Last time

- Sound waves
  - Nonlinearity: shocks
- Polytropes
  - Linear temperature profile

Lecture 22

- SBO observing sessions
- Convection experiment
- Solar convection simulations
- Questions on HW4

Rayleigh–Bénard convection

From Wikipedia, the free encyclopedia

Rayleigh-Bénard convection is a type of natural convection, occurring in a plane horizontal layer of fluid heated from below, in which the fluid develops a regular pattern of convection cells known as **Bénard cells**. Rayleigh–Bénard convection is one of the most commonly studied convection phenomena because of its analytical and experimental accessibility.[1] The convection patterns are the most carefully examined example of self-organizing nonlinear systems.[1] [2]

Bénard cells.

From lecture 14

Momentum eqn:

$$\rho_0 \frac{\partial \mathbf{u}_1}{\partial t} = -\nabla p_1 + \rho_1 \mathbf{g} \ldots$$

$$\rho_1 / \rho_0$$
$$= \delta\rho / \rho$$
$$= \delta \ln \rho$$
$$= -\delta s / c_p$$
$$= -s_1 / c_p$$

Entropy equarion:

$$\frac{\partial s_1}{\partial t} = -\mathbf{u}_1 \cdot \nabla s_0$$

Ignore pressure for now, so as to understand buoyancy effect

$$\begin{pmatrix} \mathrm{i}\omega & -g / c_p \\ -ds_0 / dz & \mathrm{i}\omega\rho_0 \end{pmatrix} \begin{pmatrix} \hat{u}_{1z} \\ \hat{s}_1 \end{pmatrix} = 0$$

no ρ factor (mistake!!)

Nonideal effects (simplified)

Momentum eqn:

$$\rho_0 \frac{\partial \mathbf{u}_1}{\partial t} = -\nabla p_1 + s_1 \mathbf{g} / c_p + \nu \nabla^2 \mathbf{u}_1 \ldots$$

Entropy equarion:

$$\frac{\partial s_1}{\partial t} = -\mathbf{u}_1 \cdot \nabla s_0 + \chi \nabla^2 s_1$$

here: χ is the thermal (radiative) diffusivity

Assume $\nu=\chi$ for simplicity

$$\begin{pmatrix} -\lambda - \nu k_z^2 & -g / c_p \\ -ds_0 / dz & -\lambda - \nu k_z^2 \end{pmatrix} \begin{pmatrix} \hat{u}_{1z} \\ \hat{s}_1 \end{pmatrix} = 0$$

$$\left(\lambda + \nu k_z^2\right)^2 - g \frac{ds_0 / c_p}{dz} = 0 \quad \left(\lambda + \nu k_z^2\right)^2 = \left(g \frac{ds_0 / c_p}{dz} \right)$$

Rayleigh-Benard convection

$$\left(\lambda + \nu k_z^2\right)^2 - g\,\frac{ds_0 / c_p}{dz} = 0$$

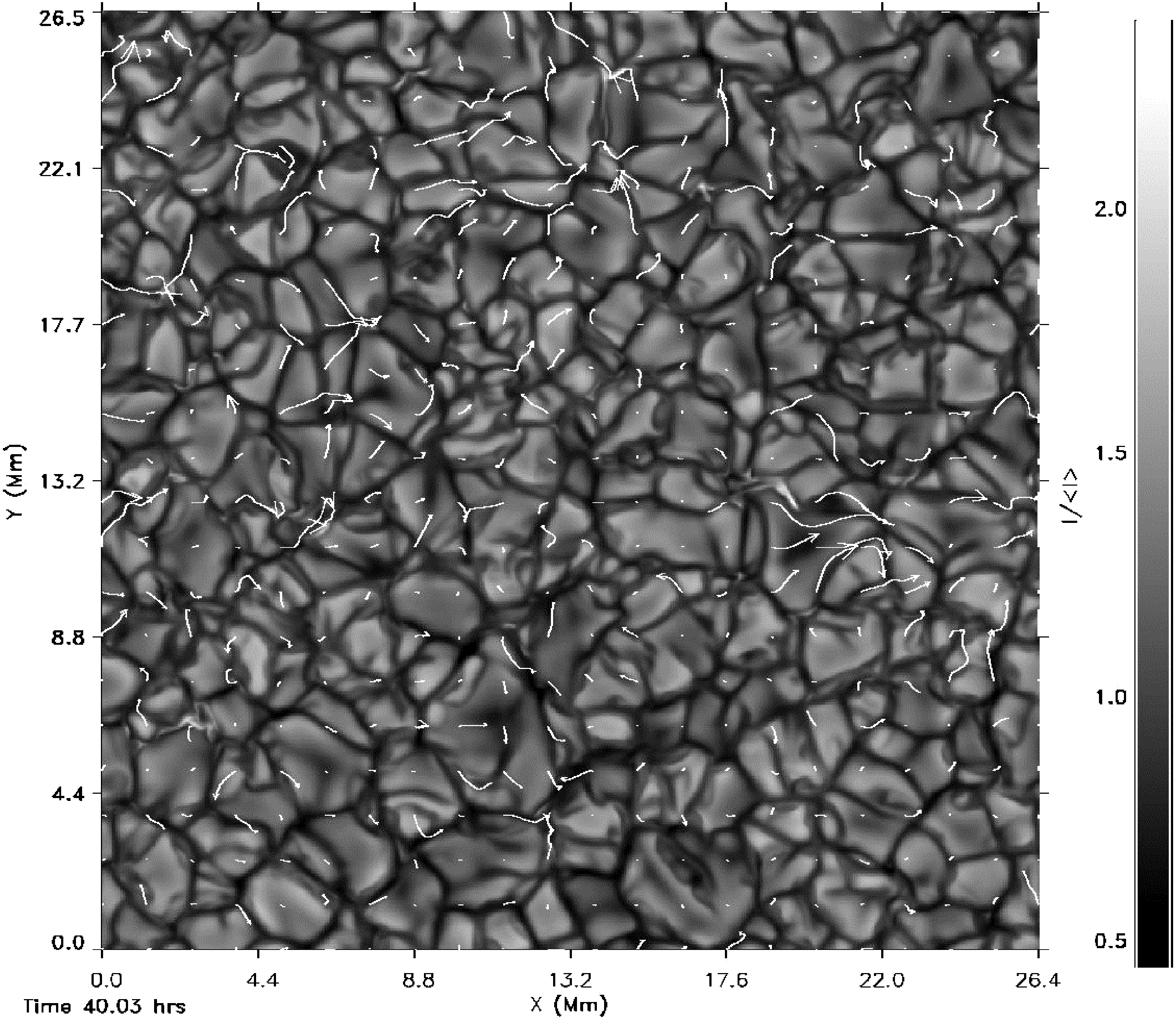
$$\lambda = -\nu k_z^2 + \left(g\,\frac{ds_0 / c_p}{dz} \right)^{1/2}$$

$$\frac{g}{(\nu k_z^2)^2}\,\frac{ds_0 / c_p}{dz} > 1$$

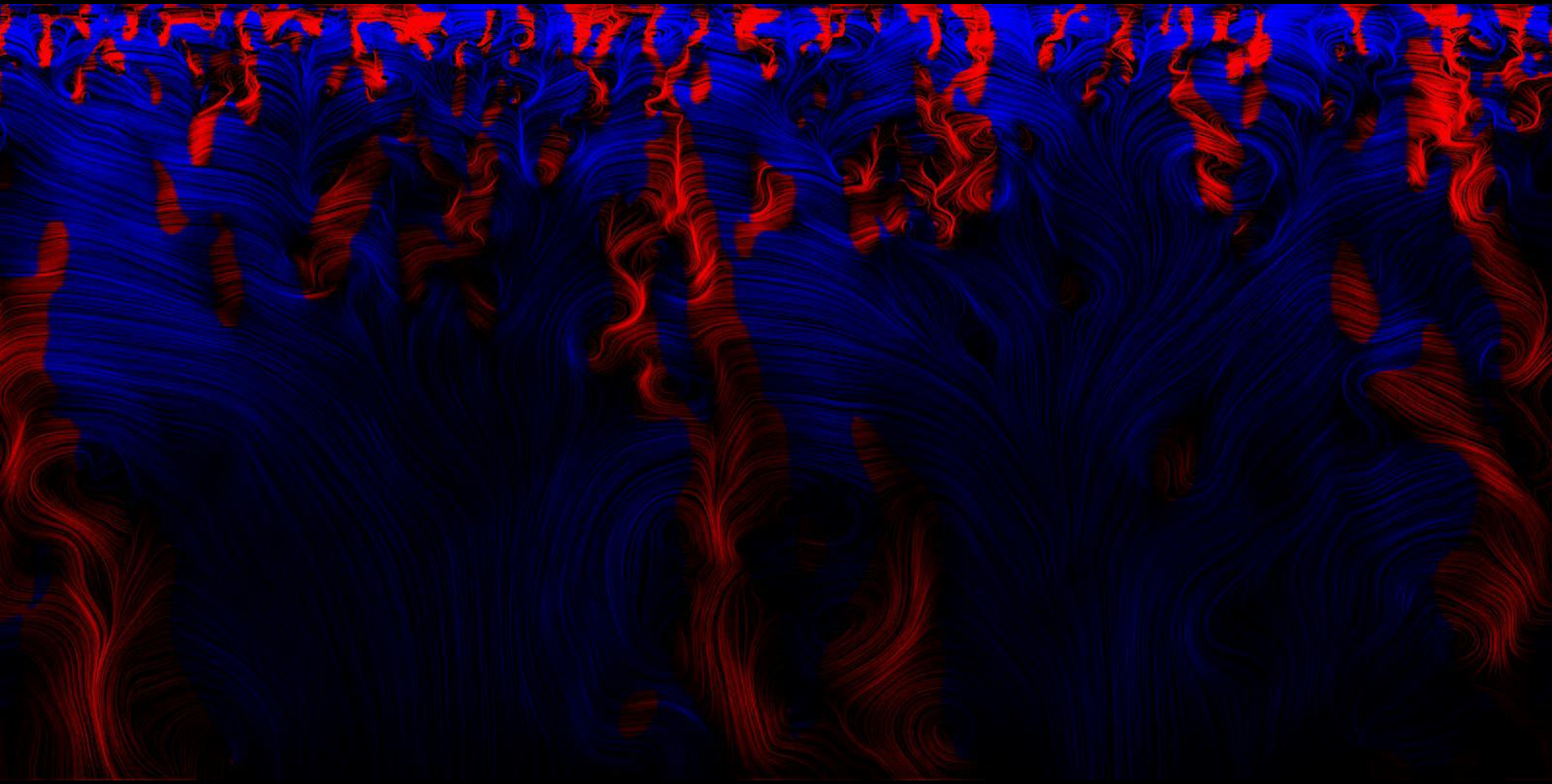
General definition of Rayleigh number

$$\mathrm{Ra} = \frac{g d^4}{\nu \chi}\,\frac{ds_0 / c_p}{dz}$$

critical value: $27\pi^4/4=657$

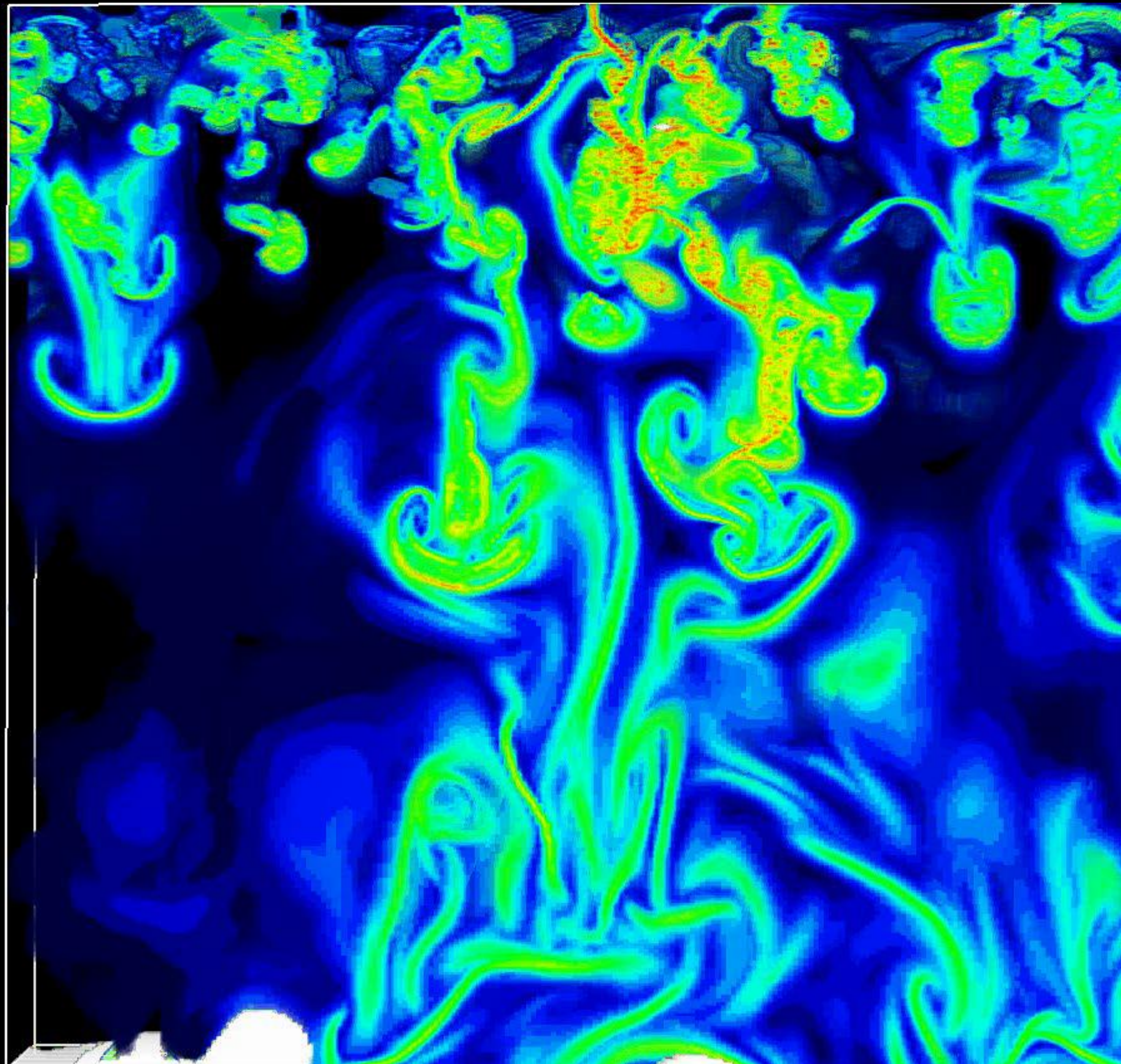

26.5
22.1
17.7
13.2
8.8
4.4
0.0
Y (Mm)
0.0
4.4
8.8
13.2
17.6
22.0
26.4
X (Mm)
Time 40.03 hrs
2.0
1.5
1.0
0.5
I/<I>

Topology of convection
(Stein & Nordlund)



Why do so slow

A. Lower parts are less unstable
B. Scale height larger
C. Spherical geometry ignored



Courtesy: Bob Stein (MSU)

What was most unclear

A. Neutrino production
B. Center to limb variation
C. Maxwell equations
D. Vector algebra
E. Magnetic pressure

What was most unclear

A. Helioseismology
B. Fourier transform
C. Thermodynamics (entropy)
D. Alfven waves
E. Sunspots

What we learned

- Convection experiment
- Solar convection simulations
- Questions on HW4
- Sign up for SBO observing sessions