Lecture 21

- Sound waves
  - Nonlinearity: shocks
- Polytropes
  - Linear temperature profile

Last time

- SBO data taking
- Ohmic diffusion
- sunspots

From lecture 11

Expand continuity eqn:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) = -\mathbf{u} \cdot \nabla \rho - \rho \nabla \cdot \mathbf{u}$$

Momntum eqn (isothermal):

$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\rho \mathbf{u} \cdot \nabla \mathbf{u} - \frac{\Re T}{\mu} \nabla \rho + \ldots$$

Linearized form

$$\frac{\partial \rho_1}{\partial t} = -\rho_0 \nabla \mathbf{u}_1$$

$$\rho_0 \frac{\partial \mathbf{u}_1}{\partial t} = -\frac{\Re T}{\mu} \nabla \rho_1$$

Trial solution

$$\rho_1(z,t) = \hat{\rho}_1 e^{ik_z z - i\omega t} + \text{c.c.}$$

$$u_{1z}(z,t) = \hat{u}_{1z} e^{ik_z z - i\omega t} + \text{c.c.}$$

$$\begin{pmatrix} i\omega & -ik_z \rho_0 \\ -ik_z \frac{\Re T}{\mu} & i\omega \rho_0 \end{pmatrix} \begin{pmatrix} \hat{\rho}_1 \\ \hat{u}_{1z} \end{pmatrix} = 0$$

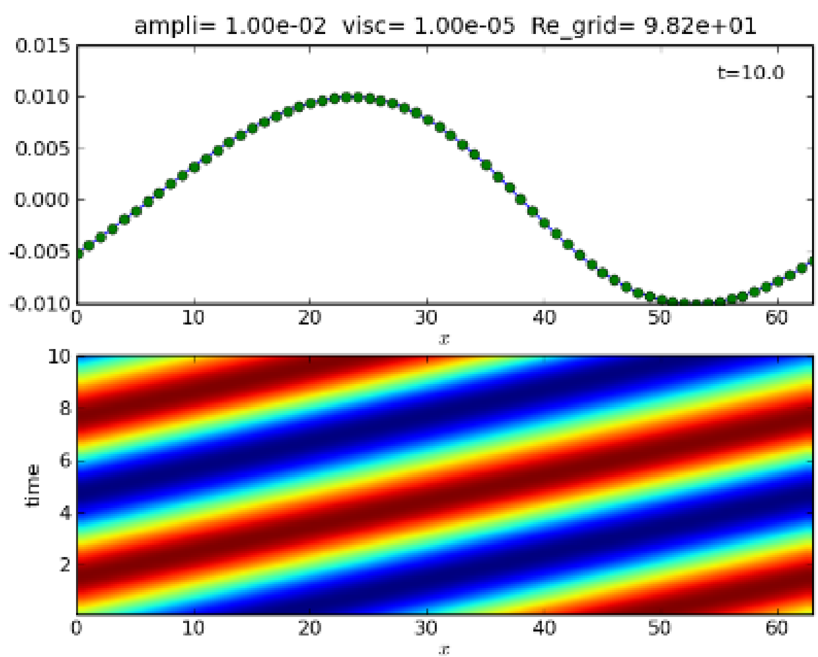
Dispersion relation $\omega^2 = \frac{\Re T}{\mu} k_z^2$

$c_s = \sqrt{\Re T / \mu}$ Sound speed

Nonlinear sound waves

→ Working material: NonlinearSound/, NonlinearSound.tar.gz [untar this file by typing tar zxf NonlinearSound.tar.gz]

In this nonlinear sound wave problem we solve the fully compressible equations in one dimension. For a weak initial amplitude you find regular sound waves. As the amplitude is increased, the initial kinetic energy becomes comparable with the thermal energy. Obviously, viscosity is required to prevent wiggles. However, this leads to a decrease in amplitude and hence a loss of kinetic energy. Since total energy is conserved, this must lead to corresponding heating. Verify that total energy is indeed conserved, and find cases where this is not the case. What went wrong in those cases?

Linear case A=1e-2	

Polytropes

$$P \propto T^{n+1} \qquad \rho \propto T^{n}$$ and $T(z)$=linear

Hydrostatic equilibrium

$$\frac{dP}{dz} = -\rho g \qquad (n+1)P_0(T/T_0)^n \frac{dT}{dz} = -\rho_0 (T/T_0)^n g$$

Thermal equilibrium

$$K\frac{dT}{dz} = \text{const}$$

obeyed for K=const

Marginal stability

$$\frac{ds/c_p}{dz} = \left[\frac{1}{\gamma}(n_{\text{crit}}+1) - n_{\text{crit}}\right]\frac{d\ln T}{dz} \qquad n_{\text{crit}} = \frac{1}{\gamma - 1} = 3/2$$

When unstable

A. $n > 3/2$
B. $n < 3/2$

Example

$$\frac{ds/c_p}{dz} = \left[\frac{1}{\gamma}(n_{\rm crit}+1) - n_{\rm crit}\right]\frac{d\ln T}{dz}$$

Put n=0 (which is less than 3/2), so

$$\frac{ds/c_p}{dz} = \frac{1}{\gamma}\frac{d\ln T}{dz}$$

Which is negative (=unstable) because dln$T$/dz<0.

What we learned

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