Lecture 32

- Comments on Eddington approx.
- Heat conduction and heat diffusion
  - Compare w/ magnetic diffusivity
- Hydrostatic corona

Last time…

- Corona: heating and wind acceleration
- Solar Wind
  - Smooth transition to supersonic
  - Critical point
  - Logarithmic differentiation

Lect. 26: Eddington approx.

ansatz (trial solution)

$$I = I_0 + \mu I_1$$

Because of $\frac{dI_0}{d\tau} = I_1 = \text{const}$

we have $I_0 = a\tau + b$

$$\int_0^1 \mu I \, d\mu = \tfrac{1}{2} I_0 + \tfrac{1}{3} I_1 = \tfrac{1}{2\pi} F_z \qquad \Rightarrow b = \tfrac{2}{3} I_1 = \tfrac{1}{2\pi} F$$

$$\int_{-1}^1 \mu I \, d\mu = \tfrac{2}{3} I_1 = \tfrac{1}{2\pi} F \qquad \Rightarrow a = \tfrac{3}{4\pi} F$$

$$I_0 = \tfrac{1}{2\pi} F\left(\tfrac{3}{2}\tau + 1\right) \qquad I_1 = \tfrac{3}{4\pi} F \qquad \Rightarrow I = \frac{3}{4\pi} F\left(\tau + \frac{2}{3} + \mu\right)$$

$$\tfrac{\sigma_B}{\pi} T^4 = S = I_0 = \tfrac{1}{2\pi} F\left(\tfrac{3}{2}\tau + 1\right) = \tfrac{\sigma_B}{2\pi} T_{\text{eff}}^4 \left(\tfrac{3}{2}\tau + 1\right) \qquad \Rightarrow T^4 = T_{\text{eff}}^4 \left(\tfrac{3}{4}\tau + \tfrac{1}{2}\right)$$

Integration of $\int I(\tau, \mu) e^{\tau/\mu} d\mu$

A. Not relevant: exclude $\mu=0$
B. Regular, because $I \rightarrow 0$
C. Have to exclude $\mu=0$.

Heat conduction

Frequent collisions: electric, kinetic, and thermal energies similar

$$k_e \frac{e^2}{r} \sim \tfrac{1}{2} m_e v_e^2 \sim k_B T$$

$$k_e = \frac{1}{4\pi\varepsilon_0}$$

permittivity

Coulomb
(1736-1806)

Coulomb cross-section

$$\sigma_{\text{cross}} = \pi r^2 = \pi \left(\frac{k_e e^2}{k_B T} \right)^2$$

Mean-free path

$$\ell = \frac{1}{n\sigma_{\text{cross}}} \propto \frac{T^2}{n}$$

Heat diffusivity

$$\chi \sim \ell v_e \sim T^{5/2}$$

Compare: electric conductivity

Acceleration until collision

$$\frac{m_e \mathbf{V}}{\tau_{ei}} = e\mathbf{E}$$

$$\tau_{ei} = \frac{T^2}{n_e v_e} \propto \frac{T^{3/2}}{n_e}$$

conductivity

$$\mathbf{J} = n_e e \mathbf{V} = \underbrace{\frac{n_e e^2}{m_e}\tau_{ei}}_{\sigma} \mathbf{E}$$

Mean-free path

$$\ell = \frac{1}{n\sigma_{cross}} \propto \frac{T^2}{n}$$

magnetic diffusivity

$$\eta = \frac{1}{\mu_0 \sigma} \sim T^{-3/2}$$

Heat diffusivity

$$\chi \sim \ell v_e \sim T^{5/2}$$

Note: n_e cancels!

Stratification w/ heat conduction

Thermal equilibrium

$$\nabla \cdot K \nabla T = 0$$

$$T^{5/2} r^2 \frac{dT}{dr} = \text{const}$$

$$K \propto T^{5/2}$$

$$\nabla \cdot (\ldots) = \frac{1}{r^2} \frac{d}{dr}\left(r^2 \ldots\right)$$

so

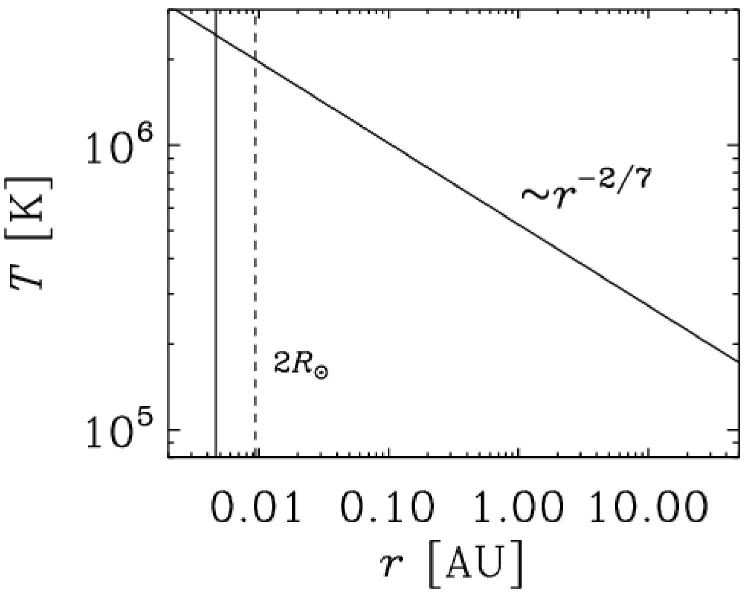
$$\int T^{5/2} dT = \text{const} \times \int r^{-2} dr$$

or

$$T = T_0 \left(\frac{r}{r_0}\right)^{-2/7}$$

combine with

$$\frac{dP}{dr} = -\rho \frac{GM}{r^2}$$



Result (Stix, p. 410)

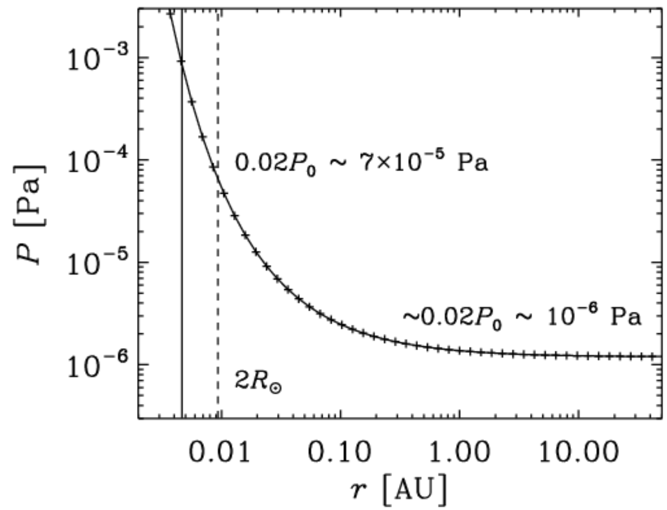
Hydrostatic equilibrium $$\frac{dP}{dr} = -\rho \frac{GM}{r^2}$$

$$\frac{d \ln P}{dr} = -\frac{\rho}{P}\frac{GM}{r^2} = -\frac{\mu}{\Re T}\frac{GM}{r^2} = -\frac{\mu}{\Re T_0}\left(\frac{r}{r_0}\right)^{2/7}\frac{GM}{r^2}$$

integrate $$\int d \ln P = \frac{\mu}{\Re T_0}\frac{GM}{r_0}\int (r/r_0)^{-12/7} d(r/r_0)$$

so $$\ln(P/P_0) = \underbrace{\tfrac{7}{5}\frac{\mu}{\Re T_0}\frac{GM}{r_0}}_{C_2 \approx 4}\left[(r/r_0)^{-5/7} - 1\right]$$

gives $$P = P_0 \exp\left\{4\left[\left(\frac{r}{r_0}\right)^{-5/7} - 1\right]\right\}$$



Homework 5, problem 3

Work with density $\frac{dP}{dr} = -\rho \frac{GM}{r^2}$

$$\frac{d}{dr}\left(\frac{\Re T}{\mu}\rho\right) = -\frac{\rho}{P}\rho\frac{GM}{r^2} = -\frac{\rho}{\Re T}\frac{GM}{r^2} = -\frac{\mu}{\Re T_0}\left(\frac{r}{r_0}\right)^{2/7}\frac{GM}{r^2}$$

integrate $$\frac{d}{dr}\left(\frac{\rho}{r^{2/7}}\right) = -\left(\frac{\rho}{r^{2/7}}\right)C_1 r^{-12/7}$$

gives $$\ln\left(\frac{\rho}{r^{2/7}}\right) = -C_1\int r^{-12/7}dr + \text{const}$$

Notes on the Solar Corona and the Terrestrial Ionosphere

By Sydney Chapman [1] 1957

Outward thermal conduction from the solar corona

We shall consider in the simplest possible way some properties that would characterize a model solar corona, static and spherically symmetrical (while recognizing that the actual corona is dynamic and asymmetric). The problem has already been discussed by many writers (Woolley and Stibbs, 1953). Van de Hulst (1953) has critically reviewed much recent work. The problem deals partly with the escape of coronal gas from the sun's gravita- denotes the thermal conductivity, for the same n and T, of a gas composed of electrons alone, half positive, half negative, supposed permanent, without recombination.

Let k denote Boltzmann's constant, e the electrostatic unit of charge, m the electron mass. Then, allowing for a factor 4/3 to correct the first approximation to the formula for K (Chapman, 1954, p. 155), we obtain

$$K_e = \frac{25}{2A_2(2)} \frac{k^{7/2}T^{5/2}}{(\pi m)^{1/2}e^4},$$

It is suggested that the hot coronal gas surrounding the earth may be the cause of the downward flow of heat below the F_2 peak in the terrestrial ionosphere; but difficult questions related to the influence of the geomagnetic field upon the fully ionized coronal gas render this suggestion speculative. It might be settled by observing the density or temperature above the F_2 peak.

The satellites to be launched during the International Geophysical Year will move in regions where such observations will be of special value in this connection.

Was Chapman right?

A. Yes: hot ionosphere thermosphere
B. No: $T^{5/2}$ law not valid beyond corona
C. Neither of the two: corona is not static
D. No: invalid if $1/n$ factor included

DYNAMICS OF THE INTERPLANETARY GAS AND MAGNETIC FIELDS*

E. N. Parker
Enrico Fermi Institute for Nuclear Studies, University of Chicago

Received January 2, 1958

We see, then, that, with the temperature varying as in equation (3) and with n
least as large as the 0.5 for neutral hydrogen, we have non-vanishing pressure
infinity,

$$p(\infty) = p_0 \exp\left[\frac{-\lambda(n+1)}{n}\right]$$

for hydrostatic equilibrium. With $n = 2.5$ (for ionized hydrogen) and $a = 10^6$ km, T_0
1.5×10^6 °K, and $M_\odot = 2 \times 10^{33}$ gm, we have $\lambda = 5.35$ and $p(\infty) = 0.55 \times 10^{-3}$
Even $n = 0.5$ (for un-ionized hydrogen) yields $p(\infty) = 10^{-7}\,p_0$. With standard coro
conditions, $N_0 = 3 \times 10^7/\text{cm}^3$, $T_0 = 1.5 \times 10^6$ °K, we have $p_0 = 2N_0kT_0 \cong 1.3$
10^{-2} dynes/cm². Hence $p(\infty) = 0.6 \times 10^{-5}$ dynes/cm² for $n = 2.5$ and 1.3×10^{-9}
$n = 0.5$.

What we learned today

- Eddington approximation
- 20% error near the surface compared with the *formal solution*
- Heat conduction and heat diffusion
  - $T^{5/2}$ law
- magnetic diffusivity different
  - $T^{-3/2}$ law
- Hydrostatic corona