Handout 15: Homogeneous Turbulence

The definition of what is turbulence is difficult. Turbulent flows are irregular in space and time and they typically extend over broad length and time scales. The standard case is vortical turbulence, but there is in principle also irrotational (or so-called acoustic) turbulence.

Turbulence is often caused by instabilities. In many theoretical studies of turbulence, turbulence is *driven* by volume forcing instead. This has the advantage of simulating homogeneous turbulence. (Convection driven by the Rayleigh-Benard or the magneto-rotational instability is not homogeneous.) However, turbulence can also be decaying and is caused just by a sufficiently irregular initial condition. An excellent text book on turbulence is that by Davidson (2015).

1 Appearance of turbulence

The standard picture of bigger vortices breaking up into smaller ones is misleading. With the advance of computer simulations, it became clear that turbulence consists instead of *tubes*; see Figure 1.

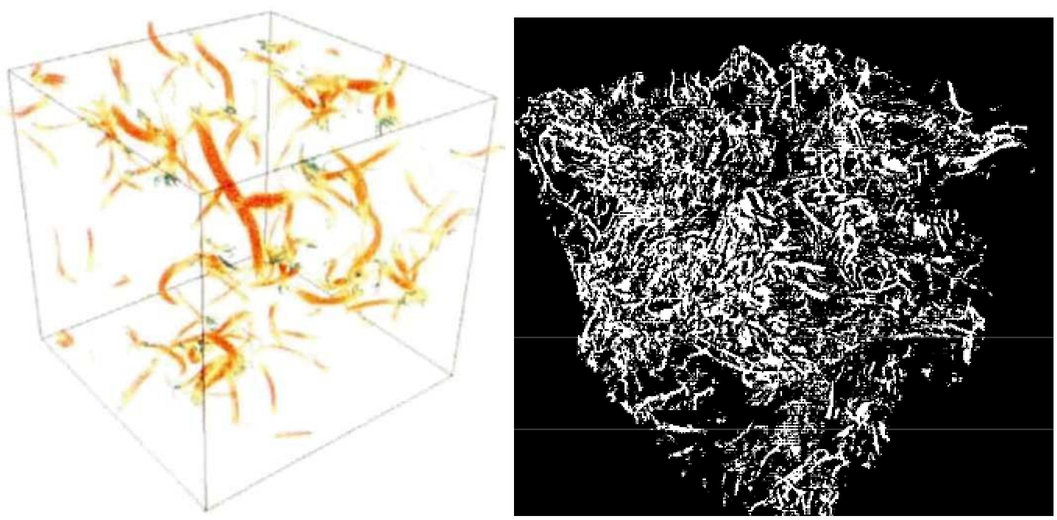

Figure 1: Examples of vortex tubes in homogeneous turbulence from She et al. (1990) (left panel) and Porter et al. (1998) (right panel).

The left-hand panel of Figure 1 shows examples of vortex tubes. Their thickness is related to the viscous scale while their length was often expected to be comparable with the integral scale. However, in subsequent years simulations at increasingly higher Reynolds numbers seem to reveal that the vortex turbulence become a less prominent feature of otherwise nebulous looking structures of variable density (see the right-hand panel of Figure 1).

The production of vortex tubes is associated with the action of local strain and thus with the strain tensor $s_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$. It is best characterized by its principle axes given by the three eigenvectors, $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, and $\boldsymbol{e}_3$, corresponding to the eigenvalues λ_1, λ_2, and λ_3. Their sum is equal to $\boldsymbol{\nabla}\cdot\boldsymbol{u}$, and vanishes for incompressible flows. The largest one is corresponds to the direction of stretching, and the smallest one to the direction of compression; see Figure 2.

It has been known for some time that in isotropic turbulence the vorticity vector tends to be aligned with the direction $\hat{\boldsymbol{e}}_2$ and is therefore normal to the plane where the flow would be two-dimensional. If the turbulence was perfectly two-dimensional, the intermediate eigenvalue of the rate-of-strain tensor would vanish. This is however not the case; see Figure 3, where we plot probability density functions (PDFs) of the three eigenvalues.

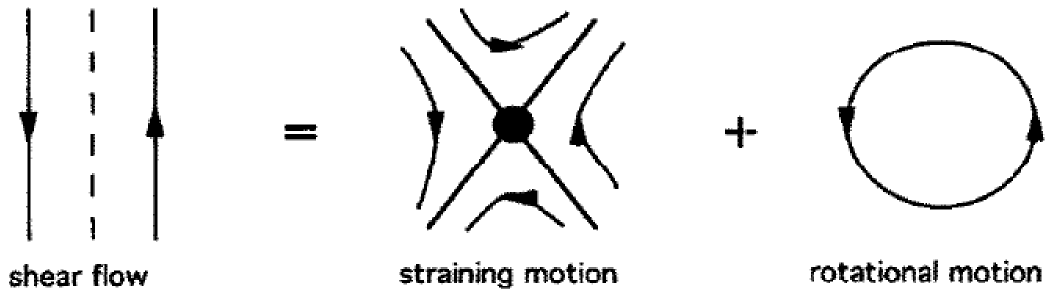

Figure 2: Sketch illustrating the principle axes in a local shear flow.

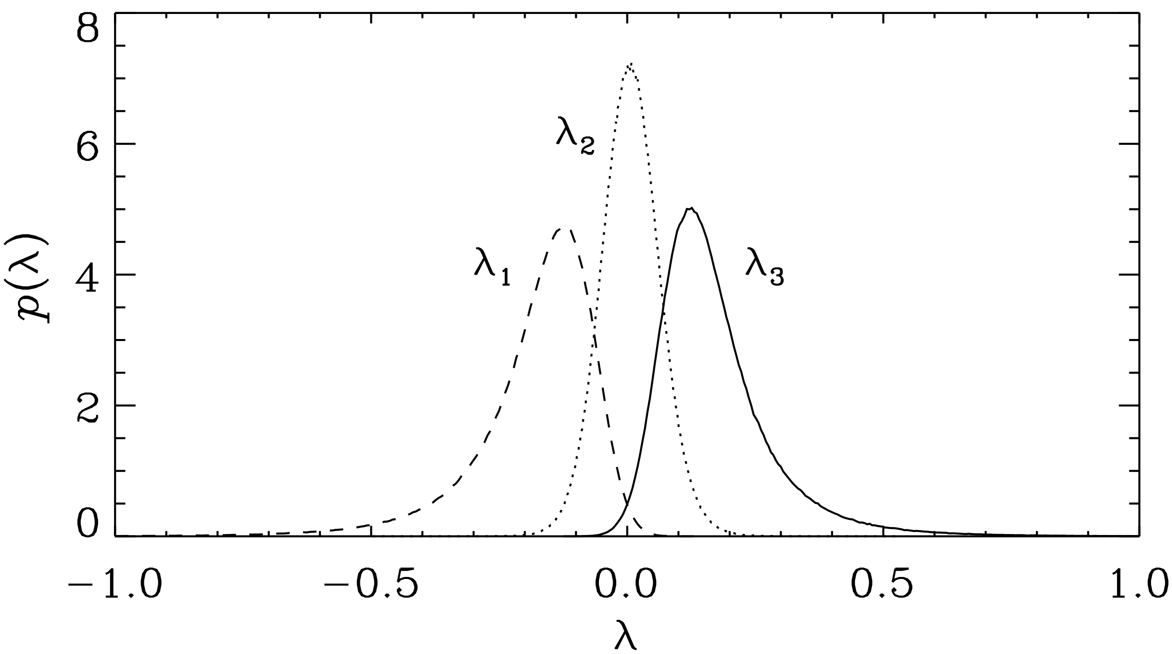

Figure 3: PDF of the eigenvalues of the rate-of-strain tensor. Note that the intermediate ones are not vanishing, as expected for two-dimensional turbulence.

2 Spectra

Incompressible forced turbulence simulations have been carried out at resolutions in excess of 4096^3 meshpoints Kaneda et al. (2003). Surprising results from this work include a strong bottleneck effect (Falkovich, 1994) near the dissipative subrange, and possibly a strong inertial range correction of about $k^{-0.1}$ to the usual $k^{-5/3}$ inertial range spectrum, so that the spectrum is $k^{-1.77}$. Similarly strong inertial range corrections have also been seen in simulations using a locally enhanced viscosity proportional to the modulus of the rate of strain matrix $|\mathbf{S}|^2$, which is also known as the Smagorinsky subgrid scale model. Here we also show the results of simulations with hyperviscosity, i.e. the $\nu\nabla^2$ diffusion operator has been replaced by a $\nu_3\nabla^6$ operator. Hyperviscosity greatly exaggerates the bottleneck effect, but it does not seem to affect the inertial range significantly; see Figure 4.

References

Davidson, P. A. *Turbulence: an introduction for scientists and engineers.* Oxford: Oxford University Press (2015).

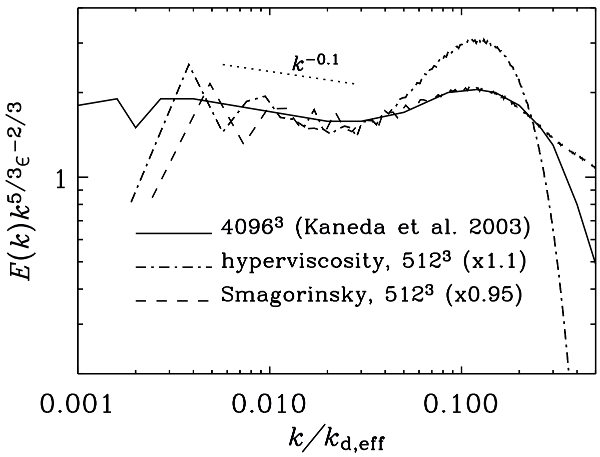

Figure 4: Comparison of energy spectra of the 4096^3 meshpoints run Kaneda et al. (2003) (solid line) and 512^3 meshpoints runs with hyperviscosity (dash-dotted line) and Smagorinsky viscosity (dashed line). (In the hyperviscous simulation we use $\nu_3 = 5 \times 10^{-13}$.) The Taylor microscale Reynolds number of the Kaneda simulation is 1201, while the hyperviscous simulation of Ref. Haugen & Brandenburg (2006) has an approximate Taylor microscale Reynolds number of $340 < \mathrm{Re}_\lambda < 730$. For the Smagorinsky simulation the value of Re_λ is slightly smaller.

Falkovich, G., "Bottleneck phenomenon in developed turbulence," *Phys. Fluids* **6**, 1411-1414 (1994).

Haugen, N. E. L., & Brandenburg, A., "Hydrodynamic and hydromagnetic energy spectra from large eddy simulations," *Phys. Fluids* **18**, 075106 (2006).

Kaneda, Y., Ishihara, T., Yokokawa, M., Itakura, K., & Uno, A., "Energy dissipation rate and energy spectrum in high resolution direct numerical simulations of turbulence in a periodic box," *Phys. Fluids* **15**, L21-L24 (2003).

Porter, D. H., Woodward, P. R., & Pouquet A., "Inertial range structures in decaying compressible turbulent flows," *Phys. Fluids* **10**, 237-245 (1998).

She, Z.-S., Jackson, E., & Orszag, S. A., "Intermittent vortex structures in homogeneous isotropic turbulence," *Nature* **344**, 226-228 (1990).

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