



PS

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2539

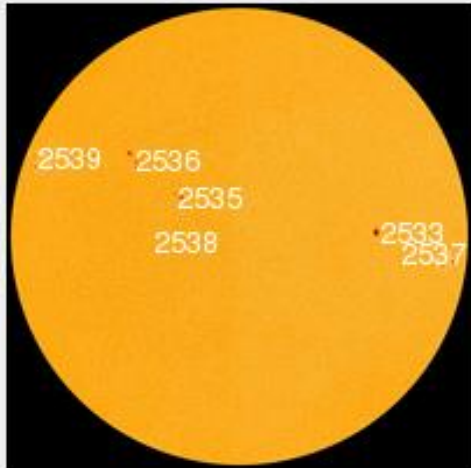
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Sunspot AR2535 is crackling with [C-class](#) solar flares. One of those flares hurled a CME in the general direction of Earth on April 28th. Forecasters say the CME could deliver a glancing blow to Earth's magnetic field on May 1st. Credit: SDO/HMI

**Sunspot number: 84**

[What is the sunspot number?](#)

Updated 29 Apr 2016

**Spotless Days**

Current Stretch: 0 days

2016 total: 0 days (0%)

2015 total: 0 days (0%)

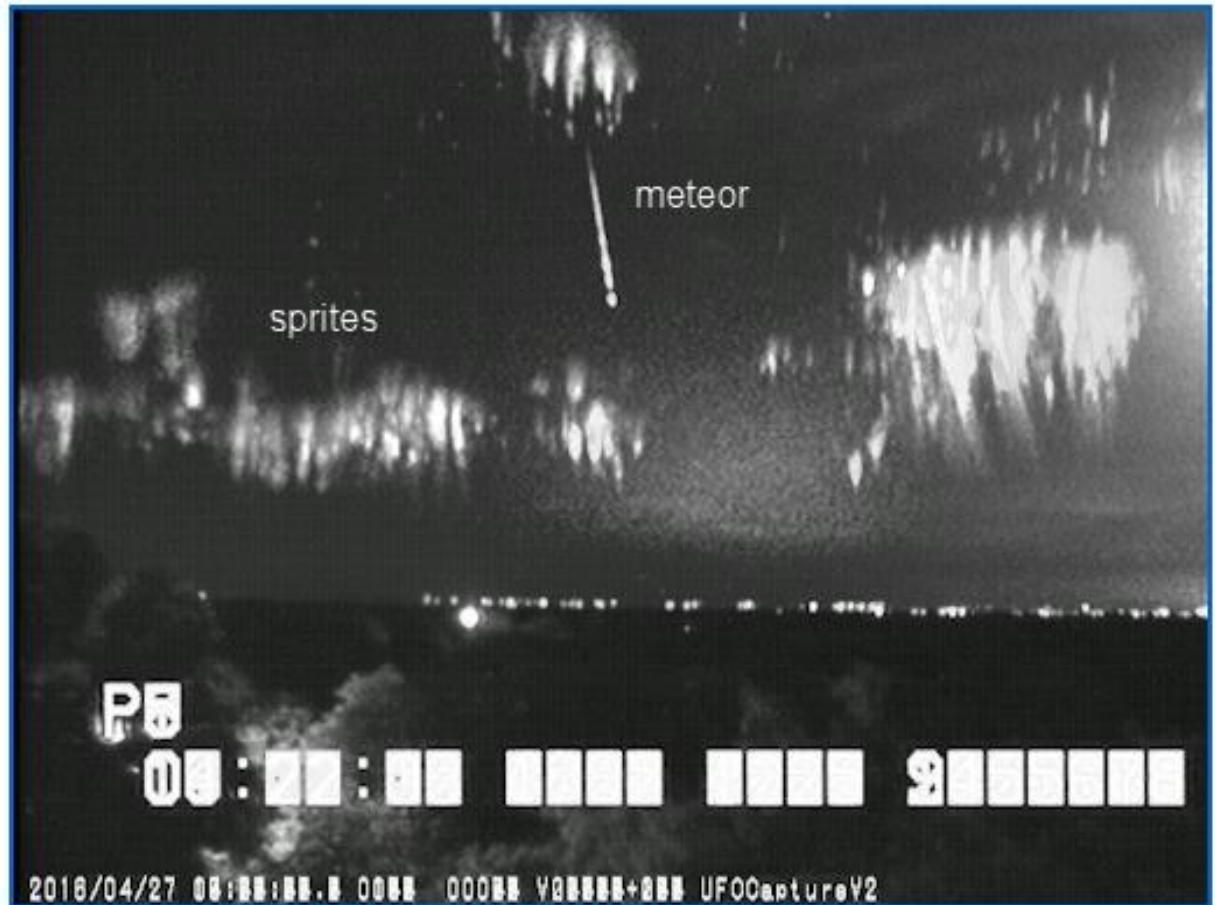
2014 total: 1 day (<1%)

2013 total: 0 days (0%)

2012 total: 0 days (0%)

2011 total: 2 days (<1%)

**"SPACE LIGHTNING" OVER TEXAS:** You know what comes out of the bottom of thunderstorms: lightning. On April 27th, Kevin Palivec of Hawley, Texas, saw something coming out of the *top*. "Storms moving across Texas produce more than just rain, wind, hail and tornadoes!" says Palivec. "They also produce a lot of space lightning called '[sprites](#).' This is a stacked image of all the sprites I caught over storms as they moved across Texas towards Dallas/Ft Worth--with one meteor thrown in!"



Because sprites are associated with thunderstorms, they tend to occur in late spring and summer. Palivec's photo shows that **sprite season is now underway**.

# *Last time...*

- More on final report
  - Relation to other work (introduction)
  - Where to go from here (conclusions)
- Results so far

# *Lecture 40*

- Review & questionnaire
- More on final report
- Results so far
- Final words

# Lecture 2

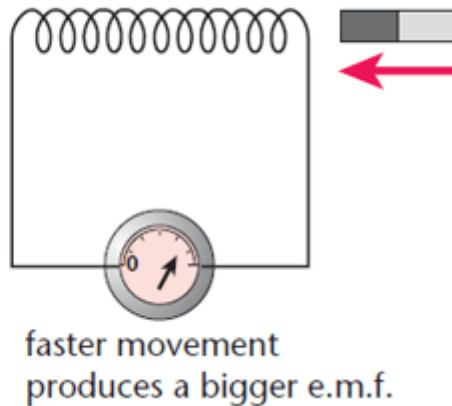
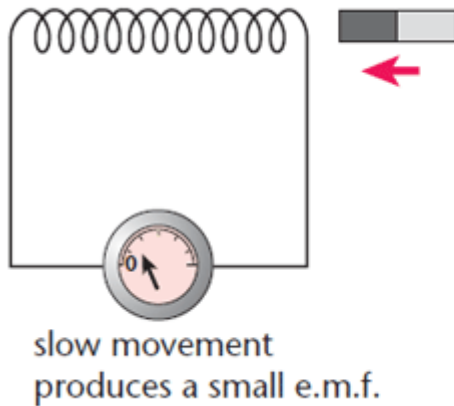
$$I_\nu(\nu, T) = \frac{2\nu^2}{c^2} k_B T$$

$$I_\lambda(\lambda, T) = I_\nu(\nu, T) \frac{c}{\lambda^2} = \frac{2\nu^2}{c^2} \frac{c}{\lambda^2} k_B T$$

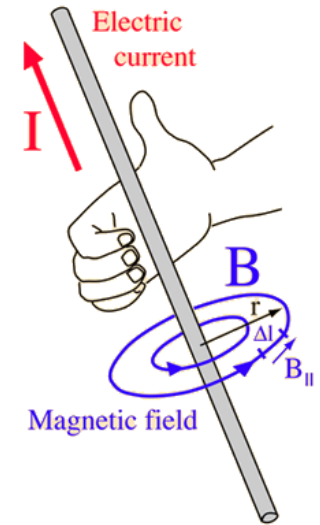
$$= \frac{2c}{\lambda^4} k_B T$$

# Lecture 5

## Q1: Faraday's law



## Q2: Ampere's law



A. 
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

B. 
$$\nabla \cdot \mathbf{B} = 0$$

C. 
$$\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = +\nabla \times \mathbf{B} - \mu_0 \mathbf{J}$$

D. 
$$\nabla \cdot \mathbf{E} = \rho_c / \epsilon_0$$

# Stokes $U = Stokes I$

$$E_x = \xi_x \cos \phi, \quad E_y = \xi_y \cos(\phi + \varepsilon)$$

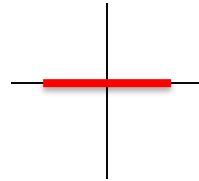
$$I = \xi_x^2 + \xi_y^2$$

$$Q = \xi_x^2 - \xi_y^2$$

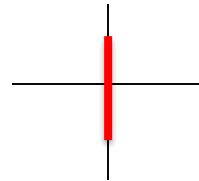
$$U = 2\xi_x \xi_y \cos \varepsilon$$

$$V = 2\xi_x \xi_y \sin \varepsilon$$

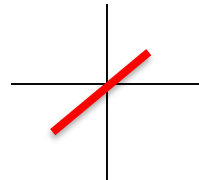
A.



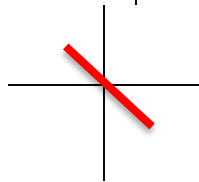
B.



C.



D.





# Lecture 6

$$\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = 1$$

$$\varepsilon_{321} = \varepsilon_{213} = \varepsilon_{132} = -1$$

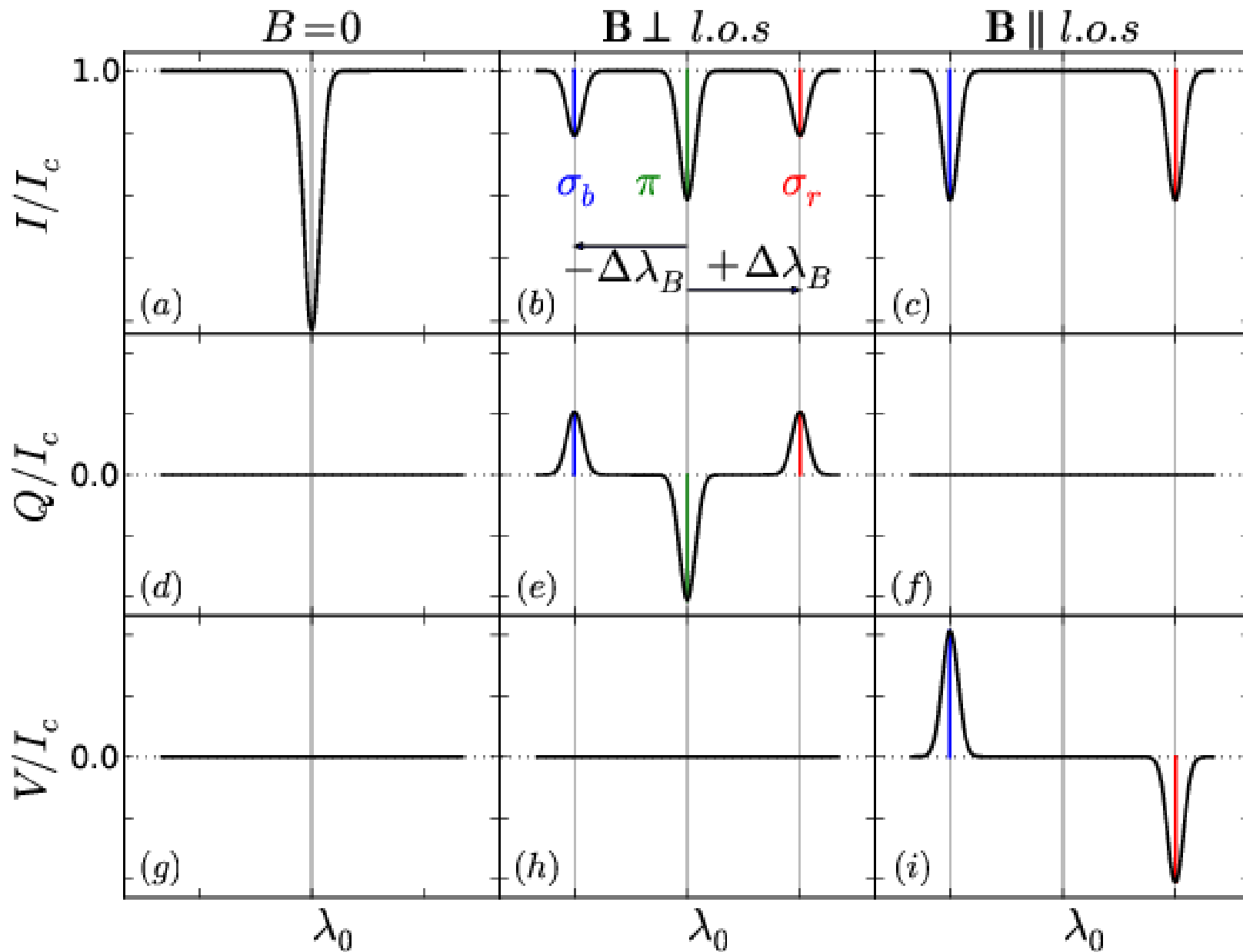
$$\varepsilon_{ijk} = 0 \text{ otherwise}$$

Also: totally  
antisymmetric tensor

what is  $\varepsilon_{ijk}\varepsilon_{ijk}$  ??

- A. 0
- B. 1
- C. 3
- D. 4
- E. 6

# Lecture 8



What is  $e^{-3i\pi/4}$  ?

A.  $(1+i)/\sqrt{2}$

B.  $(1-i)/\sqrt{2}$

C.  $(-1+i)/\sqrt{2}$

D.  $-(1+i)/\sqrt{2}$

# Lecture 11

Expand continuity eqn: 
$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) = -\mathbf{u} \cdot \nabla \rho - \rho \nabla \cdot \mathbf{u}$$

Momntum eqn (isothermal):

$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\rho \mathbf{u} \cdot \nabla \mathbf{u} - \frac{\mathcal{R}T}{\mu} \nabla \rho + \dots$$

Linearized form

$$\frac{\partial \rho_1}{\partial t} = -\rho_0 \nabla \cdot \mathbf{u}_1$$

$$\rho_0 \frac{\partial \mathbf{u}_1}{\partial t} = -\frac{\mathcal{R}T}{\mu} \nabla \rho_1$$

Trial solution

$$\rho_1(z, t) = \hat{\rho}_1 e^{ik_z z - i\omega t} + \text{c.c.}$$

$$u_{1z}(z, t) = \hat{u}_{1z} e^{ik_z z - i\omega t} + \text{c.c.}$$

$$\begin{pmatrix} i\omega & -ik_z \rho_0 \\ -ik_z \frac{\mathcal{R}T}{\mu} & i\omega \rho_0 \end{pmatrix} \begin{pmatrix} \hat{\rho}_1 \\ \hat{u}_{1z} \end{pmatrix} = 0$$

Dispersion relation 
$$\omega^2 = \frac{\mathcal{R}T}{\mu} k_z^2$$

$$c_s = \sqrt{\mathcal{R}T / \mu} \quad \text{Sound speed}$$

# Lecture 12

Dispersion relation of Lecture 11

$$\omega^2 = \frac{\Re T}{\mu} k_z^2$$

In 3-D 
$$\omega^2 = c_s^2 \left( \underbrace{k_x^2 + k_y^2}_{k_{\text{hor}}^2} + \underbrace{k_z^2}_{k_{\text{vert}}^2} \right)$$

“Solve” for  $k_z$

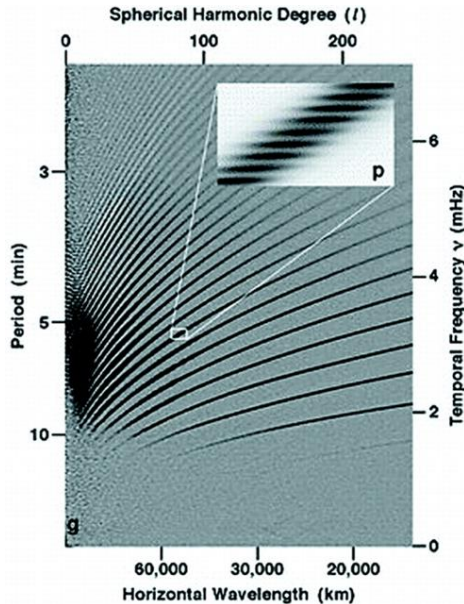
$$k_z^2 = \frac{\omega^2}{c_s^2} - k_{\text{hor}}^2$$

Consider  $c_s = c_s(r)$  [oops?]

In quantum mechanics:  
WKB approximation

- Jeffreys-Wentzel-Kramers-Brillouin
- Tunnel effect → Gamow!!

Deeper down:  $k_z$  imaginary!?



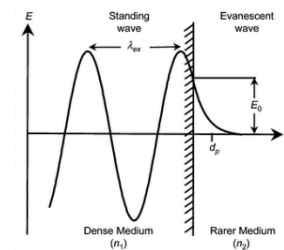
Example

$$\omega = \frac{2\pi}{300\text{s}} = 0.02\text{s}^{-1}$$

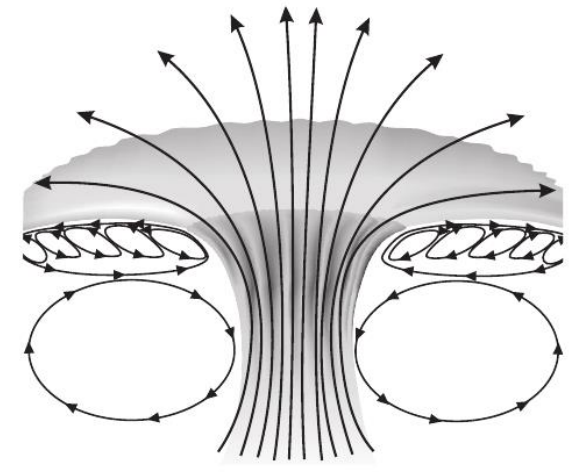
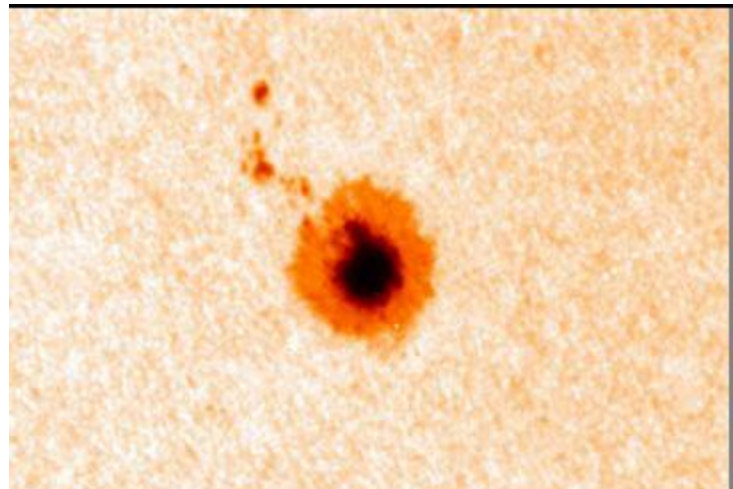
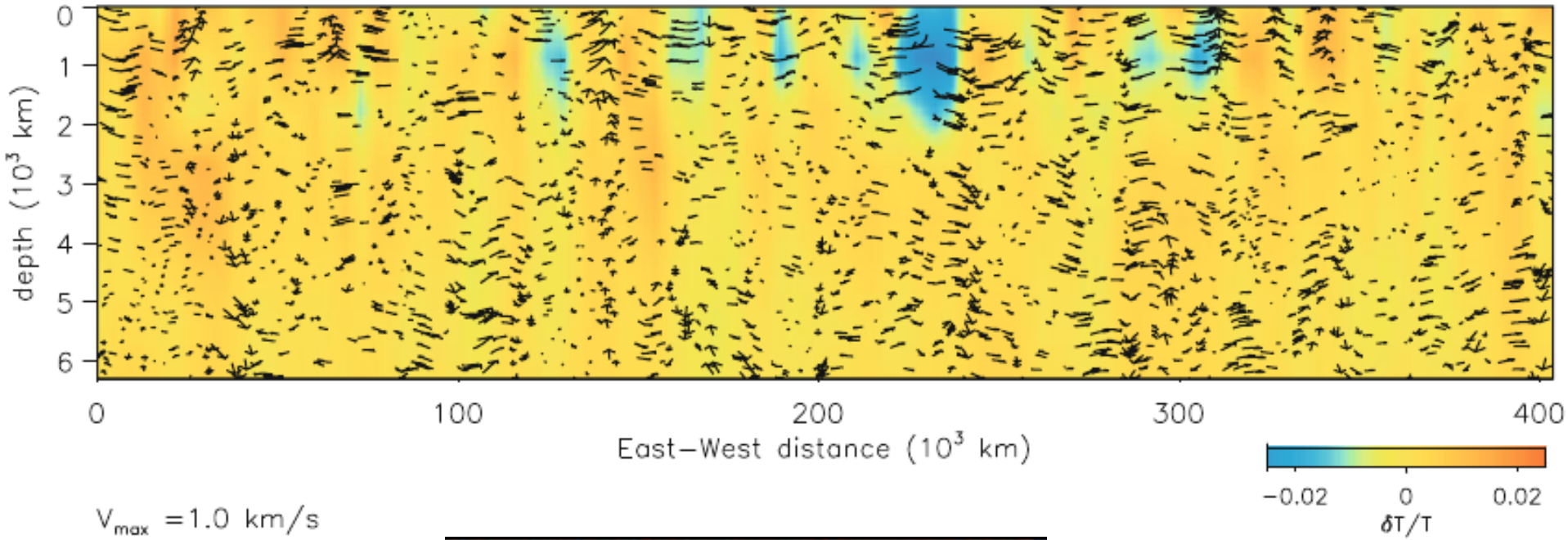
$$k_{\text{hor}} = \frac{\ell}{R} = \frac{100}{700\text{Mm}}$$

$$c_s = \frac{\omega}{k_{\text{hor}}} = 0.02 \times 7 \frac{\text{Mm}}{\text{s}} = 140 \frac{\text{km}}{\text{s}}$$

$$e^{ik_z z} = e^{\mp |k_z| z}$$



# Lecture 13

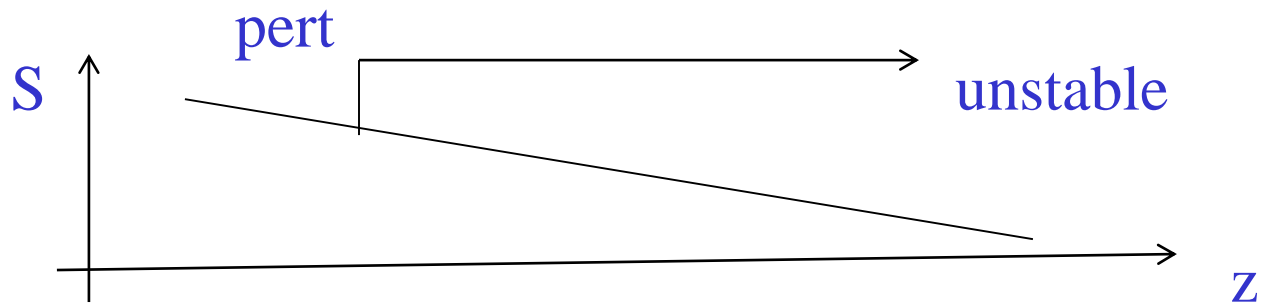
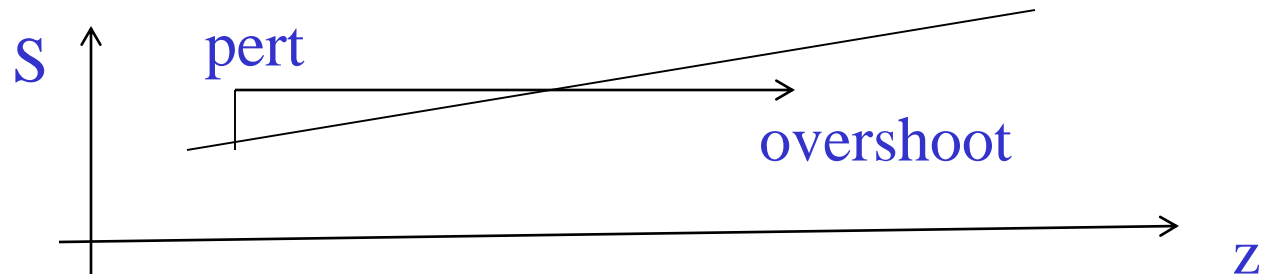


Sunspot imaging  
...but uncertain!

# Lecture 15

$$s / c_p = \frac{1}{\gamma} \ln P - \ln \rho$$

Adiabatic changes:  $S = \text{const}$   
P equilibrium:  $S+ \rightarrow$  buoyant



# Lecture 16

Momentum equation

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla P$$

Multiply by  $\mathbf{u}$

$$\frac{1}{2} \rho \frac{D\mathbf{u}^2}{Dt} = -\mathbf{u} \cdot \nabla P$$

Internal energy  
equation

$$\rho c_v \frac{DT}{Dt} = -P \nabla \cdot \mathbf{u} + \rho T \frac{Ds}{Dt}$$

Add the two to get  
one part of total  
energy equation

$$\frac{1}{2} \rho \frac{D\mathbf{u}^2}{Dt} + \rho c_v \frac{DT}{Dt} = -\nabla \cdot (\mathbf{u}P) + \rho T \frac{Ds}{Dt}$$

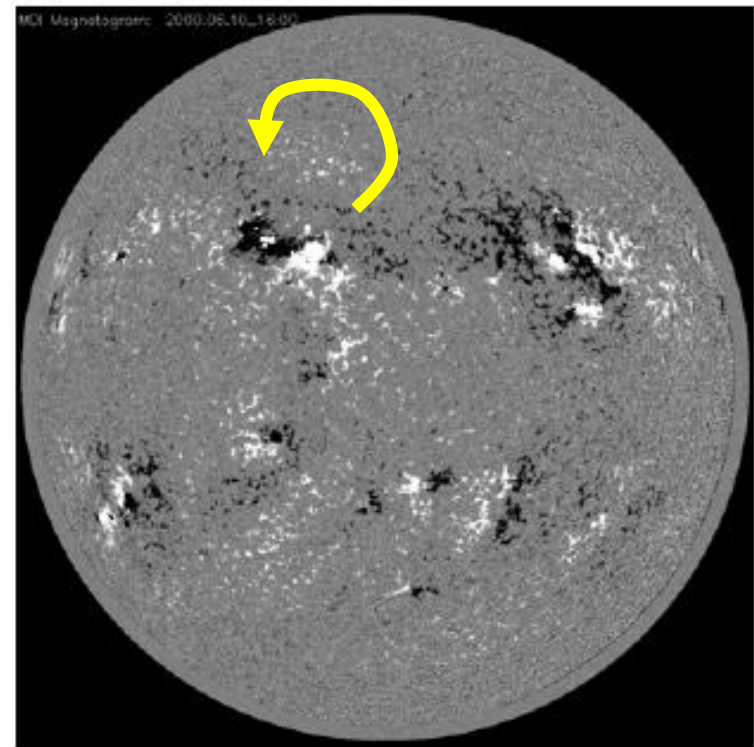


# Lecture 17

## The Sun today and 9 years ago



Solar magnetograms:  
Line of sight B-field from  
circularly polarized light



# Lecture 19

Insert  
ansatz

$$u_y = \hat{u}_y \sin(kx - \omega t)$$
$$b_y = \rho\mu_0 \hat{b}_y \sin(kx - \omega t)$$

into

$$\frac{\partial u_y}{\partial t} = B_x \nabla_x b_y / \rho\mu_0$$
$$\frac{\partial b_y}{\partial t} = B_x \nabla_x u_y$$

Linearize  $\mathbf{B} \rightarrow \mathbf{B} + \mathbf{b}$

$$-\omega \hat{u}_y \cos(k_x x - \omega t) = B_x k_x \hat{b}_y \cos(k_x x - \omega t)$$

$$-\omega \hat{b}_y \cos(k_x x - \omega t) = B_x k_x \hat{u}_y \cos(k_x x - \omega t)$$

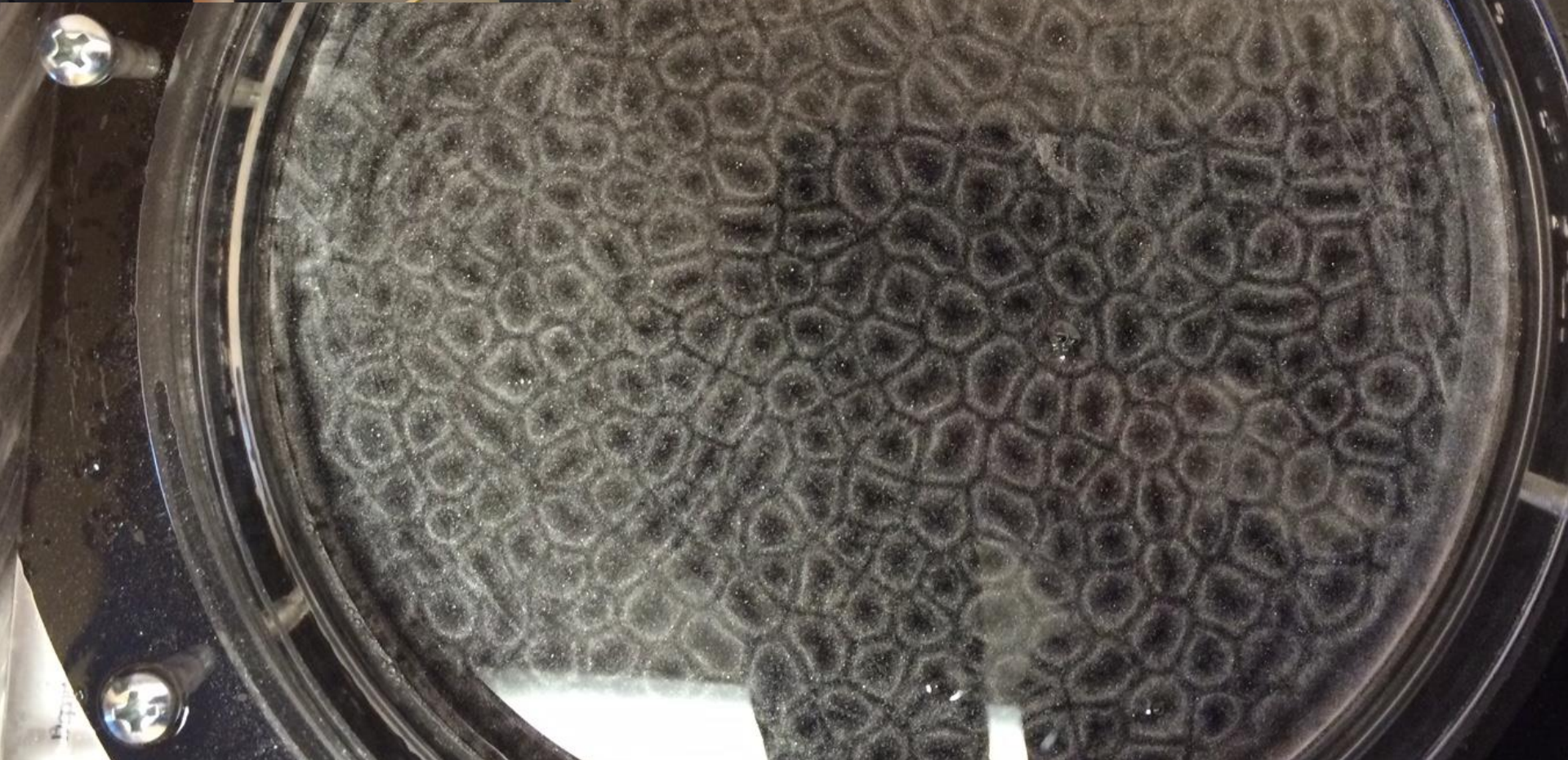
insert

$$-\omega \hat{u}_y = B_x k_x \hat{b}_y / \rho\mu_0$$

$$-\omega \hat{b}_y = B_x k_x \hat{u}_y$$

Dispersion relation

$$\omega^2 = B_x^2 k_x^2 / \rho\mu_0$$



# Lecture 22

Momentum eqn:

$$\rho_0 \frac{\partial \mathbf{u}_1}{\partial t} = -\nabla p_1 + s_1 \mathbf{g} / c_p + \nu \nabla^2 \mathbf{u}_1 \dots$$

Entropy equation:

$$\frac{\partial s_1}{\partial t} = -\mathbf{u}_1 \cdot \nabla s_0 + \chi \nabla^2 s_1$$

here:  $\chi$  is the thermal (radiative) diffusivity

Assume  $\nu = \chi$   
for simplicity

$$\begin{pmatrix} -\lambda - \nu k_z^2 & -g / c_p \\ -ds_0 / dz & -\lambda - \nu k_z^2 \end{pmatrix} \begin{pmatrix} \hat{u}_{1z} \\ \hat{s}_1 \end{pmatrix} = 0$$

$$(\lambda + \nu k_z^2)^2 - g \frac{ds_0 / c_p}{dz} = 0 \quad (\lambda + \nu k_z^2)^2 = \left( g \frac{ds_0 / c_p}{dz} \right)$$

# Lecture 26

Frequent collisions: electric, kinetic, and thermal energies similar

$$k_e \frac{e^2}{r} \sim \frac{1}{2} m_e v_e^2 \sim k_B T$$

$$k_e = \frac{1}{4\pi\epsilon_0}$$

permittivity



Coulomb  
(1736-1806)

Coulomb cross-section

$$\sigma_{\text{cross}} = \pi r^2 = \pi \left( \frac{k_e e^2}{k_B T} \right)^2$$

Mean-free path

$$\ell = \frac{1}{n\sigma_{\text{cross}}} \propto \frac{T^2}{n}$$

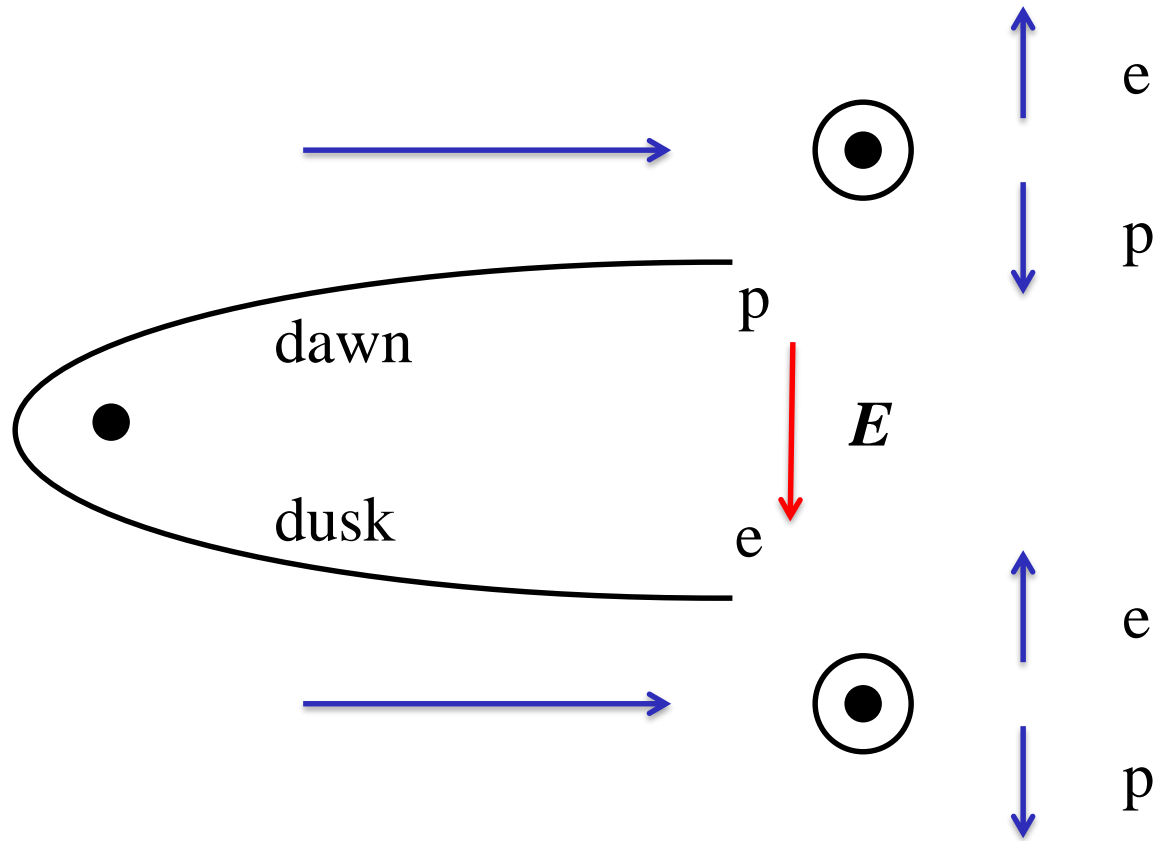
Heat diffusivity

$$\chi \sim \ell v_e \sim T^{5/2}$$

Also true for viscosity

$$\nu \sim \ell v_e \sim T^{5/2}$$

# Lecture 34: Cross-tail electric field



# *What we did today*

- Review & questionnaire
- Final report & Results so far
- Final words:
  - Comment about straylight
  - Deadline: May 5, 11:59pm, Q&A session 4:30-7
  - Thanks to Baylee for her help & dedication
    - Thanks also to Fabio for preparations behind the scene
  - For future: recommendation letters
  - Summer projects?