Lecture 3

- About solar interior
- Radiation transport
- Effective (Rosseland) opacity
- Nuclear burning
- # of neutrinos

Summary of previous lecture

- Long-term solar variability
  - Grant minima/maxima
  - Total solar irradiance
- Spectral irradiance
  - Black body, gray body
- Internal structure of the Sun
  - *Y*-dependence
  - Intensity & radiation transport

Conversion of spectral distribution function

$$F(T) = \int I_{\nu}(\nu, T)\, d\nu$$

or

$$F(T) = \int I_{\lambda}(\lambda, T)\, d\lambda$$

$$\nu(\lambda) = \frac{c}{\lambda}$$

Hint: compute $\frac{d\nu}{d\lambda}$

Dimensional analysis

$$\left[I_\nu(\nu,T)\right]=\frac{\mathrm{W}}{\mathrm{m}^2\mathrm{Hz}} \qquad k_\mathrm{B}=1.38\times10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$$

$$I_\lambda(\lambda,T)=\lambda^a T^b c^c k_\mathrm{B}^d$$

$$I_\lambda(\lambda,T)=\frac{2c}{\lambda^4}k_\mathrm{B}T$$

What was this law called?

A. Fokker-Planck law
B. Rayleigh-Benard law
C. Rayleigh-Wien law
D. Rayleigh-Jeans law
E. Wien law

Internal structure of the Sun

- No way to look into the Sun, except…
- Seeing deeper
  - Different wavelengths
  - Ca, Fe lines (slightly higher up)
  - infrared (slightly deeper)
- Helioseismology
- Neutrinos
- Theory (depends on Y and mixing length)
  - $X+Y+Z=1$

Dependence on Y

- Solve time-dependent stellar structure eqns
- Produce more *Y* via burning of *X*

$$\ln L = \ln L_\odot + a(Y_0 - Y_{0\odot}) + b(\alpha - \alpha_\odot)$$

$$\ln r = \ln r_\odot + c(Y_0 - Y_{0\odot}) + d(\alpha - \alpha_\odot)\ ,$$

R and *L* grow (faint sun paradox)

$$a \equiv \frac{\partial \ln L}{\partial Y_0} = 8.6 \qquad b \equiv \frac{\partial \ln L}{\partial \alpha} = 0.04$$

$$c \equiv \frac{\partial \ln r}{\partial Y_0} = 2.1 \qquad d \equiv \frac{\partial \ln r}{\partial \alpha} = -0.13$$

Significance of Z

- Affects the opacity
- Affects so-called CNO energy generation
- Through ^{12}C and ^{14}N
- Constrained observationally
- Produced in SNe of earlier generations

More on intensity

$$I_\nu(\mathbf{x}, \hat{\mathbf{n}}, t)$$

depends also on direction

for each ray path…

$$\frac{dI_\nu}{ds} = -\rho\kappa_\nu (I_\nu - S_\nu)$$

$$\hat{\mathbf{n}} \cdot \nabla I_\nu = -\rho\kappa_\nu (I_\nu - S_\nu)$$

or

$$\frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$$

with $d\tau_\nu = -\rho\kappa_\nu ds$

optical depth

3. A Not-So-Ordinary Differential Equation. Consider a one-dimensional "slab" of gas that starts at $x = 0$ and ends at $x = D$, and is surrounded by empty space. A ray of light with intensity I_0 hits the slab at $x = 0$ and shines through it parallel to the x axis. Inside the slab, the intensity obeys

$$\frac{dI}{dx} = \alpha (S - I)$$

where α and S are constants.

(a) Solve this equation for $I(x)$ at all points between $x = 0$ and $x = D$.

(b) Define the quantity $\tau = \alpha D$. Give an approximate solution for the "emergent intensity" $I(D)$ under the three limiting cases:

- $\tau \ll 1$.
- $\tau \gg 1$ and $S \gg I_0$.
- $\tau \gg 1$ and $S \ll I_0$.

(c) Each of the three above cases matches with one of the following three physical analogies. Which do you think corresponds to which, and why?

- Shining a flashlight through a piece of dark smoky quartz.
- Shining a flashlight through the bright flame of a welder's torch.
- Shining a flashlight through a glass window pane.

Hint: The quantity τ can be thought of as the "optical depth" or opaqueness of the slab—i.e., how efficiently does the gas absorb (or otherwise eliminate) the incoming beam. The quantity S is a "source function" that describes how the gas in the slab generates its own light.

Radiation

Assume that source function given by just the Planck (or Kirchhoff-Planck) function

$$S_\nu = B_\nu \equiv \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_{\rm B}T} - 1}$$

$$\hat{\mathbf{n}} \cdot \nabla I_\nu = -\rho\kappa_\nu \left(I_\nu - S_\nu \right)$$

Spherical coordinates

$$\cos\theta \frac{dI_\nu}{dr} = -\rho\kappa_\nu \left(I_\nu - B_\nu \right)$$

Approximate solution

Leading order $$I_\nu = B_\nu$$

Insert

$$\cos\theta \frac{dB_\nu}{dr} = -\rho\kappa_\nu \left(I_\nu - B_\nu\right)$$

so

$$I_\nu = B_\nu - \frac{\cos\theta}{\rho\kappa_\nu}\frac{dB_\nu}{dr}$$

Interested in flux

$$\int_{4\pi} I_\nu \cos\theta \, d\Omega = 2\pi \int_{-1}^{1} I_\nu \cos\theta \, d\cos\theta$$

Angular integration

Interested in flux

$$\int_{4\pi} I_\nu \cos\theta \, d\Omega = 2\pi \int_{-1}^{1} I_\nu \cos\theta \, d\cos\theta$$

insert

$$I_\nu = B_\nu - \frac{\cos\theta}{\rho\kappa_\nu}\frac{dB_\nu}{dr}$$

get

$$F_\nu = -\frac{2\pi}{\rho\kappa_\nu}\frac{dB_\nu}{dr}\underbrace{\int_{-1}^{1}\mu^2 d\mu}_{=2/3}$$

Frequency integration

get

$$F_\nu = -\frac{4\pi}{3\rho\kappa_\nu}\frac{dB_\nu}{dr} = -\frac{4\pi}{3\rho\kappa_\nu}\frac{dB_\nu}{dT}\frac{dT}{dr}$$

get

$$\int_0^\infty F_\nu d\nu = -\frac{4\pi}{3\rho}\frac{dT}{dr}\int_0^\infty \frac{1}{\kappa_\nu}\frac{dB_\nu}{dT}d\nu$$

Finally

$$\int_0^\infty F_\nu d\nu = -K\frac{dT}{dr}$$

Rosseland mean opacity

Write as

$$\int_0^\infty \frac{1}{\kappa_\nu}\frac{dB_\nu}{dT}d\nu = \frac{1}{\kappa}\int_0^\infty \frac{dB_\nu}{dT}d\nu$$

which defines the so-called Rosseland mean opacity

$$\frac{1}{\kappa} = \int_0^\infty \frac{1}{\kappa_\nu}\frac{dB_\nu}{dT}d\nu \bigg/ \int_0^\infty \frac{dB_\nu}{dT}d\nu$$

and which enters

$$K = \frac{16\sigma_{\mathrm{SB}}T^3}{3\rho\kappa}$$

Auxiliary formulae

Need to know

$$\int_0^\infty \frac{x^3}{e^x - 1}\,dx = \frac{\pi^4}{15}$$

definition

$$\frac{2\pi k_B^4}{15 h^3 c^2} = \sigma_{SB}$$

so

$$S = \frac{\sigma_{SB}}{\pi} T^4$$