Lecture 18

- Discussion on Homework 3
- Conversion between kinetic and magnetic energy forms
- Alfven waves

Last time

- Buoyant rise of flux tubes
- Ohm’s law in lab frame
- Conversion of kinetic into magnetic energy
- How magnetic fields get advected

Plasma begins

A. below the convection zone
B. below the photosphere
C. inside the heliosphere (50 AU,say)
D. none of the above

Other aspects

- Familiarity with underlying research article
- Sunspot number, $R=s*(10*G+S)$, area, B-field
- Helical fields, twisted, parallel current
- Gravity waves
- Other?

Lab (S) and comoving (S') frames

Observer in S sees charge moving

$$\mathbf{F} = q\mathbf{u} \times \mathbf{B}$$

Observer in S' sees charge moving sideways

$$\mathbf{F}' = q\mathbf{E}'$$

Therefore, because F=F',

$$\mathbf{E}' = \mathbf{u} \times \mathbf{B}$$

With additional background E-field

$$\mathbf{E}' = \mathbf{E} + \mathbf{u} \times \mathbf{B}$$

Alfven waves

Neglect diffusivity term
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{u} \times \mathbf{B} \right)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{B} + \mathbf{B} \cdot \nabla \mathbf{u} + \mathbf{u} \nabla \cdot \mathbf{B} - \mathbf{B} \nabla \cdot \mathbf{u}$$

Linearize **B → B+b**

$$\frac{\partial u_y}{\partial t} = B_x \nabla_x b_y / \rho \mu_0$$

$$\frac{\partial b_y}{\partial t} = B_x \nabla_x u_y$$

Alfven waves

Insert ansatz

$$u_y = \hat{u}_y \sin(kx - \omega t)$$

$$b_y = \rho\mu_0 \hat{b}_y \sin(kx - \omega t)$$

into

$$\frac{\partial u_y}{\partial t} = B_x \nabla_x b_y / \rho\mu_0$$

$$\frac{\partial b_y}{\partial t} = B_x \nabla_x u_y$$

Linearize **B → B+b**

$$-\omega \hat{u}_y \cos(k_x x - \omega t) = B_x k_x \hat{b}_y \cos(k_x x - \omega t)$$

$$-\omega \hat{b}_y \cos(k_x x - \omega t) = B_x k_x \hat{u}_y \cos(k_x x - \omega t)$$

insert

$$-\omega \hat{u}_y = B_x k_x \hat{b}_y$$

$$-\omega \hat{b}_y = B_x k_x \hat{u}_y$$

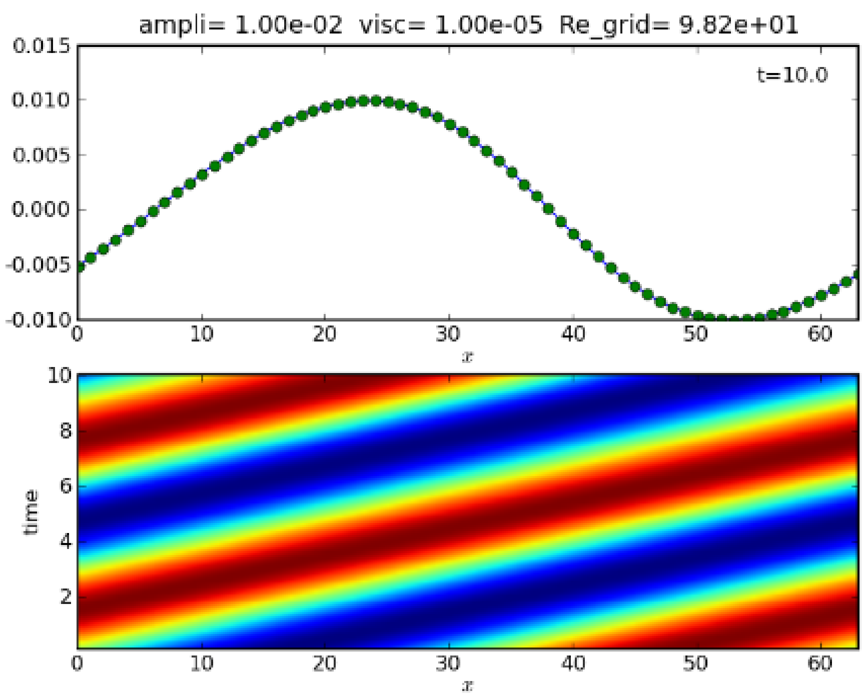
Dispersion relation

$$\omega^2 = B_x^2 k_x^2$$

Nonlinear Alfven waves

→ Working material: NonlinearAlfven/, NonlinearAlfven.tar.gz [untar this file by typing tar zxf NonlinearAlfven.tar.gz]

In this nonlinear Alfven wave problem we solve the fully compressible equations in one dimension. For a weak initial amplitude you find regular Alfven waves. As the amplitude is increased, the initial kinetic energy becomes comparable with the thermal energy. Obviously, viscosity is required to prevent wiggles. However, this leads to a decrease in amplitude and hence a loss of kinetic energy. Since total energy is conserved, this must lead to corresponding heating. Verify that total energy is indeed conserved, and find cases where this is not the case. What went wrong in those cases?

Linear case A=1e-2	

What we learned

- How to read science news
- Alfven waves in action
- Pencil Code