Lecture 26

- Eddington approximation
  - Equation & boundary condition
  - Trial solution
  - T and effective T

Eddington approximation

Radiative transfer

$$\mu \frac{dI}{d\tau} = I - S$$

non-gray $\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$

Thermal equilibrium

$$F_z = \int_{4\pi} n_z I \, d\Omega = 2\pi \int_{-1}^{1} \mu I \, d\mu = \text{const}$$

Boundary condition at the top (τ=0)

$$F_z = \int_{\substack{2\pi \\ \text{upperhalf}}} n_z I \, d\Omega = 2\pi \int_{0}^{1} \mu I \, d\mu = \text{const}$$

ansatz (trial solution)

$$I = I_0 + \mu I_1$$

Eddington approximation

Note that

$$\int_0^1 \mu I \, d\mu = \int_0^1 \mu\left(I_0 + \mu I_1\right) d\mu = \tfrac{1}{2} I_0 + \tfrac{1}{3} I_1$$

$$\int_{-1}^1 \mu I \, d\mu = \int_{-1}^1 \mu\left(I_0 + \mu I_1\right) d\mu = \tfrac{2}{3} I_1$$

$$\int_{-1}^1 I \, d\mu = \int_{-1}^1 \left(I_0 + \mu I_1\right) d\mu = 2 I_0$$

integrate over μ

$$\mu \frac{dI}{d\tau} = I - S \qquad \frac{1}{3}\frac{dI_1}{d\tau} = I_0 - S$$

integrate over μ

$$\mu^2 \frac{dI}{d\tau} = \mu\left(I - S\right) \qquad \frac{1}{3}\frac{dI_0}{d\tau} = \frac{1}{3} I_1 = \text{const}$$

Eddington approximation

Because of $\frac{dI_0}{d\tau} = I_1 = \text{const}$

we have $I_0 = a\tau + b$

$$\int_0^1 \mu I \, d\mu = \tfrac{1}{2} I_0 + \tfrac{1}{3} I_1 = \tfrac{1}{2\pi} F_z \qquad \Rightarrow b = \tfrac{2}{3} I_1 = \tfrac{1}{2\pi} F$$

$$\int_{-1}^1 \mu I \, d\mu = \tfrac{2}{3} I_1 = \tfrac{1}{2\pi} F \qquad \Rightarrow a = \tfrac{3}{4\pi} F$$

$$I_0 = \tfrac{1}{2\pi} F\left(\tfrac{3}{2}\tau + 1\right) \qquad I_1 = \tfrac{3}{4\pi} F \qquad \Rightarrow I = \tfrac{3}{4\pi} F\left(\tau + \tfrac{2}{3} + \mu\right)$$

$$\tfrac{\sigma_B}{\pi} T^4 = S = I_0 = \tfrac{1}{2\pi} F\left(\tfrac{3}{2}\tau + 1\right) = \tfrac{\sigma_B}{2\pi} T_{\text{eff}}^4 \left(\tfrac{3}{2}\tau + 1\right) \qquad \Rightarrow T^4 = T_{\text{eff}}^4 \left(\tfrac{3}{4}\tau + \tfrac{1}{2}\right)$$

What we learned

- Equation & boundary condition
  - No radiation from the top
- Trial solution
  - Explains center-to-limb darkening to first order
- *T* and effective *T*
  - $T_{\mathrm{min}} = 0.84\ T_{\mathrm{eff}}$