Lecture 24

- SBO observing sessions
- More on mean-field dynamos
- Second sample midterm exam

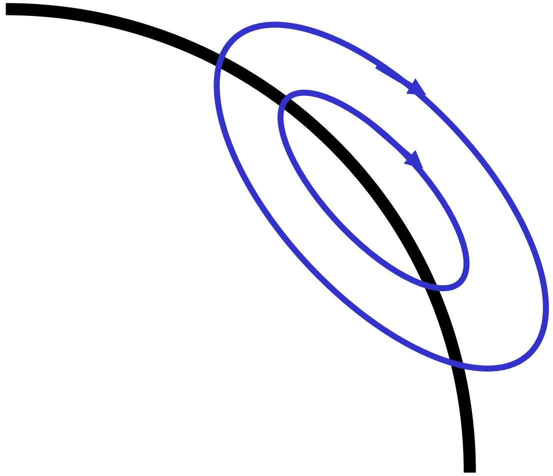
Last time

- SBO observing sessions
- First sample midterm exam
- Questions on HW4

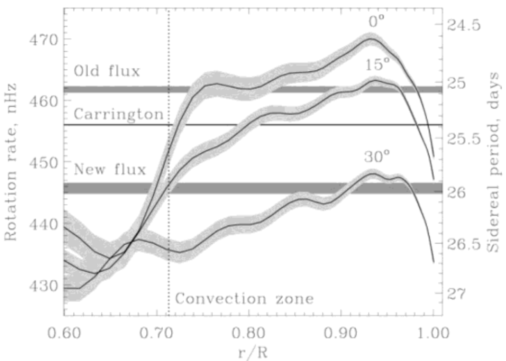
Next SBO session

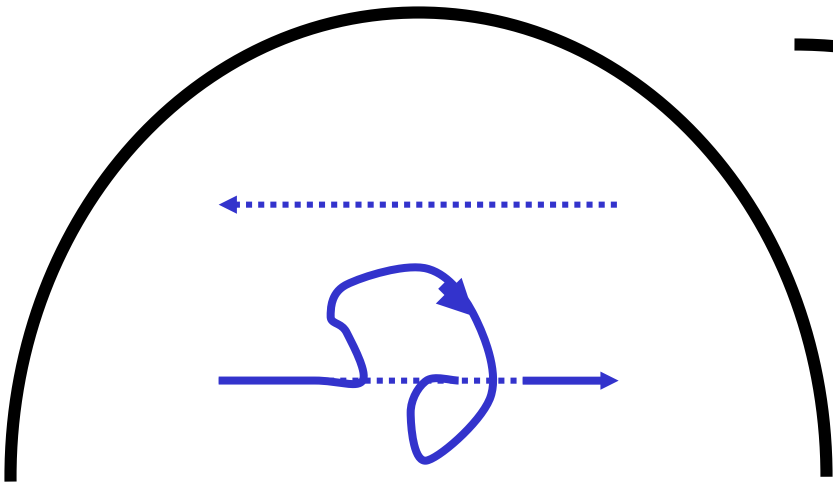
- Friday 11-12
  - Kyle & Shashank
- Friday 1-2
  - Cathy

α-effect dynamos (large scale)



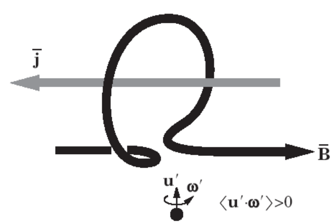
Differential rotation
(faster inside)

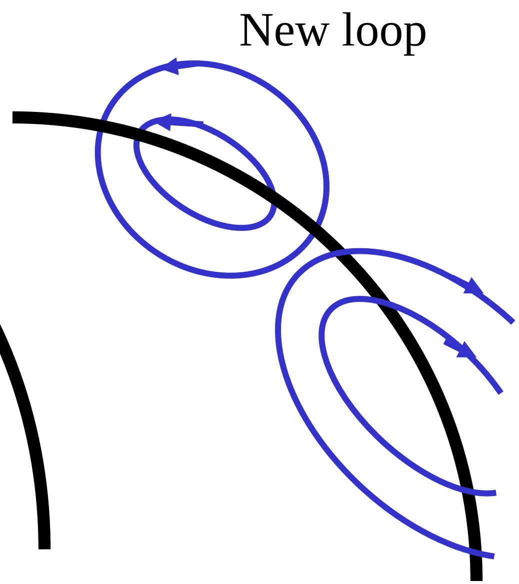




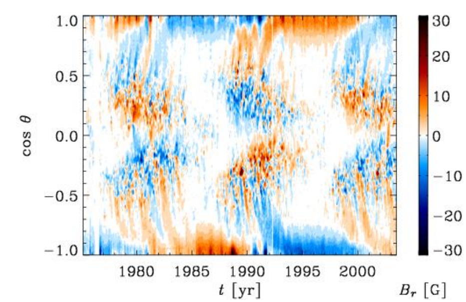
Cyclonic convection;
Buoyant flux tubes

→ α-effect





Equatorward
migration

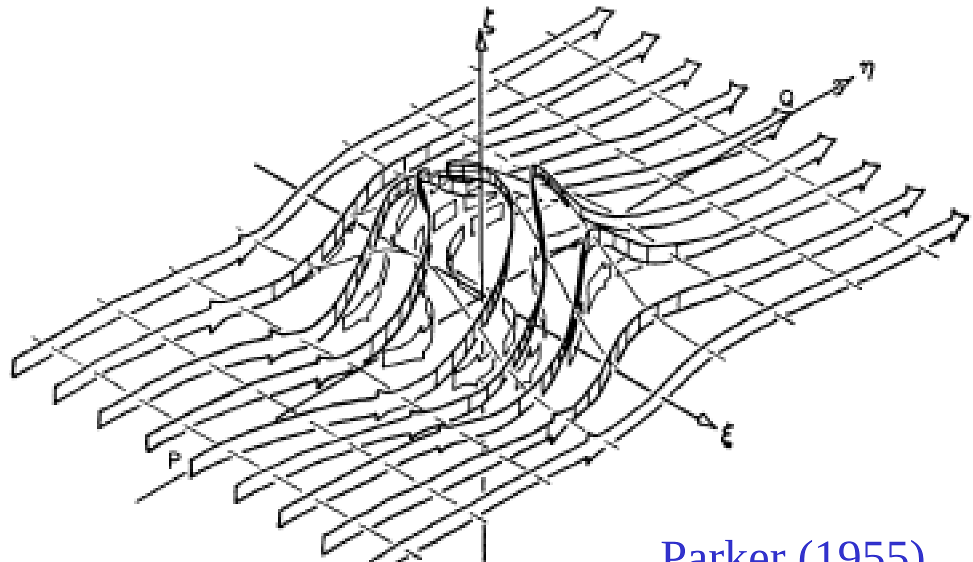


The beginnings

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times\left(\mathbf{U}\times\mathbf{B}\right)+\eta\nabla^2\mathbf{B}$$

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla\times\left(\overline{\mathbf{U}}\times\overline{\mathbf{B}}+\overline{\mathbf{u}\times\mathbf{b}}\right)+\eta\nabla^2\overline{\mathbf{B}}$$

$$\mathbf{U} = \overline{\mathbf{U}}+\mathbf{u}, \quad \mathbf{B} = \overline{\mathbf{B}}+\mathbf{b}$$



Parker (1955)

$$\overline{\mathbf{v}\times\mathbf{H}'} = \gamma\mu\sigma\overline{\mathbf{v}^2}\,L^2\Big\{-\frac{1}{3}\nabla(\log\sqrt{\overline{\mathbf{v}^2}})\times\bar{\mathbf{H}} - \frac{4T}{15}\Big[4\bar{\mathbf{H}}\,\omega\cdot\nabla(\log\rho\sqrt{\overline{\mathbf{v}^2}}) - (\bar{\mathbf{H}}\cdot\omega)\nabla(\log\rho\sqrt{\overline{\mathbf{v}^2}}) - \omega\,\bar{\mathbf{H}}\cdot\nabla(\log\rho\sqrt{\overline{\mathbf{v}^2}})\Big]\Big\}.$$

Steenbeck, Krause, & Rädler (1966)

α^2 dynamo (no shear)

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left(\alpha \overline{\mathbf{B}}\right) + \eta_{\mathrm{T}} \nabla^2 \overline{\mathbf{B}}$$

1-D case

$$\frac{\partial \overline{B}_x}{\partial t} = -\alpha \frac{\partial \overline{B}_y}{\partial z} + \eta_{\mathrm{T}} \frac{\partial^2 \overline{B}_x}{\partial z^2}$$

$$\frac{\partial \overline{B}_y}{\partial t} = +\alpha \frac{\partial \overline{B}_x}{\partial z} + \eta_{\mathrm{T}} \frac{\partial^2 \overline{B}_y}{\partial z^2}$$

Dispersion relation

$$\lambda \hat{\mathbf{B}} = \begin{pmatrix} -\eta_{\mathrm{T}} k^2 & -\mathrm{i}\alpha k_z & 0 \\ \mathrm{i}\alpha k_z & -\eta_{\mathrm{T}} k^2 & 0 \\ 0 & 0 & -\eta_{\mathrm{T}} k^2 \end{pmatrix} \hat{\mathbf{B}}$$

$$\eta_{\mathrm{T}} k^2 \left(\left(\lambda + \eta_{\mathrm{T}} k^2\right)^2 - (\alpha k)^2 \right) = 0$$

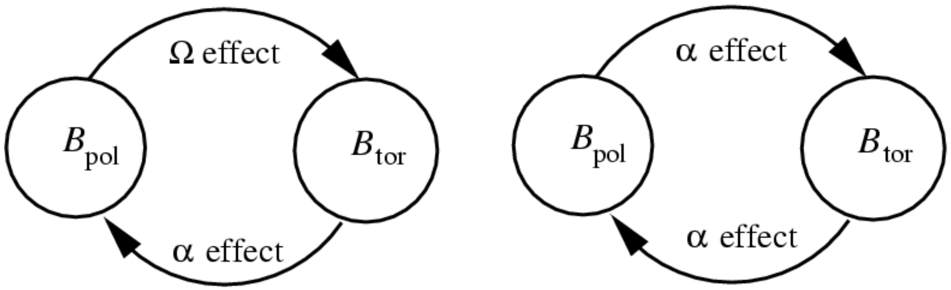
$$\lambda = -\eta_{\mathrm{T}} k^2 \pm |\alpha k|$$

non-oscillatory
exponential growth

αΩ dynamo

$$\frac{\partial \overline{B}_x}{\partial t} = -\alpha \frac{\partial \overline{B}_y}{\partial z} + \eta_{\mathrm{T}} \frac{\partial^2 \overline{B}_x}{\partial z^2}$$

$$\frac{\partial \overline{B}_y}{\partial t} = \frac{\partial \overline{U}_y}{\partial x} \overline{B}_x + \eta_{\mathrm{T}} \frac{\partial^2 \overline{B}_y}{\partial z^2}$$



$$\lambda \hat{\mathbf{B}} = \begin{pmatrix} -\eta_{\mathrm{T}} k^2 & -\mathrm{i}\alpha k_z & 0 \\ \overline{U}_y{}' & -\eta_{\mathrm{T}} k^2 & 0 \\ 0 & 0 & -\eta_{\mathrm{T}} k^2 \end{pmatrix} \hat{\mathbf{B}}$$

$$\lambda = -\eta_{\mathrm{T}} k^2 \pm \sqrt{ik\alpha \overline{U}_y{}'}$$

$$= -\eta_{\mathrm{T}} k^2 \pm (1+i)\sqrt{\tfrac{1}{2} k\alpha \overline{U}_y{}'}$$

oscillatory
traveling wave
exponential growth

What we learned

- More on mean-field dynamos
- Second sample midterm exam