Lecture 19

- Alfven wave dispersion relation
- Ohmic diffusion
- Reynolds numbers
- Prandtl numbers

Last time

- How to read science news
- Alfven waves in action
- Pencil Code

Alfven waves

Neglect diffusivity term

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{B} + \mathbf{B} \cdot \nabla \mathbf{u} + \mathbf{u} \nabla \cdot \mathbf{B} - \mathbf{B} \nabla \cdot \mathbf{u}$$

Linearize **B** → **B+b**

$$\frac{\partial u_y}{\partial t} = B_x \nabla_x b_y / \rho \mu_0$$

$$\frac{\partial b_y}{\partial t} = B_x \nabla_x u_y$$

Alfven waves

Insert ansatz

$$u_y = \hat{u}_y \sin(kx - \omega t)$$

$$b_y = \rho\mu_0 \hat{b}_y \sin(kx - \omega t)$$

into

$$\frac{\partial u_y}{\partial t} = B_x \nabla_x b_y / \rho\mu_0$$

$$\frac{\partial b_y}{\partial t} = B_x \nabla_x u_y$$

Linearize **B → B+b**

$$-\omega \hat{u}_y \cos(k_x x - \omega t) = B_x k_x \hat{b}_y \cos(k_x x - \omega t)$$

$$-\omega \hat{b}_y \cos(k_x x - \omega t) = B_x k_x \hat{u}_y \cos(k_x x - \omega t)$$

insert

$$-\omega \hat{u}_y = B_x k_x \hat{b}_y / \rho\mu_0$$

$$-\omega \hat{b}_y = B_x k_x \hat{u}_y$$

Dispersion relation

$$\omega^2 = B_x^2 k_x^2 / \rho\mu_0$$

Alfven speed

2 equations with two unknowns

$$\frac{\partial u_y}{\partial t} = B_x \nabla_x b_y / \rho\mu_0$$

$$\frac{\partial b_y}{\partial t} = B_x \nabla_x u_y$$

$$v_{\mathrm{A}}^2 = B^2 / \rho\mu_0$$

B=2000 G, ρ=10^{-6} g/cm^3: v_{A}=6 km/s

What is $\rho = 10^{-6}$ g/cm^3*?*

A. 10^3 kg/m^3
B. 1 kg/m^3
C. 10^{-3} kg/m^3
D. 10^{-6} kg/m^3
E. 10^{-9} kg/m^3

Ohmic diffusion

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

No flow: **u=0**

$$B_x = B_0 e^{ik_x x - i\omega t}$$

Dispersion relation

$$-i\omega = -\eta k_x^2$$

Advection vs. diffusion

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

Magnetic Reynolds number

$$\frac{uB/\ell}{\eta B/\ell^2} = \frac{u\ell}{\eta} = \mathrm{Re}_M$$

Fluid Reynolds number

$$\frac{u^2/\ell}{\nu u/\ell^2} = \frac{u\ell}{\nu} = \mathrm{Re}$$

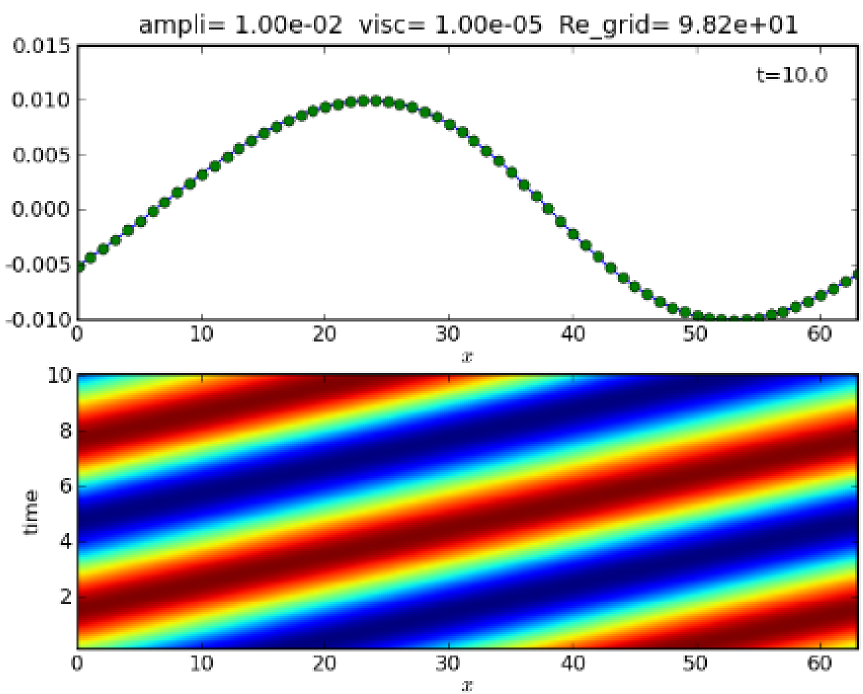
Astrophysical conditions

	T [K]	ρ [g cm^{-3}]	P_m	u_{rms} [cm s^{-1}]	L [cm]	R_m
Solar CZ (upper part)	10^4	10^{-6}	10^{-7}	10^6	10^8	10^6
Solar CZ (lower part)	10^6	10^{-1}	10^{-4}	10^4	10^{10}	10^9
Protostellar discs	10^3	10^{-10}	10^{-8}	10^5	10^{12}	10
CV discs and similar	10^4	10^{-7}	10^{-6}	10^5	10^7	10^4
AGN discs	10^7	10^{-5}	10^4	10^5	10^9	10^{11}
Galaxy	10^4	10^{-24}	(10^{11})	10^6	10^{20}	(10^{18})
Galaxy clusters	10^8	10^{-26}	(10^{29})	10^8	10^{23}	(10^{29})

Nonlinear Alfven waves

→ Working material: NonlinearAlfven/, NonlinearAlfven.tar.gz [untar this file by typing tar zxf NonlinearAlfven.tar.gz]

In this nonlinear Alfven wave problem we solve the fully compressible equations in one dimension. For a weak initial amplitude you find regular Alfven waves. As the amplitude is increased, the initial kinetic energy becomes comparable with the thermal energy. Obviously, viscosity is required to prevent wiggles. However, this leads to a decrease in amplitude and hence a loss of kinetic energy. Since total energy is conserved, this must lead to corresponding heating. Verify that total energy is indeed conserved, and find cases where this is not the case. What went wrong in those cases?

Linear case A=1e-2	

What we learned

- Alfven wave dispersion relation
- Ohmic diffusion
- Reynolds numbers
- Prandtl numbers