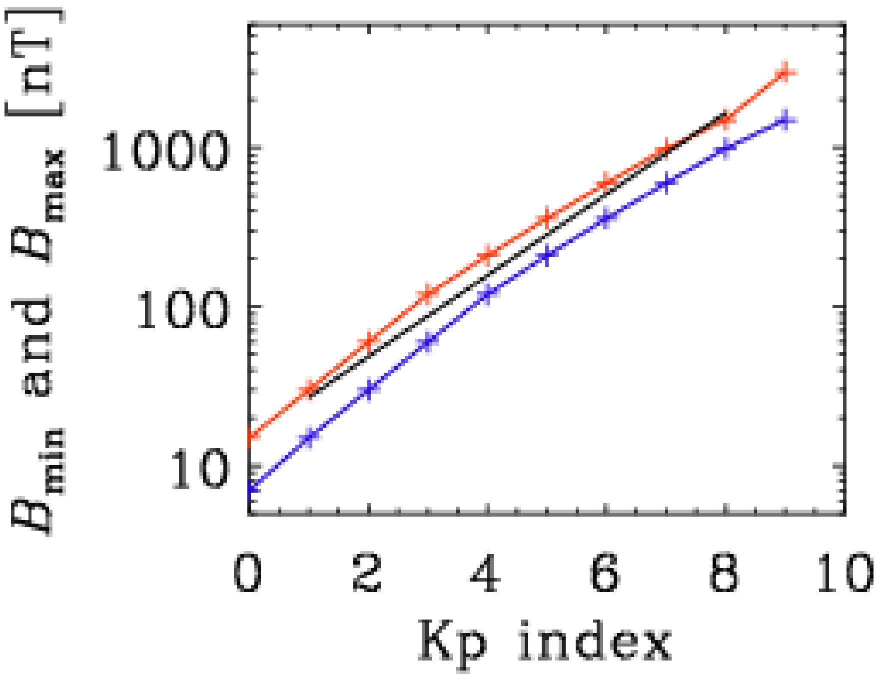
Lecture 8

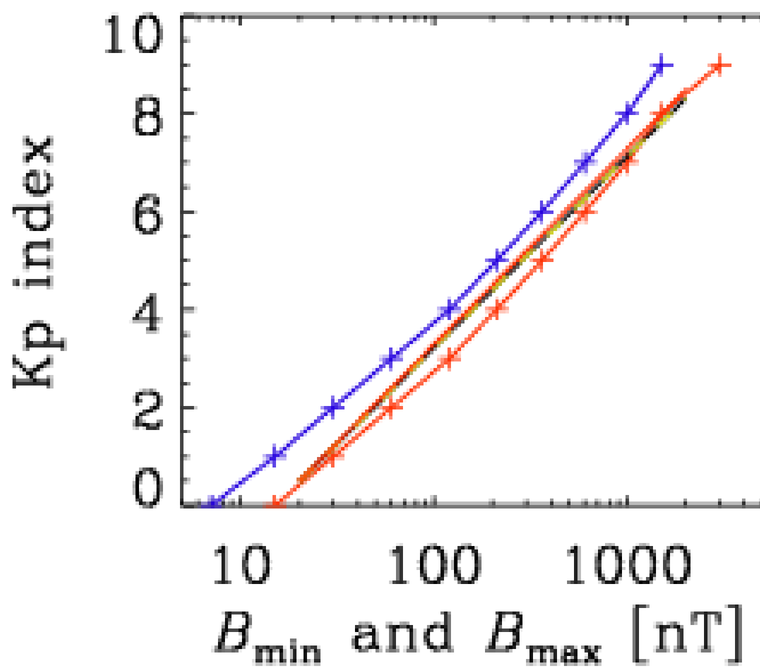
- Polarimetry
- Zeeman splitting
- Polarized light
- Stokes parameter
- Stokes radiative transfer

Summary of previous lecture

- About LASP visit
- Kronecker and Levi-Civita
- Homework
- Polarimetry → on Monday

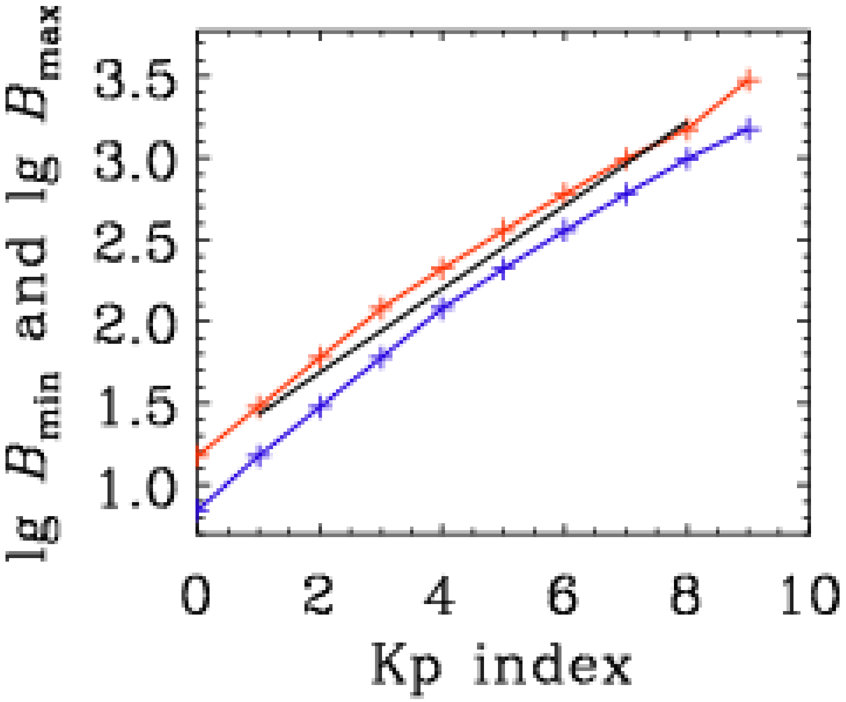
Goal: as linear as possible

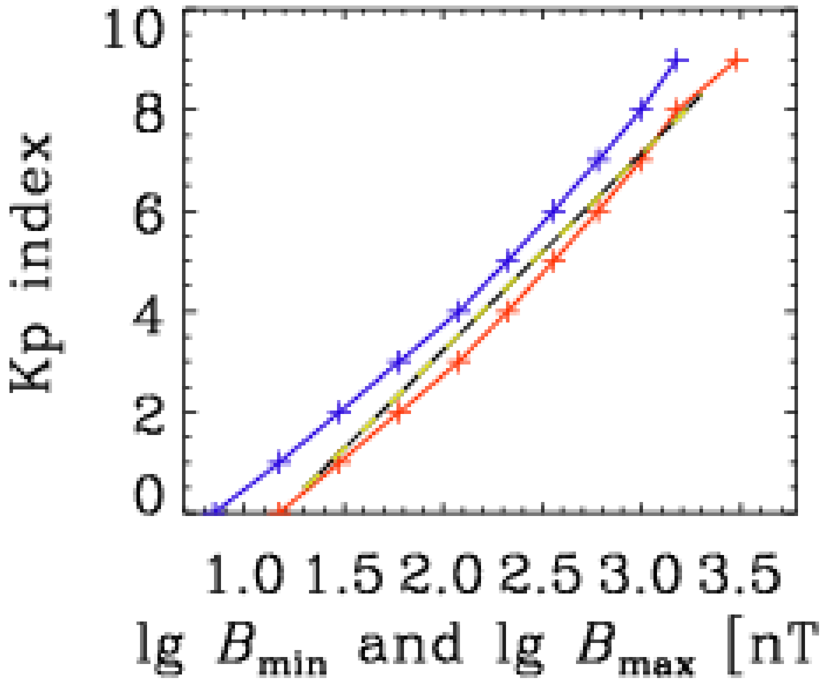




- other examples: finite time blowup

Take slope from log





Transfer eqn w/ integrating factor

We can sort of separate the variables to get

$$\frac{dI}{dx} + \alpha I = \alpha S,$$

but we can't quite integrate this equation yet unless we introduce a so-called integrating factor $e^{-\alpha x}$ and substitute

$$I(x) = e^{-\alpha x}\tilde{I}(x).$$

Inserting this gives

$$-\alpha I + e^{-\alpha x}\frac{d\tilde{I}}{dx} + \alpha I = \alpha S$$

where the αI cancels, so we have

$$e^{-\alpha x}\frac{d\tilde{I}}{dx} = \alpha S, \tag{1}$$

or

$$\frac{d\tilde{I}}{dx} = \alpha S e^{\alpha x}, \tag{2}$$

which can now be solved by separation of variables, i.e.

$$\tilde{I} - \tilde{I}_0 = \alpha S \int_0^x e^{\alpha x'}\, dx',$$

Optical depth τ=2

A. $I/I_0 = 1/2$
B. $I/I_0 = 1/2e$
C. $I/I_0 = 1/e^2$
D. $I/I_0 = e^2$

Visible optical depth: 90% gets to ground

A. $\tau = \ln 0.9$

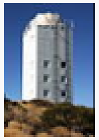
B. $\tau = \ln (1/0.9)$

C. $\tau = -\ln 0.9$

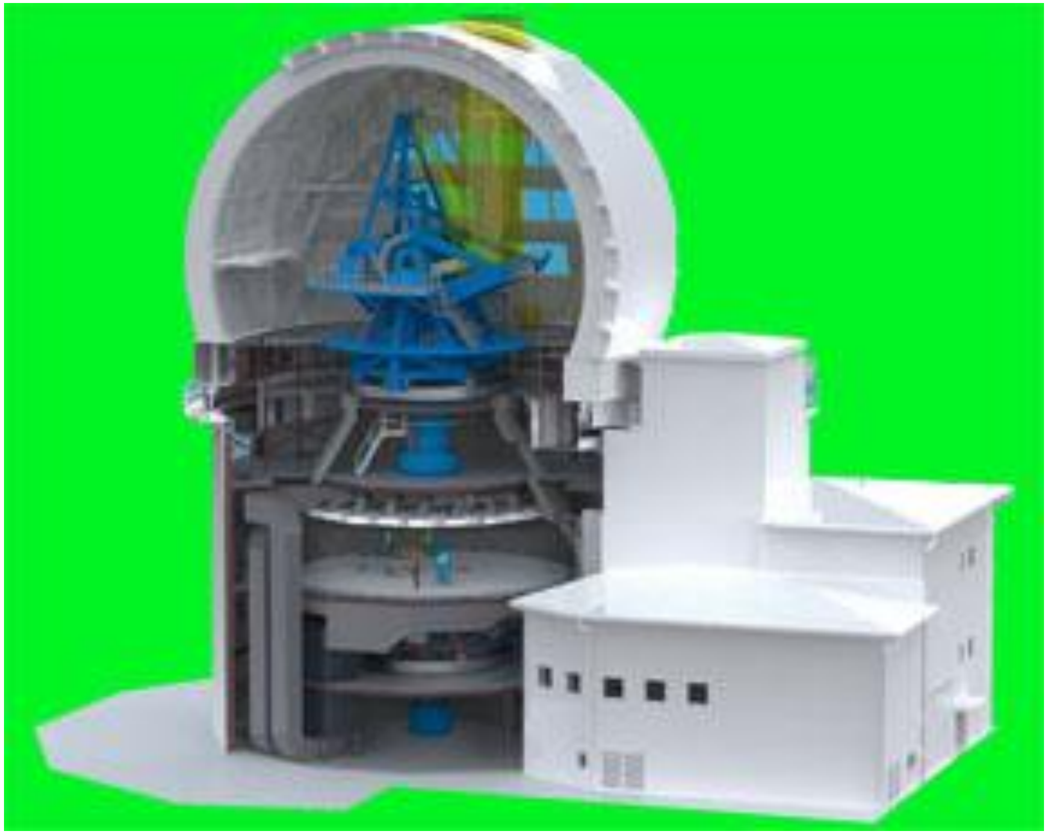
D. $\tau = 0.9$

E. $\tau = 1/0.9$

Observational techniques

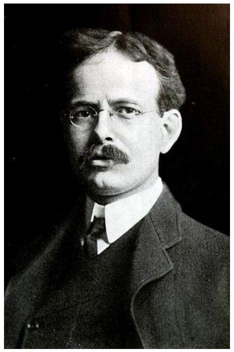
Name/Observatory	Image	Aperture d.	Year(s)	Location	Country(s)	
COronal Solar Magnetism Observatory (COSMO)[1]	-	150 cm	proposed	Hawaii, USA	United States	
Chinese Large Solar Telescope	-	180 cm	constructing	Western part of China	China	
National Large Solar Telescope	-	200 cm	proposed[2]	Merak Village, Ladakh, India	India	
Chinese Giant Solar Telescope	-	500-800 cm	planned	Western part of China	China	Could be the world's largest s
European Solar Telescope (EST)[4]	-	400+ cm	planned	Canary Islands	15 European countries[5]	
Daniel K. Inouye Solar Telescope (formerly Advanced Technology Solar Telescope (ATST))	-	424 cm[6]	under construction [7]	Maui, Hawaii, USA	United States	
GREGOR solar telescope, Teide Obs.		150 cm	2012-	Tenerife, Spain	Germany	[8]
BBO NST, BBS Obs.		160 cm	2008-	California, USA	United States	Largest aperture solar telesc
New Vacuum Solar Telescope (NVST)	-	100 cm	2010-	Yunnan Astronomical Observatory, China	China	100 cm vacuum solar telesco
ONSET (Optical and Near-Infrared Solar Eruption Tracer)	-	3x27,5 cm	2010-	Yunnan Astronomical Observatory, China	China	The ONSET consists of four tu aperture of 27.5 cm, (2) a ch 27.5 cm, (3) a WL vacuum tub tube.[10]
Bulgarian 15-cm Solar Coronagraph,[11] NAO - Rozhen	-	100 cm	2005-	Rozhen, Bulgaria	Template:BG	
Swedish 1-m Solar Telescope[12](SST), ORM		100 cm	2002-	La Palma, Spain	Sweden	

Solar Telescopes

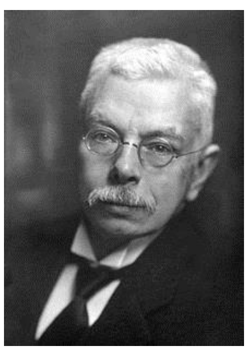


- Daniel K. Inouye Solar Telecope (DKIST)
- Heating, seeing, active optics, polarization

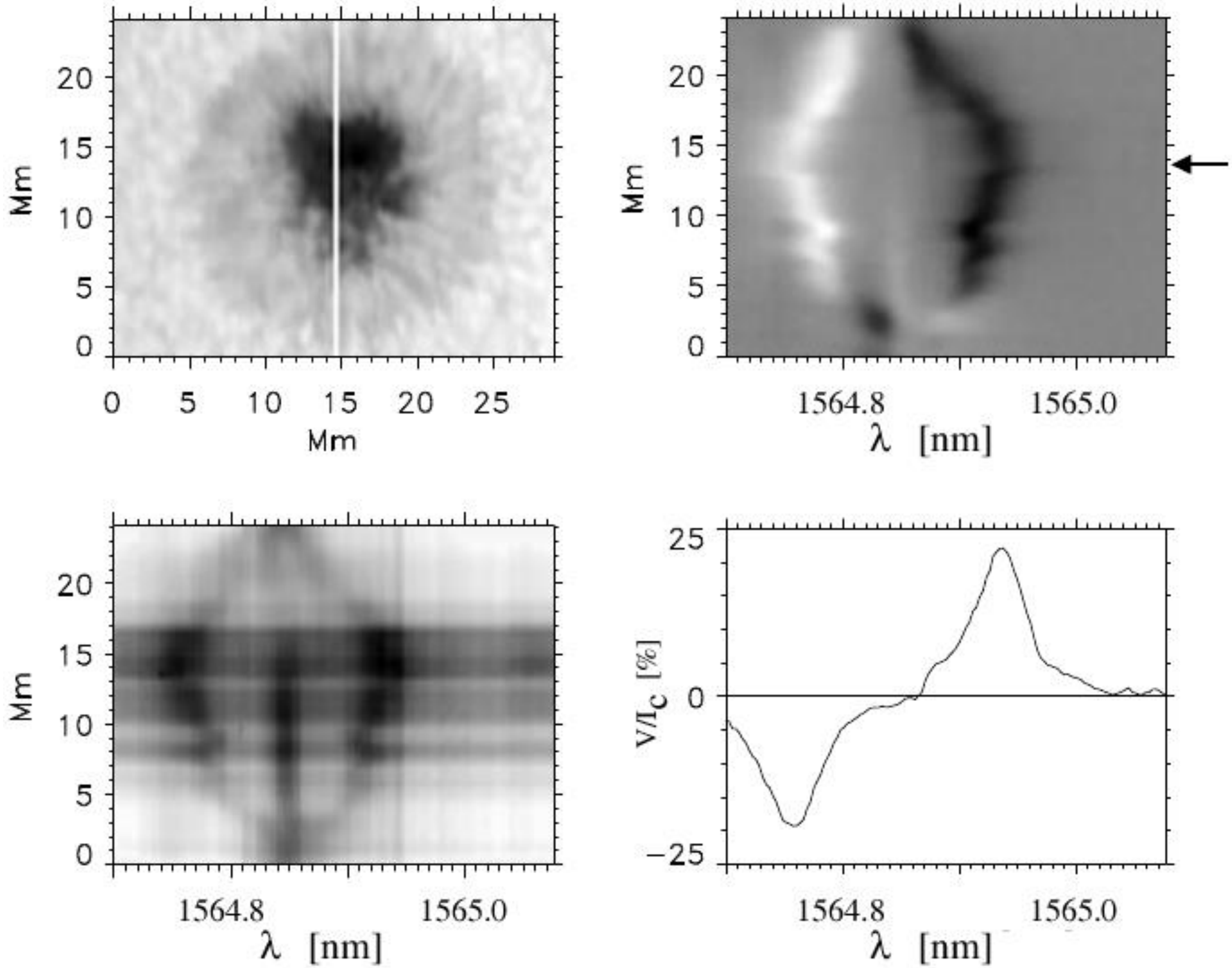
Zeeman splitting in sunspot



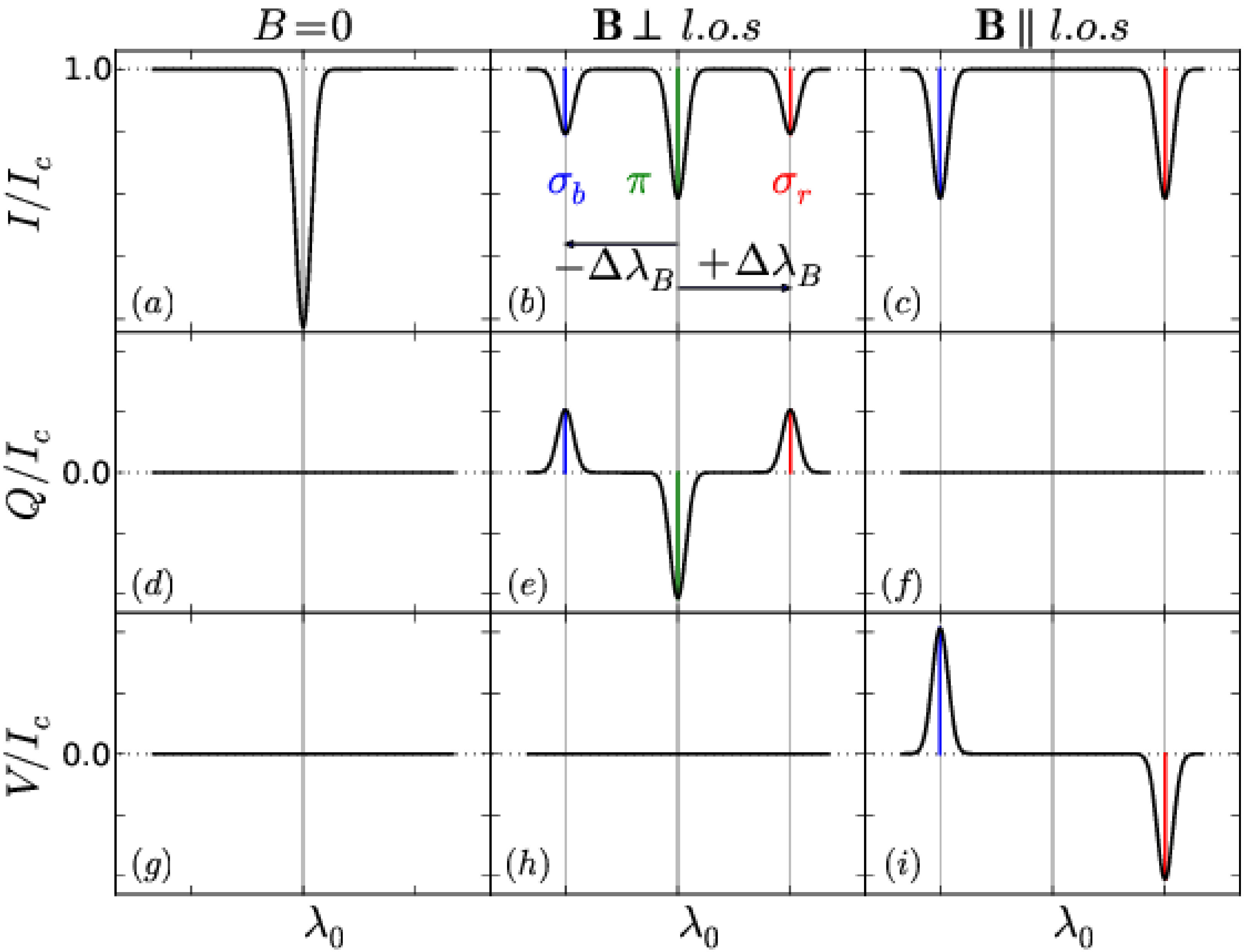
George Ellery Hale (1908)
~1000G=0.1T



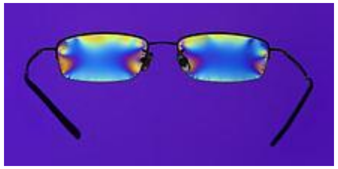
Pieter Zeeman (1897)



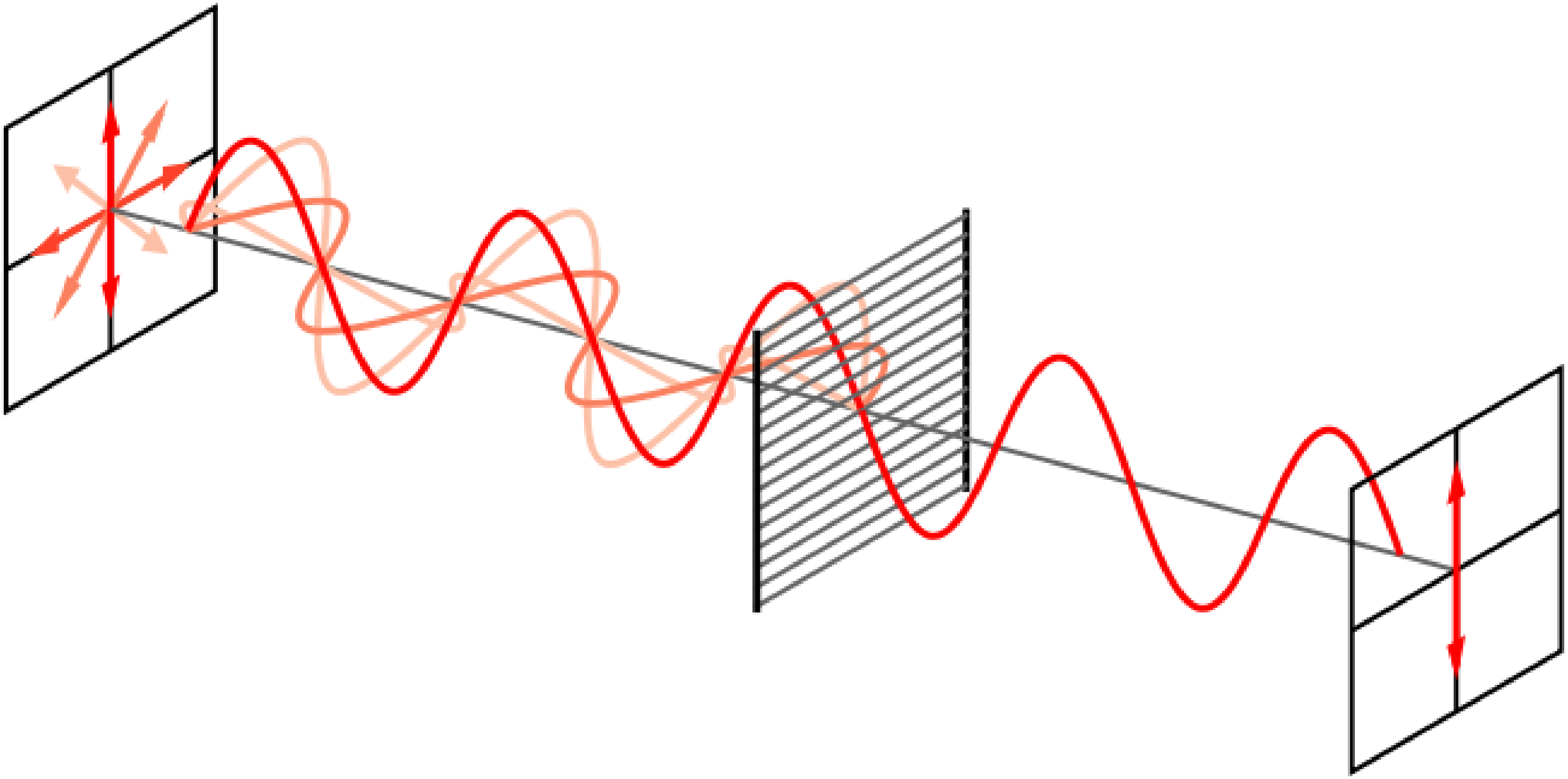
Zeeman triplet: polarized



Linearly polarized light



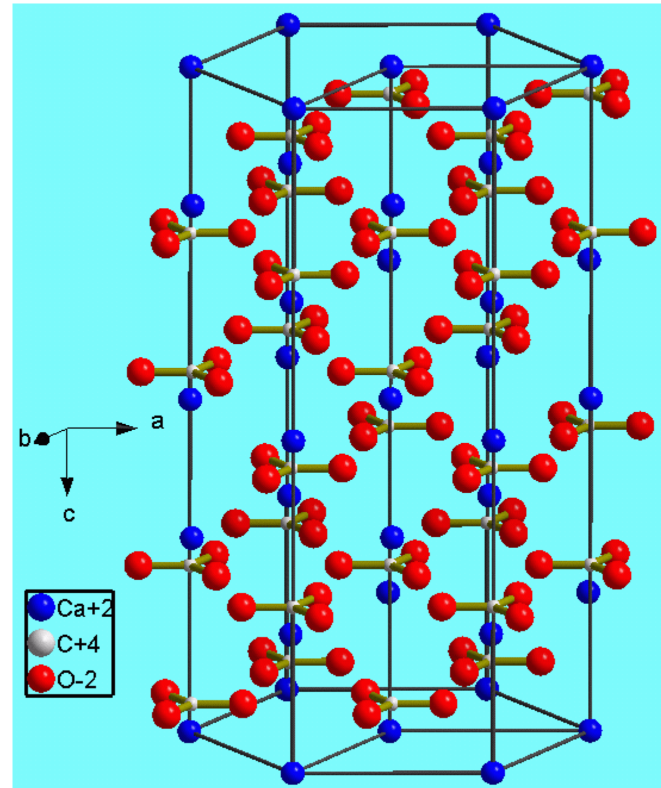
Grid-wire polarizer



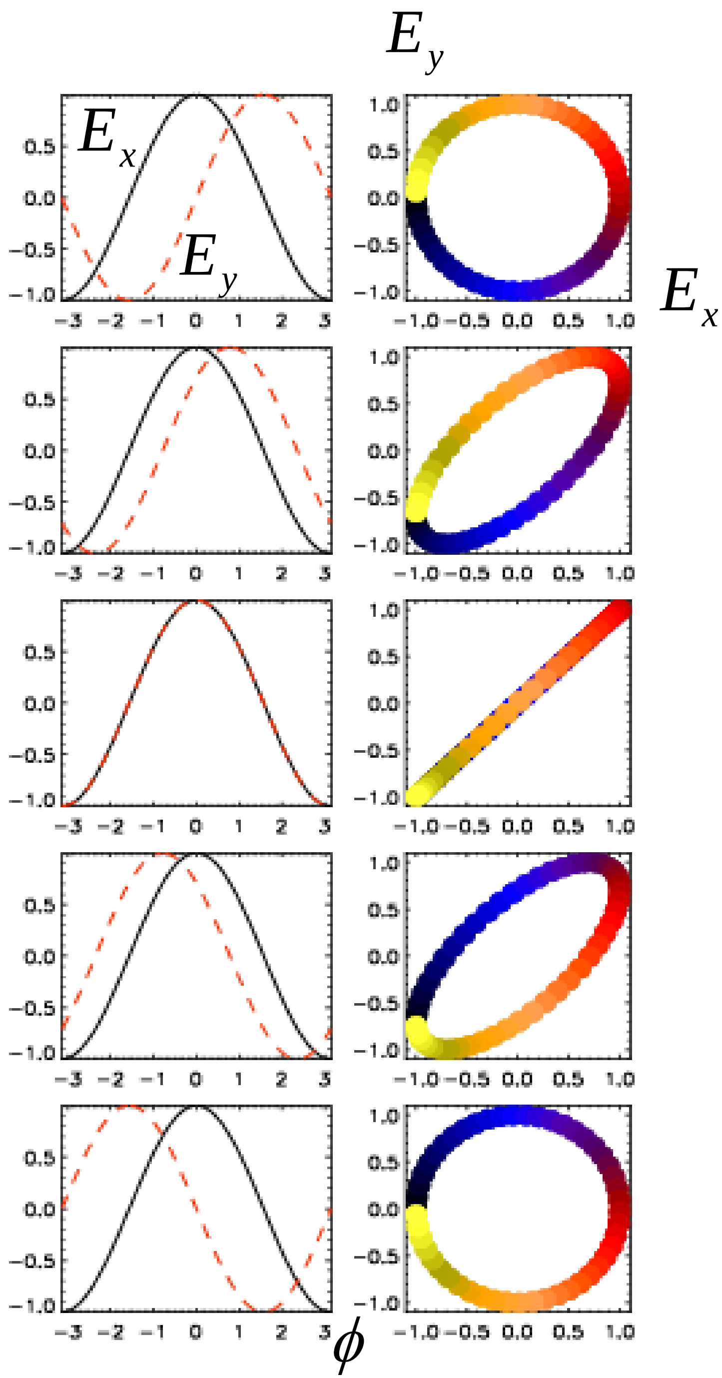
- ***E***-field parallel to wire: like metal → reflected
- Perp to wire: transmitted

Birefringence (=double refraction)

Rasmus Bartholin
Birefrigence:1669



- Different refractive index in different directions
- Possibly used by Vikings in 13^{th}-14^{th} centuries

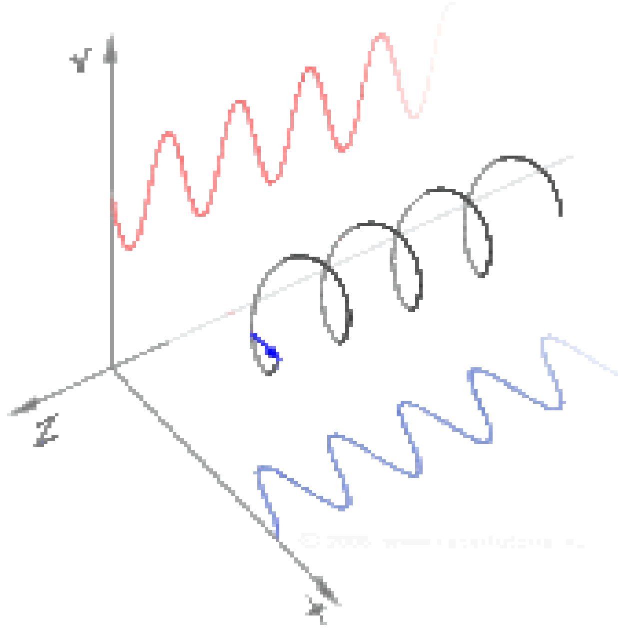


Circularly polarized light

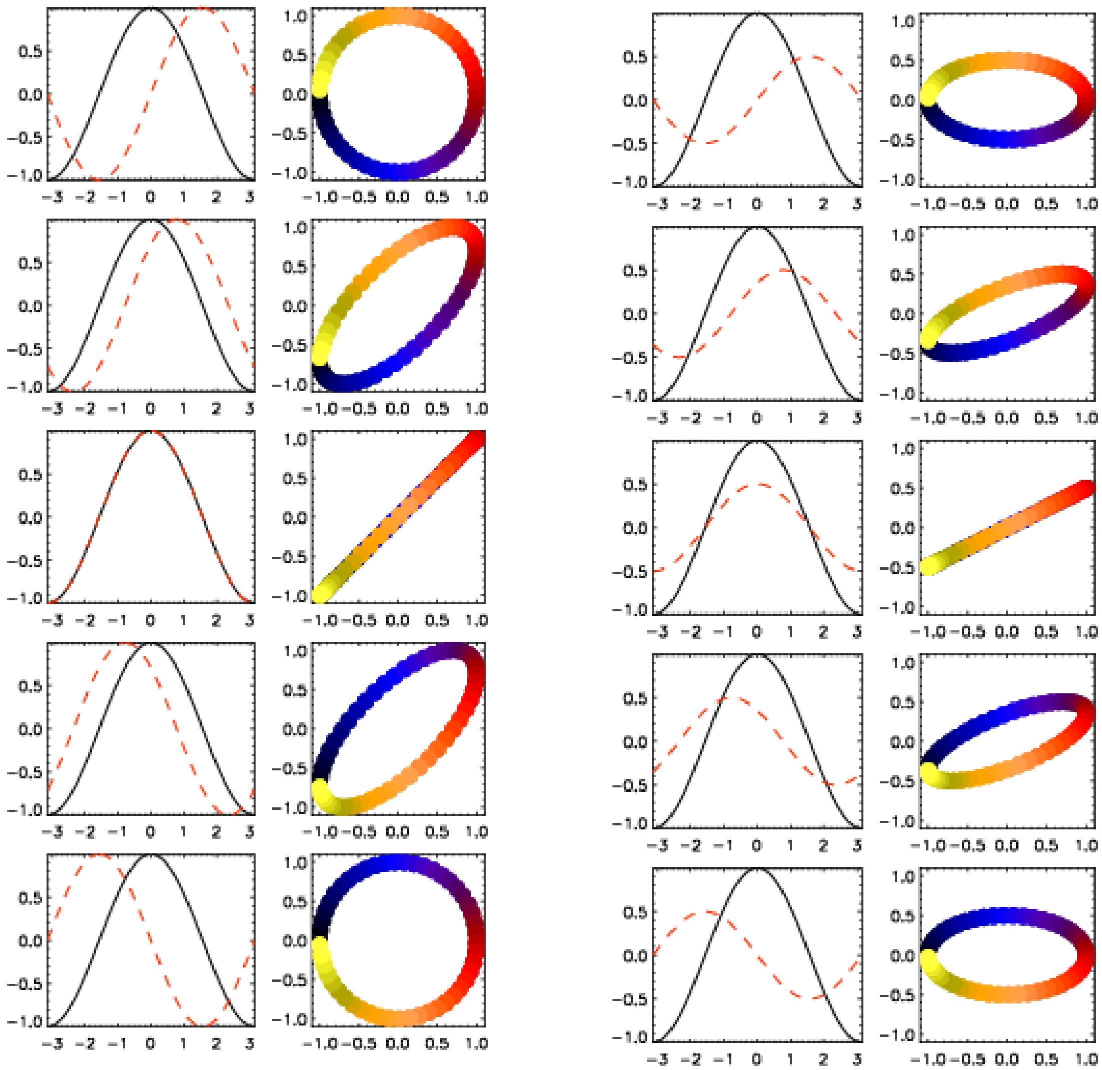
- One component is phase-shifted
- Polarization plane appears rotating

$$E_x = \xi_x \cos\phi$$

$$E_y = \xi_y \cos(\phi + \varepsilon)$$



Circularly & elliptically polarized light



- Different amplitudes for *x* and *y* components

- Even at 90º appears as ellipse

Stokes parameters

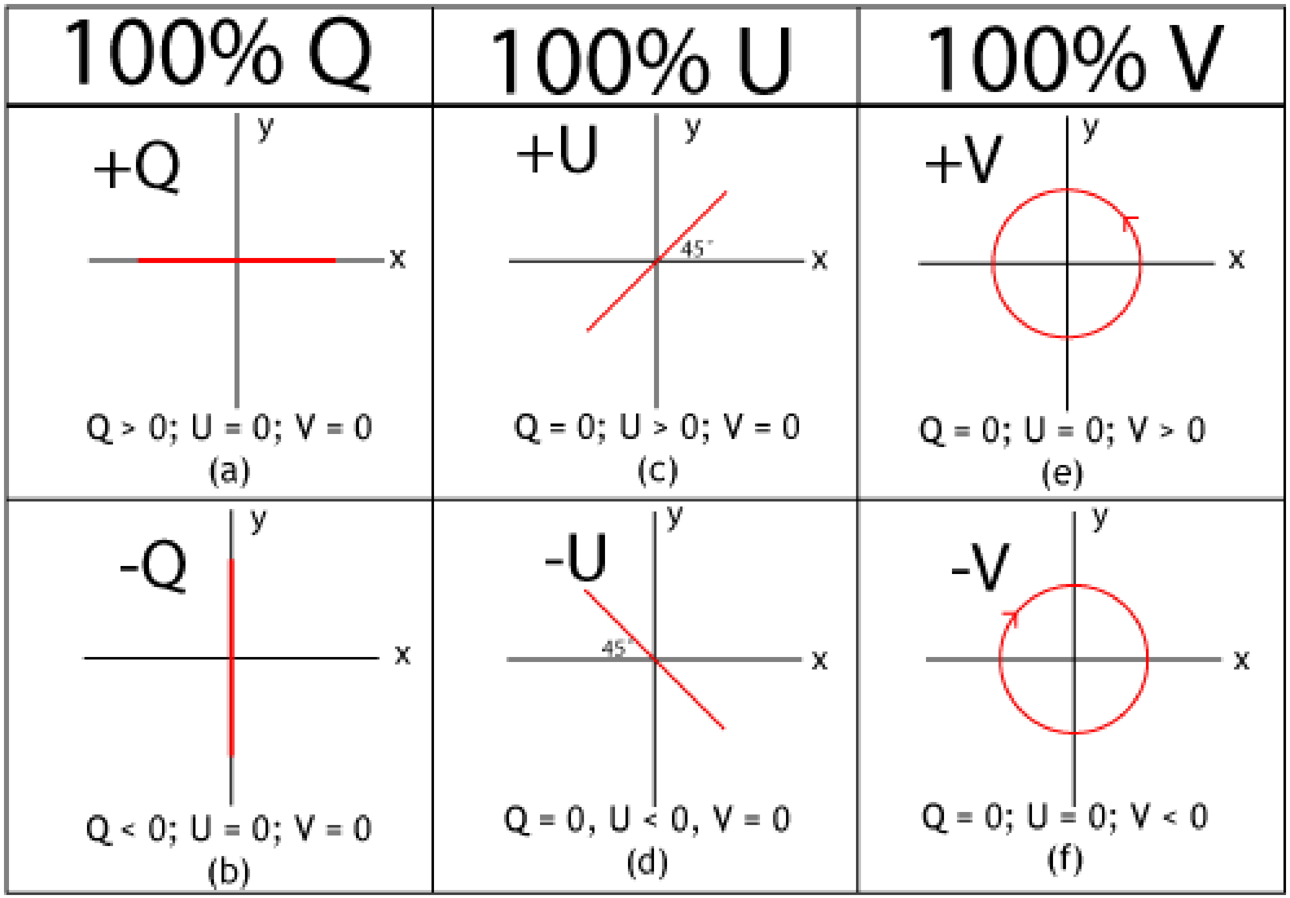
$$E_x = \xi_x \cos\phi, \quad E_y = \xi_y \cos(\phi + \varepsilon)$$

$$I = \xi_x^2 + \xi_y^2$$

$$Q = \xi_x^2 - \xi_y^2$$

$$U = 2\xi_x \xi_y \cos\varepsilon$$

$$V = 2\xi_x \xi_y \sin\varepsilon$$



π ambiguity: vectors w/o arrow

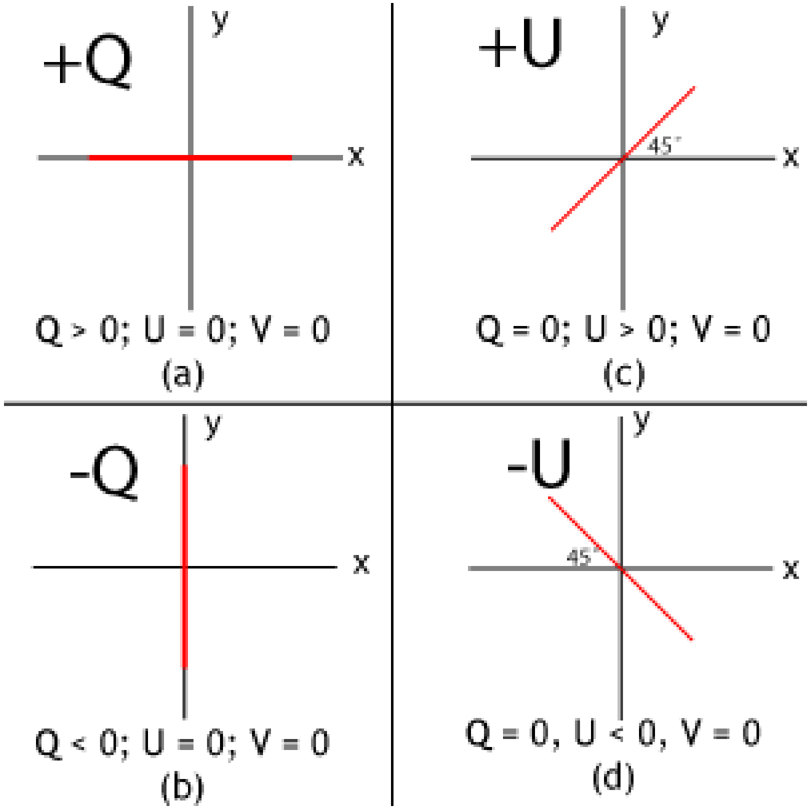
$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = |\mathbf{E}|\, e^{i\psi}, \qquad \begin{pmatrix} Q \\ U \end{pmatrix} = |\mathbf{P}|\, e^{2i\psi}$$

$\psi = 0^{\circ}$

$2\psi = 0^{\circ}$

$\psi = 45^{\circ}$

$2\psi = 90^{\circ}$



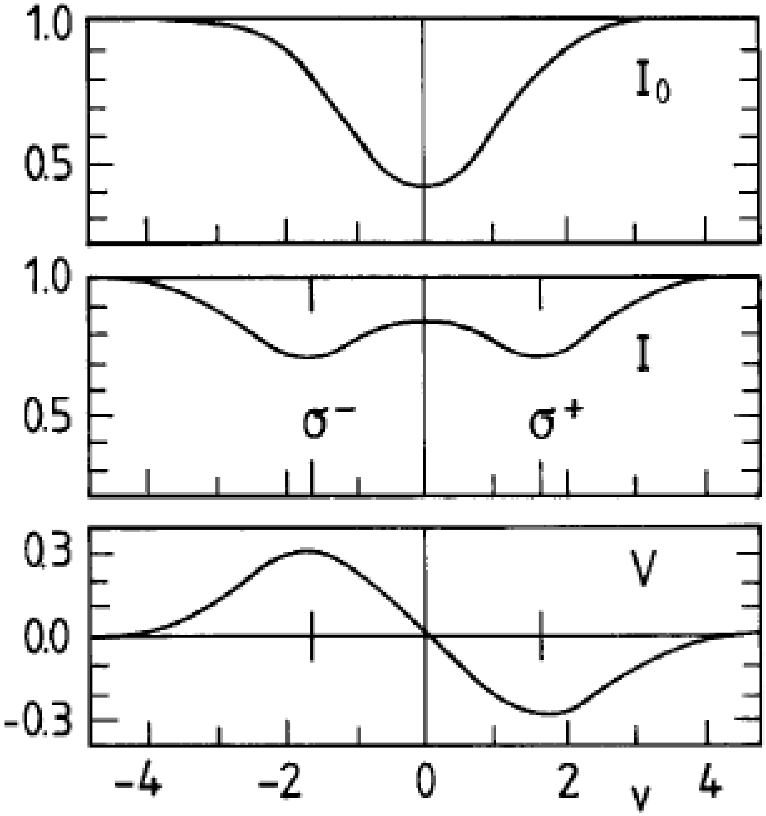
$\psi = 90^{\circ}$

$2\psi = 180^{\circ}$

$\psi = 135^{\circ}$

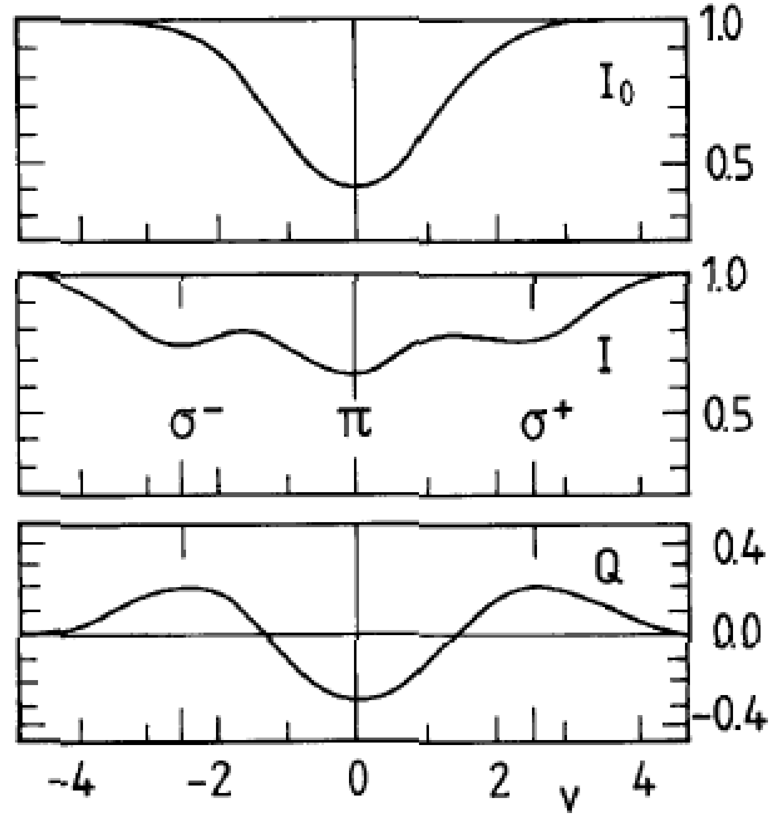
$2\psi = 270^{\circ}$

Longitudinal & transverse fields



$$I = I_{\mathrm{C}}(1 - \tau) -$$

$$V(\lambda) = -I_{\mathrm{C}}\tau(\eta^{+} - \eta^{-})/2$$



$$I = I_{\mathrm{C}}(1 - \tau) - I_{\mathrm{C}}\tau\left(\frac{1}{2}\eta + \frac{1}{4}(\eta^{+} + \eta^{-})\right)$$

$$Q = -I_{\mathrm{C}}\tau\left(\frac{1}{2}\eta - \frac{1}{4}(\eta^{+} + \eta^{-})\right) ,$$

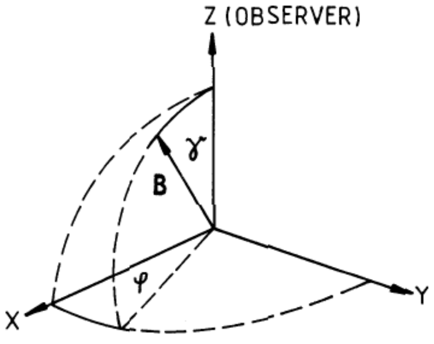
Radiative transfer → Stokes transfer

scalar I $$\mu \frac{dI}{d\tau} = I - S$$

vector $\mathbf{I}$ $$\mu \frac{d\mathbf{I}}{d\tau} = (\mathbf{1} + \boldsymbol{\eta})(\mathbf{I} - \mathbf{S})$$

$$\boldsymbol{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} \qquad \boldsymbol{\eta} = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & 0 & 0 \\ \eta_U & 0 & \eta_I & 0 \\ \eta_V & 0 & 0 & \eta_I \end{pmatrix}$$

Relation to B field



$$\eta_I = \frac{1}{2}\eta \sin^2\gamma + \frac{1}{4}(\eta^+ + \eta^-)(1 + \cos^2\gamma) \ ,$$

$$\eta_Q = \left(\frac{1}{2}\eta - \frac{1}{4}(\eta^+ + \eta^-)\right) \sin^2\gamma \cos 2\phi \ ,$$

$$\eta_U = \left(\frac{1}{2}\eta - \frac{1}{4}(\eta^+ + \eta^-)\right) \sin^2\gamma \sin 2\phi \ ,$$

$$\eta_V = \frac{1}{2}(\eta^+ - \eta^-) \cos\gamma \ .$$