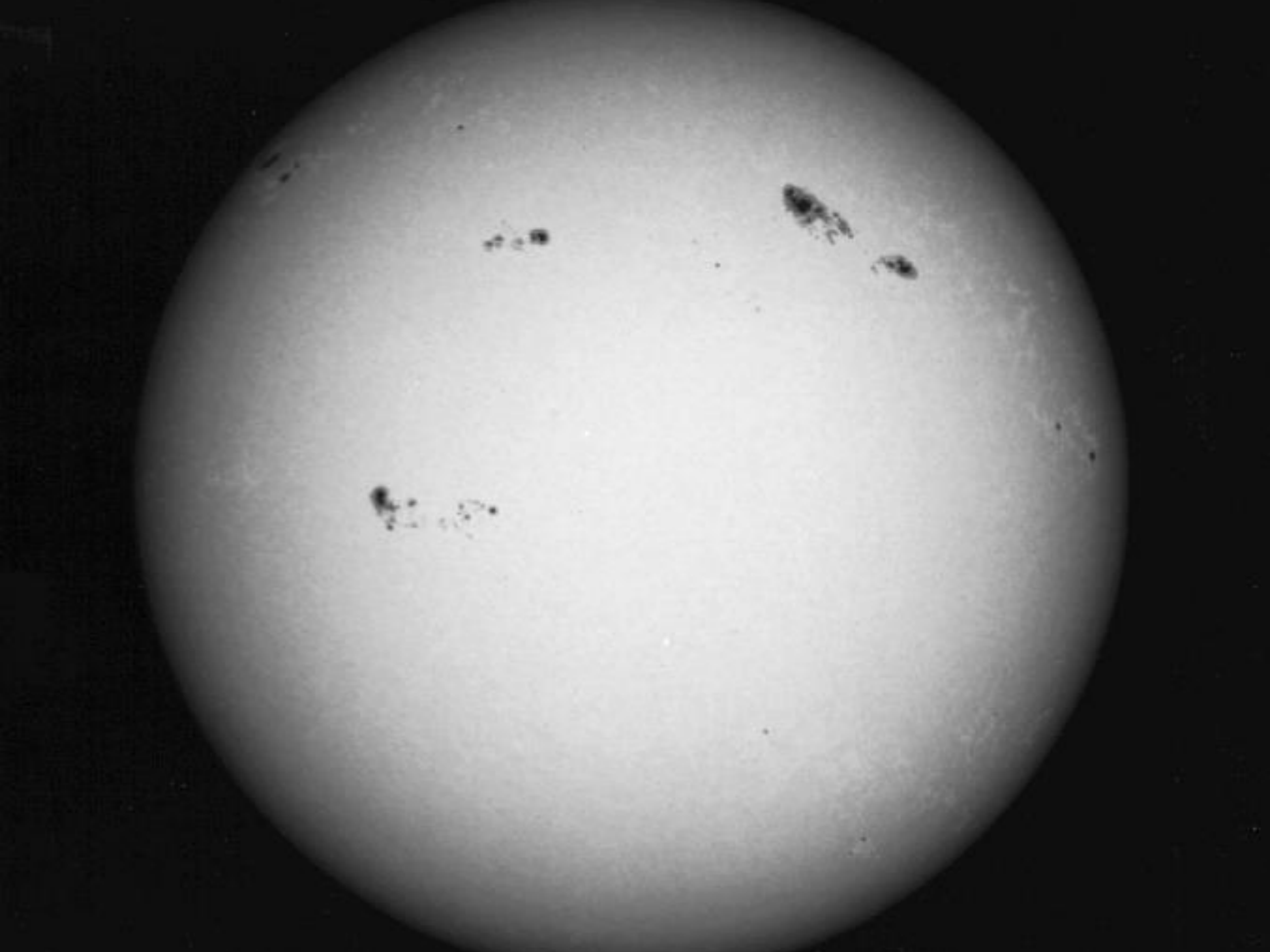
Leading order $I_\nu = B_\nu$

Insert

$$\cos\theta \frac{dB_\nu}{dr} = -\rho\kappa_\nu \left(I_\nu - B_\nu\right)$$

so

$$I_\nu = B_\nu - \frac{\cos\theta}{\rho\kappa_\nu}\frac{dB_\nu}{dr}$$

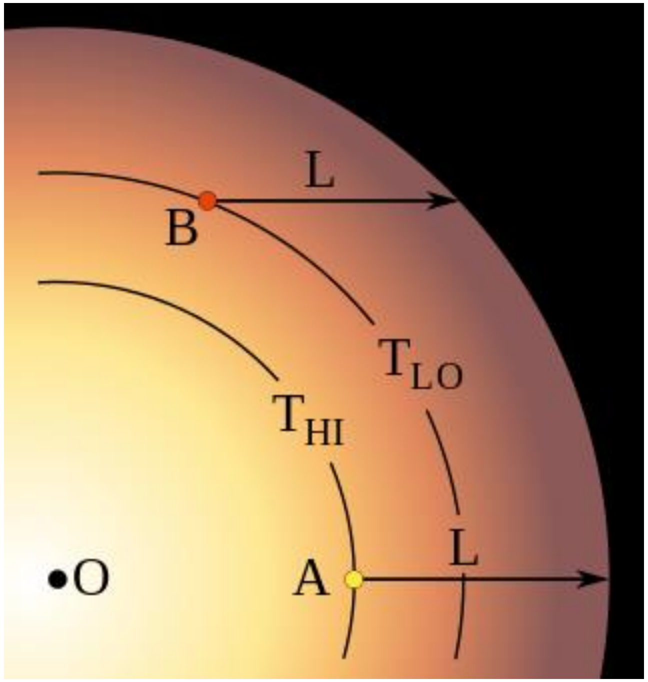


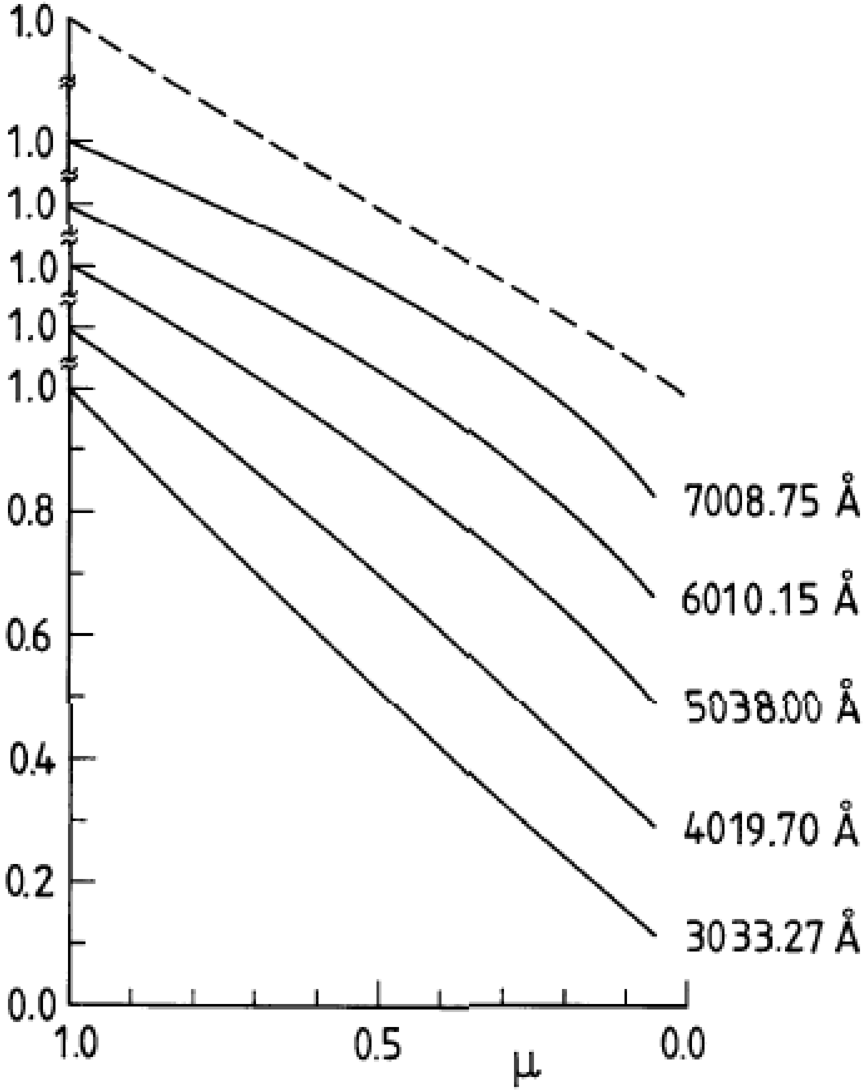
Why dimmer toward limb

A. Refraction, less bright in red
B. Emission maximum normal to surface
C. Temperature increases with depths
D. Edge is further away from us

Limb darkening

- Stix Sect. 4.3.1
- See deeper





Optionally: use SBO to measure this at different λ ?

A. Yes, if possible
B. Yes, probably
C. Probably not
D. no

Changes in entropy?

$$\rho T \frac{Ds}{Dt} = \underbrace{\oint_{4\pi} (I - S)\, d\Omega}_{-\nabla \cdot \mathbf{F}_{\mathrm{rad}}} + Q_{\mathrm{visc}} + Q_{\mathrm{Joule}} + Q_{\mathrm{nuclear}}$$

Here:

$$\frac{Ds}{Dt} = \frac{\partial s}{\partial t} + \mathbf{u} \cdot \nabla s$$

Buoyancy oscillations

Momentum eqn:

$$\rho_0 \frac{\partial \mathbf{u}_1}{\partial t} = -\nabla p_1 + \rho_1 \mathbf{g} \ldots$$

$$\rho_1 / \rho_0$$
$$= \delta\rho / \rho$$
$$= \delta \ln \rho$$
$$= -\delta s / c_p$$
$$= -s_1 / c_p$$

Entropy equarion:

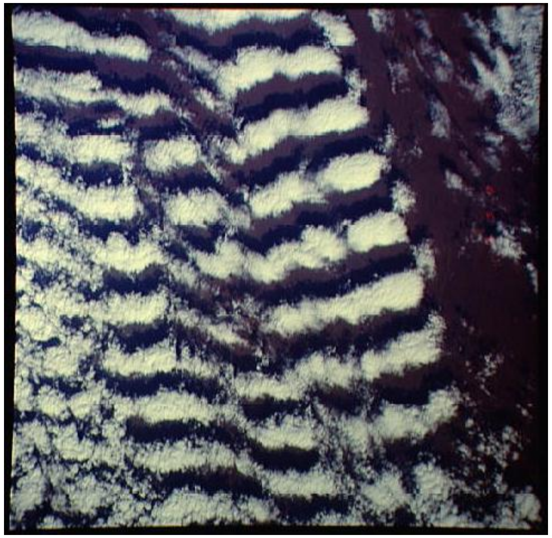
$$\frac{\partial s_1}{\partial t} = -\mathbf{u}_1 \cdot \nabla s_0$$

Ignore pressure for now,
so as to understand
buoyancy effect

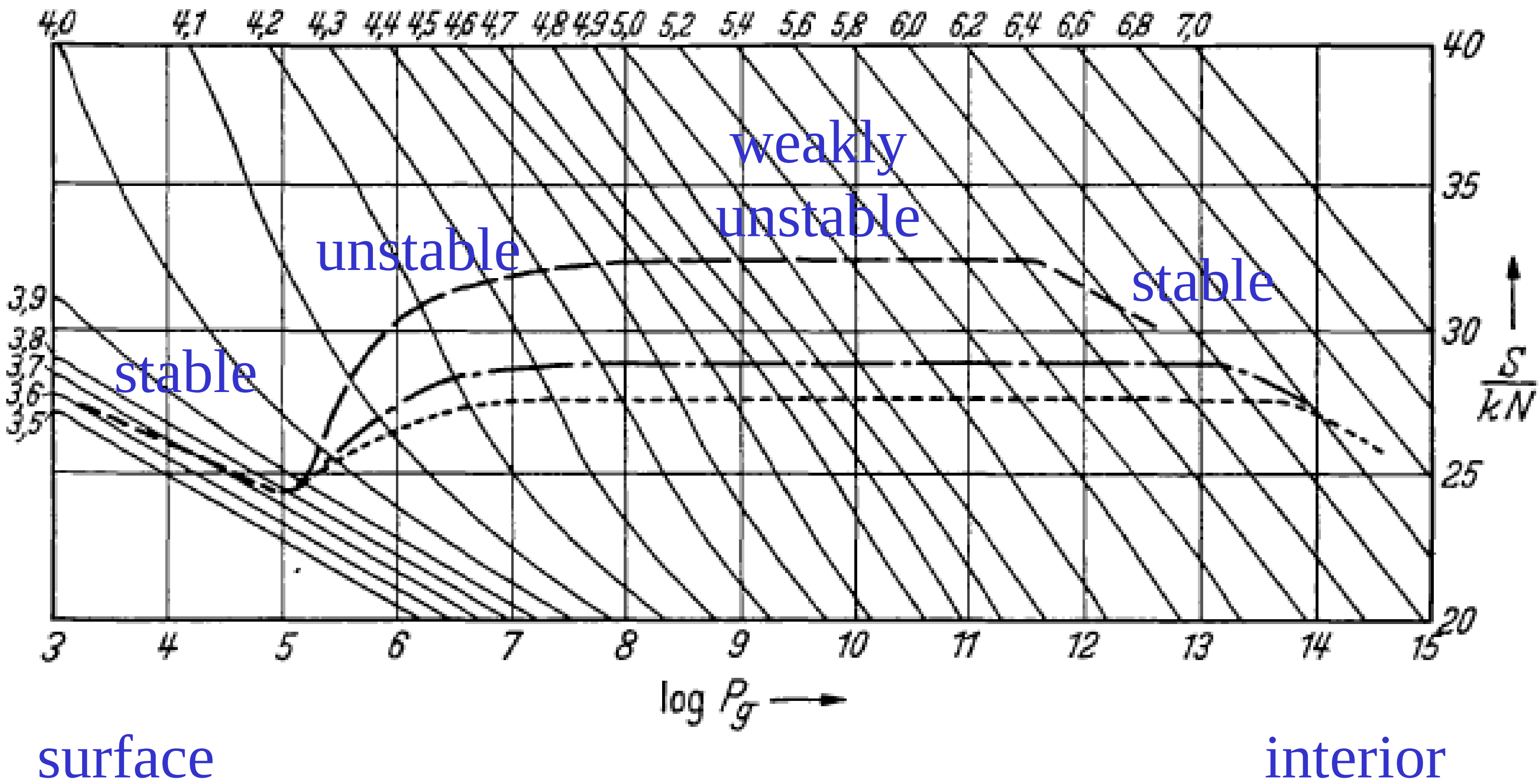
$$\begin{pmatrix} \mathrm{i}\omega & g / c_p \\ -ds_0 / dz & \mathrm{i}\omega \end{pmatrix} \begin{pmatrix} \hat{u}_{1z} \\ \hat{s}_1 \end{pmatrix} = 0$$

g-modes

- Would probe the center
- Are evanescent in the convection zone



Stratification in the sun



lg P used as convenient depth coordinate

Instructor's evaluation from

A. Electronically?
B. On paper?

Internal energy & specific heat

$$c_v dT = \frac{P}{\rho^2} d\rho + T ds$$

Internal energy equation

$$\rho c_v \frac{DT}{Dt} = P \frac{D \ln \rho}{Dt} + \rho T \frac{Ds}{Dt}$$

Use continuity equation

$$\rho c_v \frac{DT}{Dt} = -P \nabla \cdot \mathbf{u} + \rho T \frac{Ds}{Dt}$$

Total energy equation

Momentum equation

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla P$$

Multiply by ***u***

$$\tfrac{1}{2}\rho \frac{D\mathbf{u}^2}{Dt} = -\mathbf{u}\cdot\nabla P$$

Internal energy equation

$$\rho c_v \frac{DT}{Dt} = -P\nabla\cdot\mathbf{u} + \rho T \frac{Ds}{Dt}$$

Add the two to get one part of total energy equation

$$\tfrac{1}{2}\rho \frac{D\mathbf{u}^2}{Dt} + \rho c_v \frac{DT}{Dt} = -\nabla\cdot(\mathbf{u}P) + \rho T \frac{Ds}{Dt}$$

What we learned

- Entropy: what makes it change
- Buoyancy oscillations
- Energy equation