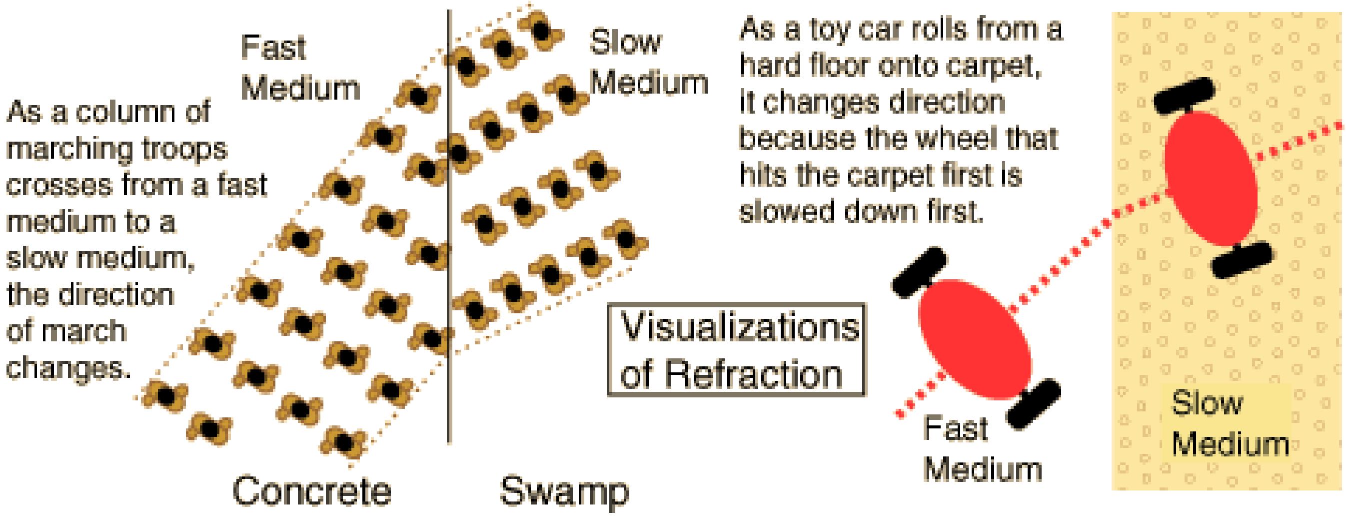
Lecture 9

- Refraction
- Wave equation (light waves)

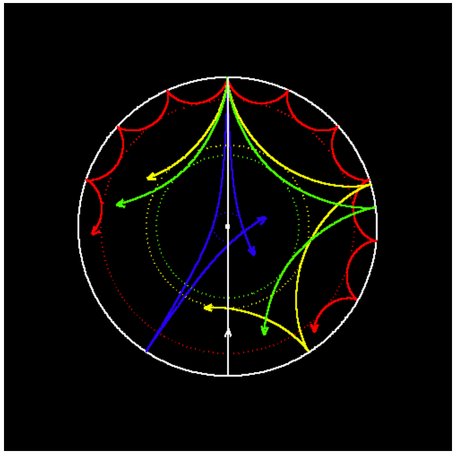
Summary of previous lecture

- Polarimetry
- Zeeman splitting
- Polarized light
- Stokes parameter
- Stokes radiative transfer

Refraction analogy



Application to the Sun:
Upward bending



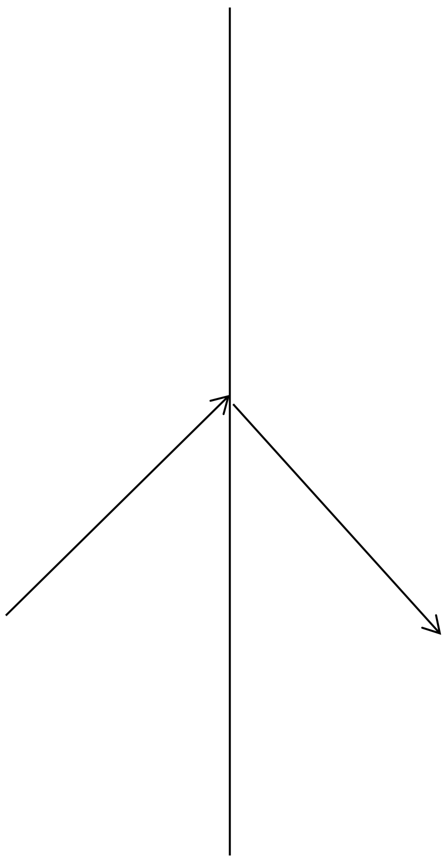
Deeper down:
Sound speed large

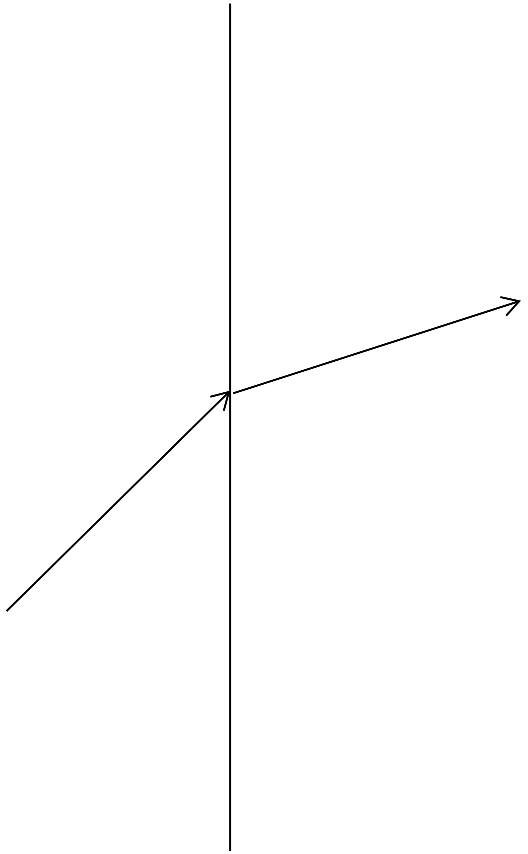
$$c_s^2 = \frac{RT}{\mu}$$

at the top: reflection when wavenlength
~ density scale height

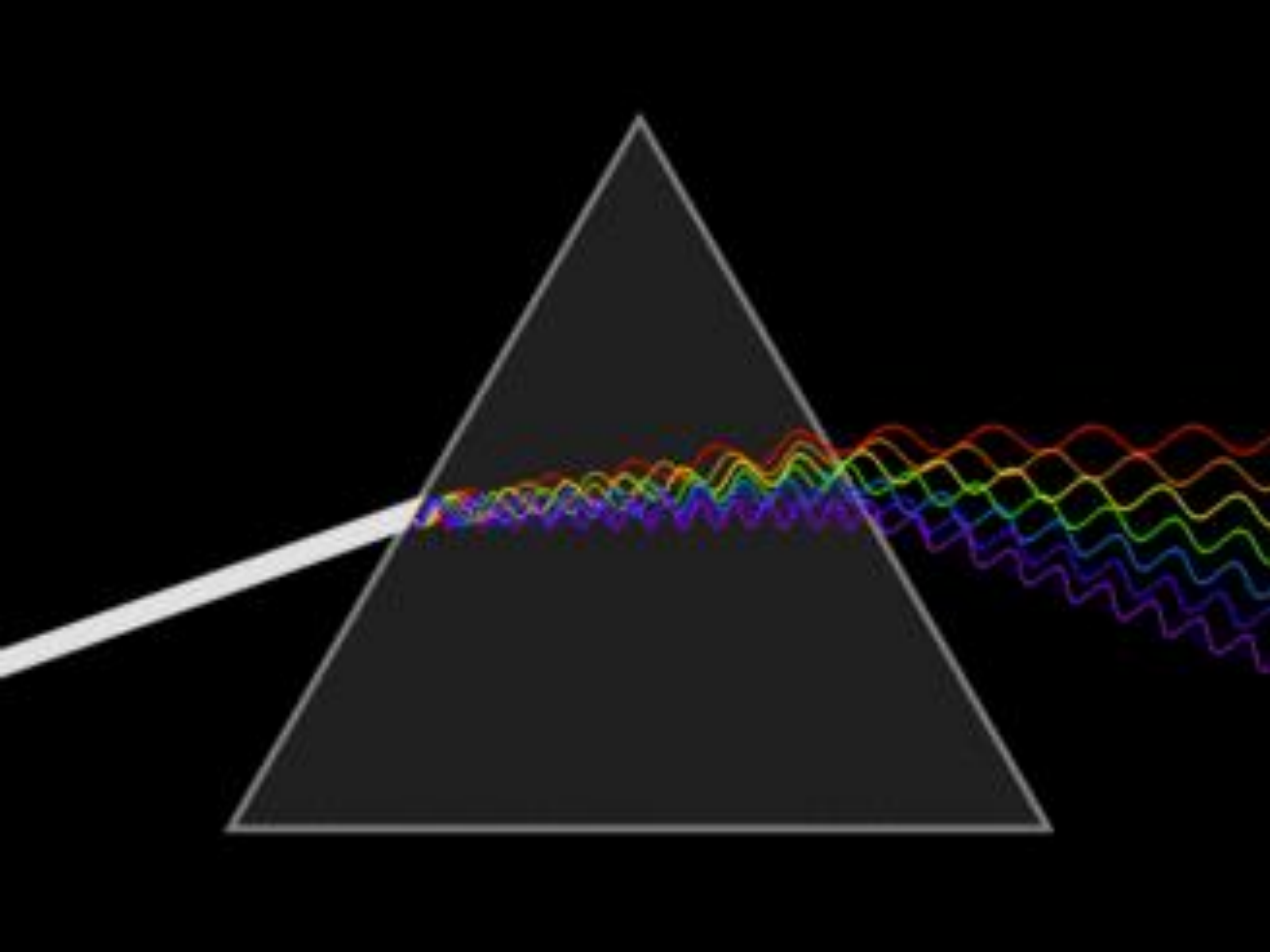
Not like so

but like so





and blue is slowed down more
…. at least in glass and water



Wave equation

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \qquad \nabla \cdot \mathbf{B} = 0$$

$$\mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = +\nabla \times \mathbf{B} - \cancel{\mu_0 \mathbf{J}} \qquad \nabla \cdot \mathbf{E} = \cancel{\rho_c / \varepsilon_0}$$

For example

$$\frac{\partial}{\partial t}\begin{pmatrix} 0 \\ B_y(x,t) \\ 0 \end{pmatrix} = -\begin{pmatrix} \partial_x \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ E_z \end{pmatrix} = ?$$

$$\mu_0 \varepsilon_0 \frac{\partial}{\partial t}\begin{pmatrix} 0 \\ 0 \\ E_z(x,t) \end{pmatrix} = +\begin{pmatrix} \partial_x \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ B_y \\ 0 \end{pmatrix} = ?$$

$\mathrm{A}: \quad +\partial_x E_z$ and $+\partial_x B_y$

$\mathrm{B}: \quad +\partial_x E_z$ and $-\partial_x B_y$

$\mathrm{C}: \quad -\partial_x E_z$ and $+\partial_x B_y$

$\mathrm{D}: \quad -\partial_x E_z$ and $-\partial_x B_y$

Imaginary unit: $i^2=-1$

what is $e^{i\pi}$?

A. 1
B. -1
C. i
D. -i

What is $e^{-3i\pi/4}$?

A. $(1+i)/\sqrt{2}$

B. $(1-i)/\sqrt{2}$

C. $(-1+i)/\sqrt{2}$

D. $-(1+i)/\sqrt{2}$

→ *Eigenvalue problem*

$$-i\omega\hat{B}_y = ik_x\hat{E}_z$$

$$-i\omega\hat{E}_z = ik_x\hat{B}_y / \mu_0\varepsilon_0$$

Write as matrix equation

$$-i\omega\begin{pmatrix}\hat{B}_y \\ \hat{E}_z\end{pmatrix} = \begin{pmatrix} 0 & ik_x \\ ik_x/\mu_0\varepsilon_0 & 0 \end{pmatrix}\begin{pmatrix}\hat{B}_y \\ \hat{E}_z\end{pmatrix}$$

Eigenvalue problem with eigenvalue ω

→ Eigenvalue problem

Vanishing determinant

$$\det\begin{pmatrix} \omega & k_x \\ k_x/\mu_0\varepsilon_0 & \omega \end{pmatrix} = 0$$

$$\omega^2 = k_x^2/\mu_0\varepsilon_0$$

Eigenvalue problem with eigenvalue ω

$$\omega = \pm k_x/\sqrt{\mu_0\varepsilon_0}$$

Speed of light

$$B_y = 2\hat{B}_y \cos k_x\left(x \mp ct\right) \qquad c = 1/\sqrt{\mu_0\varepsilon_0}$$

Wave equation

$$\frac{\partial}{\partial t}\begin{pmatrix} 0 \\ B_y(x,t) \\ 0 \end{pmatrix} = -\begin{pmatrix} \partial_x \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ E_z \end{pmatrix} = \partial_x E_z$$

$$\mu_0 \varepsilon_0 \frac{\partial}{\partial t}\begin{pmatrix} 0 \\ 0 \\ E_z(x,t) \end{pmatrix} = +\begin{pmatrix} \partial_x \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ B_y \\ 0 \end{pmatrix} = \partial_x B_y$$

insert

$$B_y(x,t) = \hat{B}_y e^{ik_x x - i\omega t} + \text{c.c.} \quad E_z(x,t) = \hat{E}_z e^{ik_x x - i\omega t} + c.c.$$

Stokes U = Stokes I

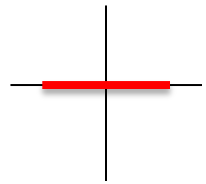
$$E_x = \xi_x \cos\phi, \quad E_y = \xi_y \cos(\phi + \varepsilon)$$

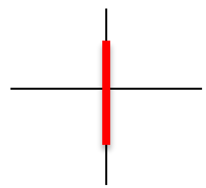
$$I = \xi_x^2 + \xi_y^2$$

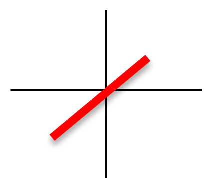
$$Q = \xi_x^2 - \xi_y^2$$

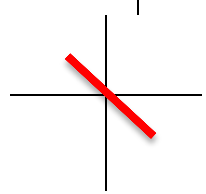
$$U = 2\xi_x \xi_y \cos\varepsilon$$

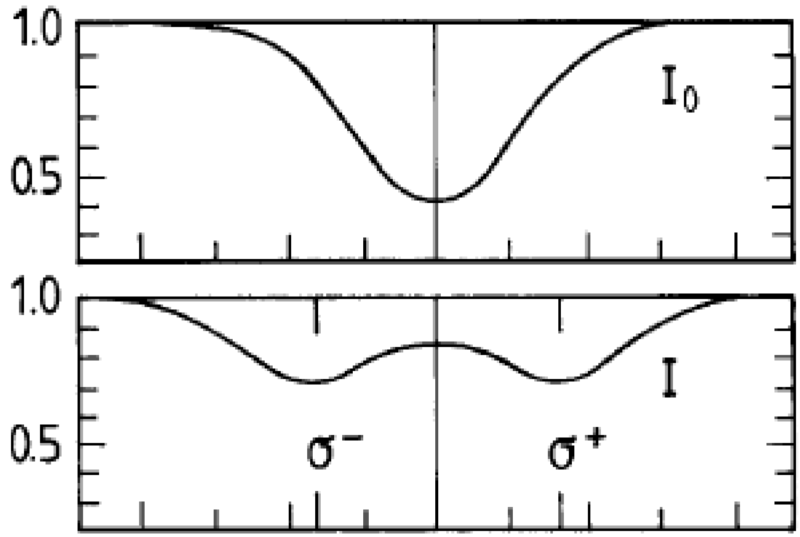
$$V = 2\xi_x \xi_y \sin\varepsilon$$

A. 

B. 

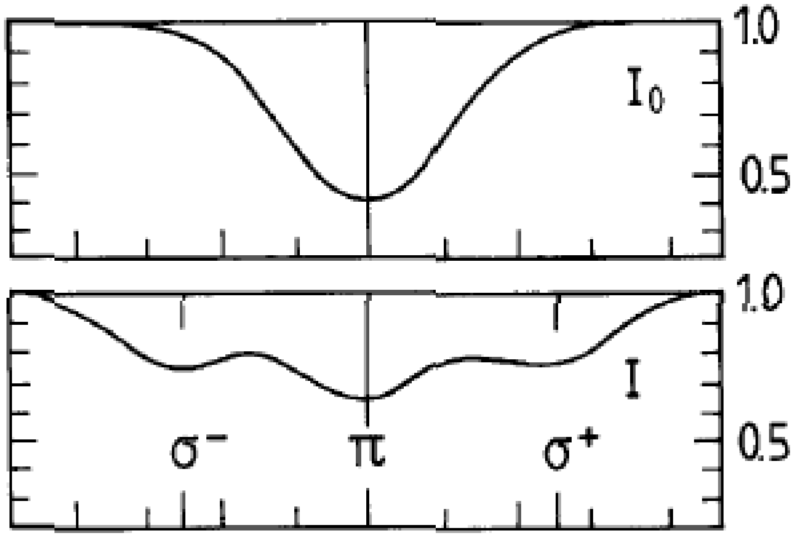
C. 

D. 



Absorption lines

A. Inclined magnetic field
B. Radial magnetic field
C. Transverse magnetic field
D. Longitudinal magnetic field
E. Rotating magnetic field



Absorption lines

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Zeeman triplet: polarized