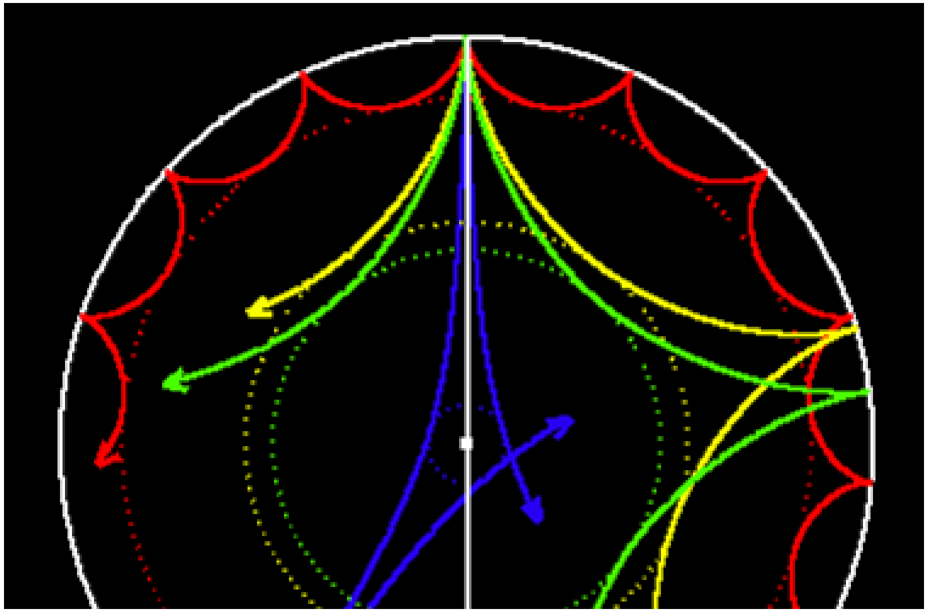
Helioseismology

- Solar 5 min oscillations
- Discrete frequencies
- Standing waves (Stix pp. 181-189)
- Helioseimology (Stix pp. 213, 214)

k_z^2

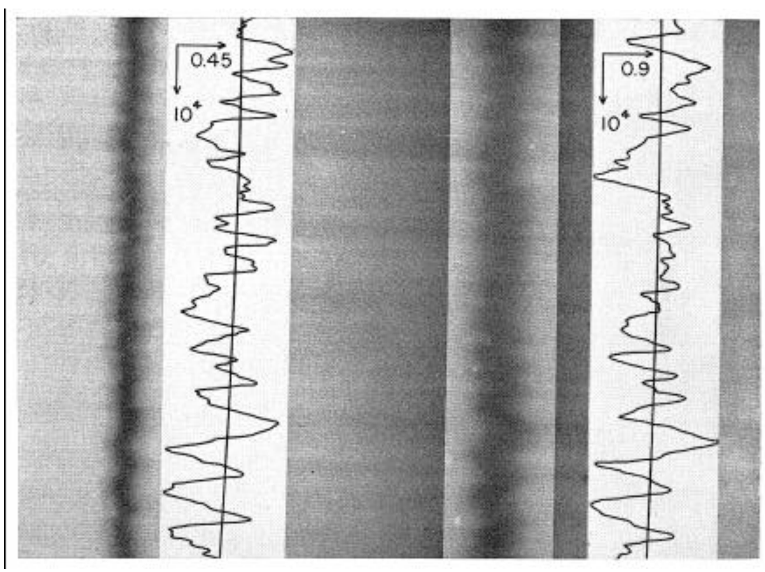


IAU meeting of 1960

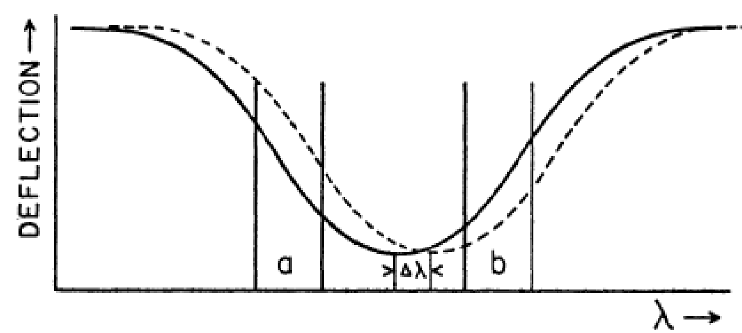
International Astronomical Union

— R. B. LEIGHTON:

We have been spending about a week here discussing velocity fields, so I would like to take the liberty of showing you some as they appear on the surface of the sun. Let me first outline briefly the results which our observations have indicated to us. First, we have definite evidence for *horizontal motion* (*i.e.*, tangential to the solar surface) whose magnitude lies somewhere in the range 0.2 to 0.5 km/s, on a scale of about 30 000 km. This size is relatively large compared with the solar granulation. These motions represent



Discovered supergranulation (slow) and ?random vertical motion (fast)



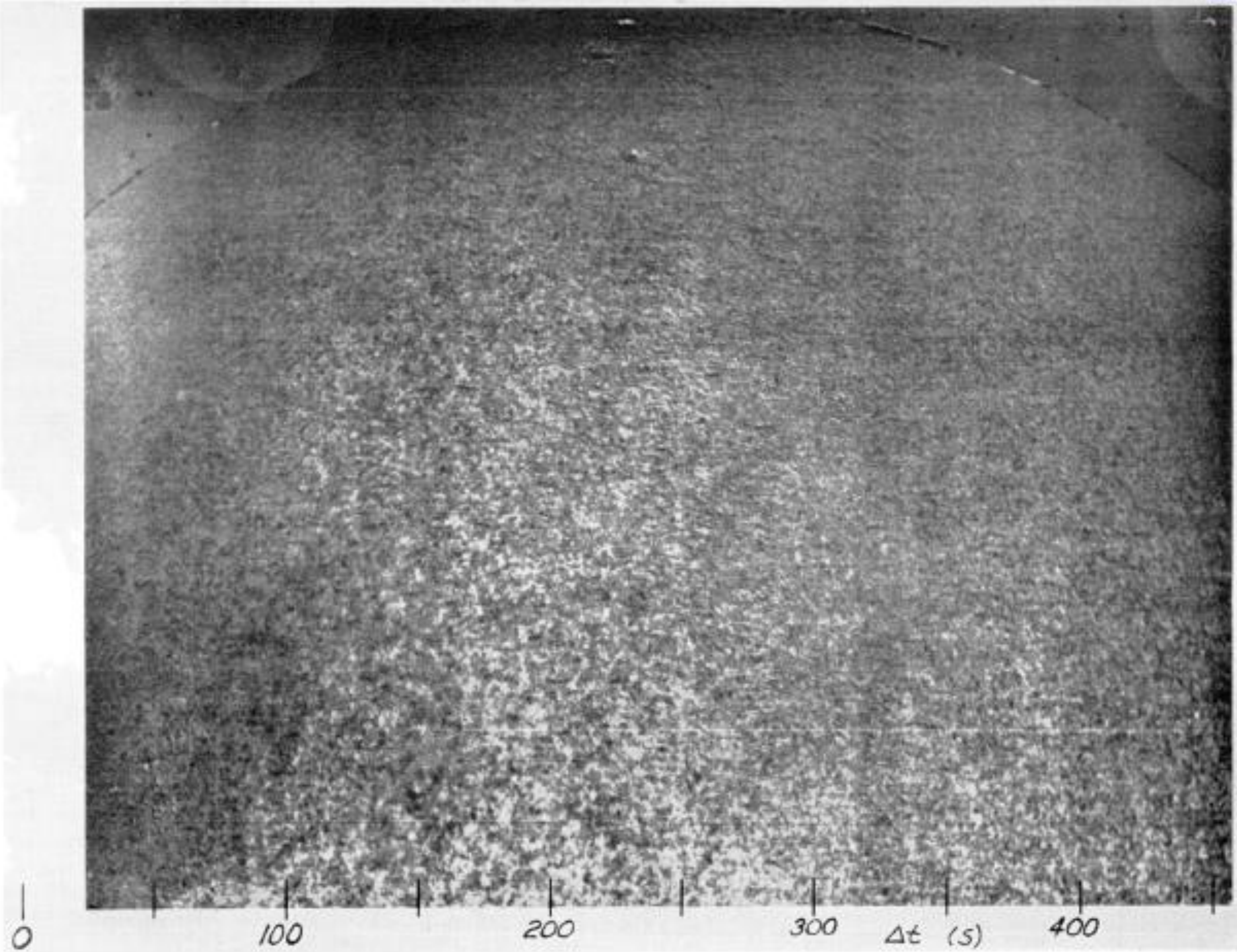


FIG. 14.—Doppler difference plate, showing the oscillatory time correlation of the small-scale velocity field. Ca 6103. June 10, 1960, 13^h40^m U.T.

Fourier transform: space & time

$$f(\mathbf{x},t) = \int_{-\infty}^{\infty} \hat{f}(\mathbf{k},\omega)\, e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t} \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{d\omega}{2\pi}$$

Familiar from
Fourier ansatz
(=trial solution)

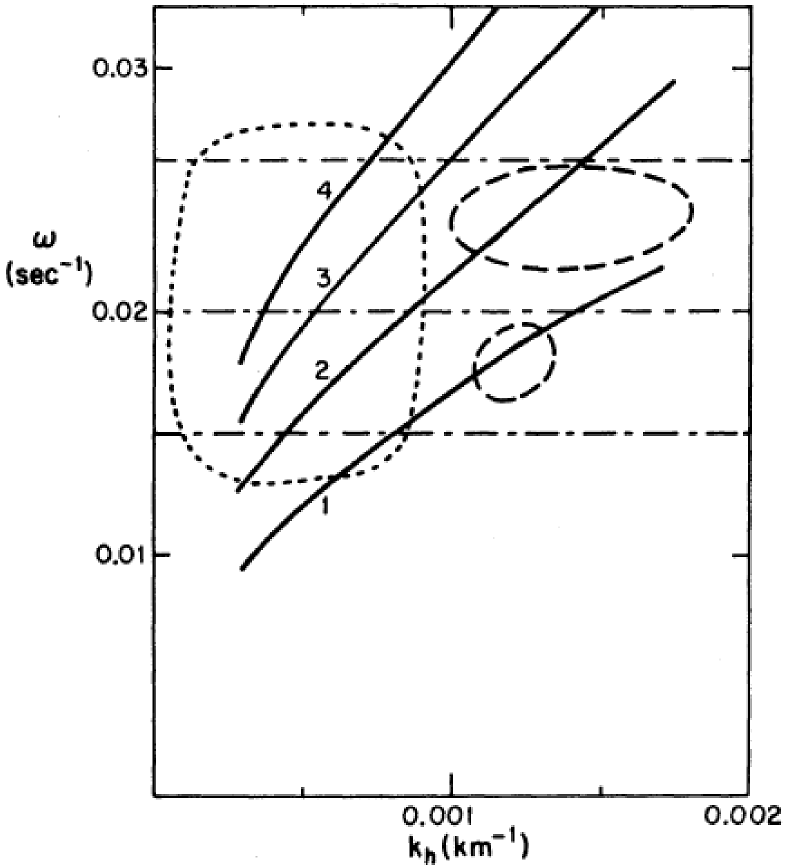
$$\rho_1(z,t) = \hat{\rho}_1 e^{ik_z z - i\omega t} + \text{c.c.}$$

$$u_{1z}(z,t) = \hat{u}_{1z} e^{ik_z z - i\omega t} + \text{c.c.}$$

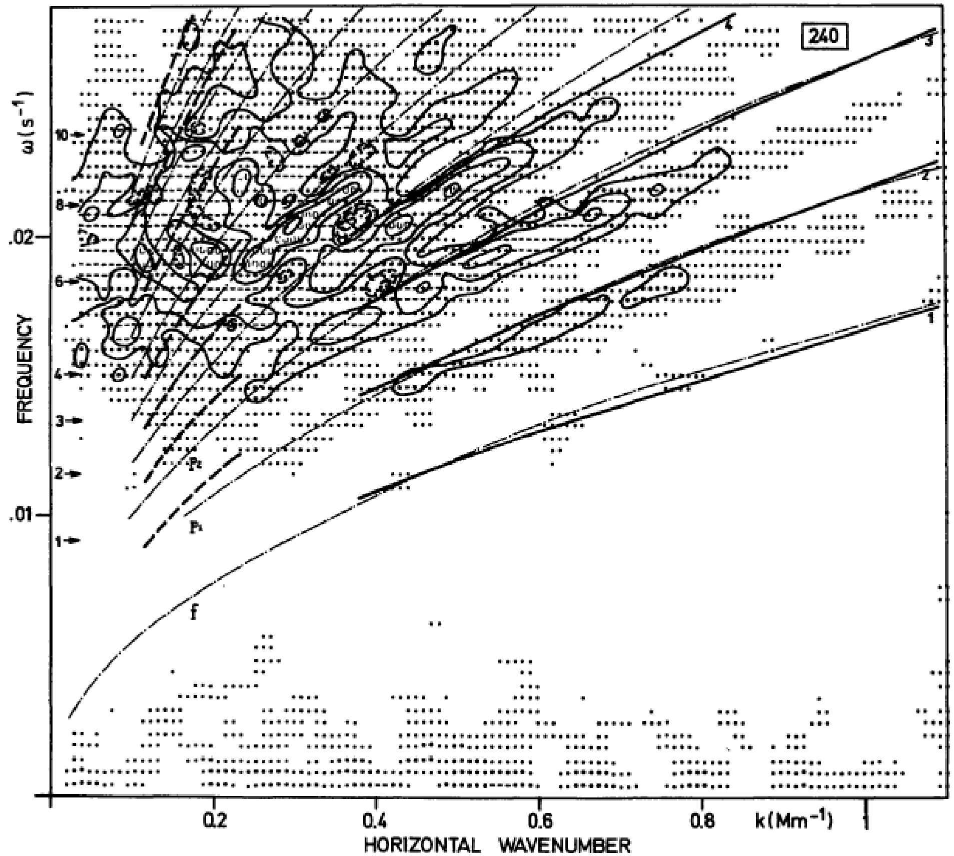
Computer routines: FFT (fast Fourier transform), forward & backward

$$\hat{f}(\mathbf{k},\omega) = \int_{-\infty}^{\infty} f(\mathbf{x},t)\, e^{-i(\mathbf{k}\cdot\mathbf{x} - \omega t)}\, d^2\mathbf{x}\, dt$$

5 min osc are *global*



Roger Ulrich
(1970)



Franz-Ludwig Deubner
(1974)

Vertical wavenumber

Dispersion relation of Lecture 11 $\omega^2 = \frac{\Re T}{\mu} k_z^2$

In 3-D $\omega^2 = c_s^2 (\underbrace{k_x^2 + k_y^2}_{k_{\mathrm{hor}}^2} + \underbrace{k_z^2}_{k_{\mathrm{vert}}^2})$

"Solve" for k_z

$$k_z^2 = \frac{\omega^2}{c_s^2} - k_{\mathrm{hor}}^2$$

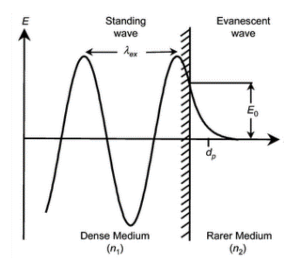
Consider $c_s = c_s(\mathrm{r})$ [oops?]

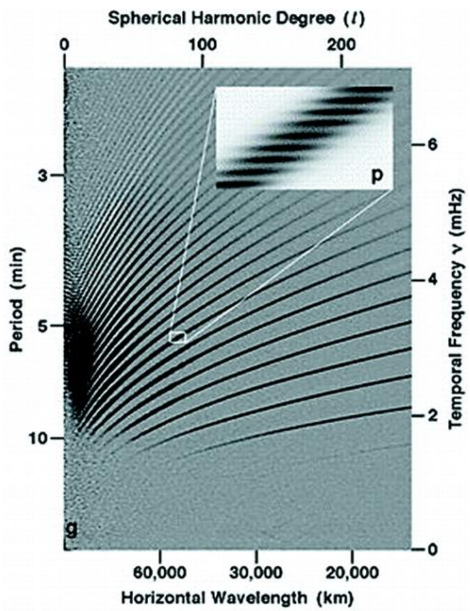
In quantum mechanics:
WKB approximation

- → Jeffreys-Wrentzel-Kramers-Brilloin
- → Tunnel effect → Gamow!!

Deeper down: k_z imaginary!?

$$e^{ik_z z} = e^{\mp |k_z| z}$$





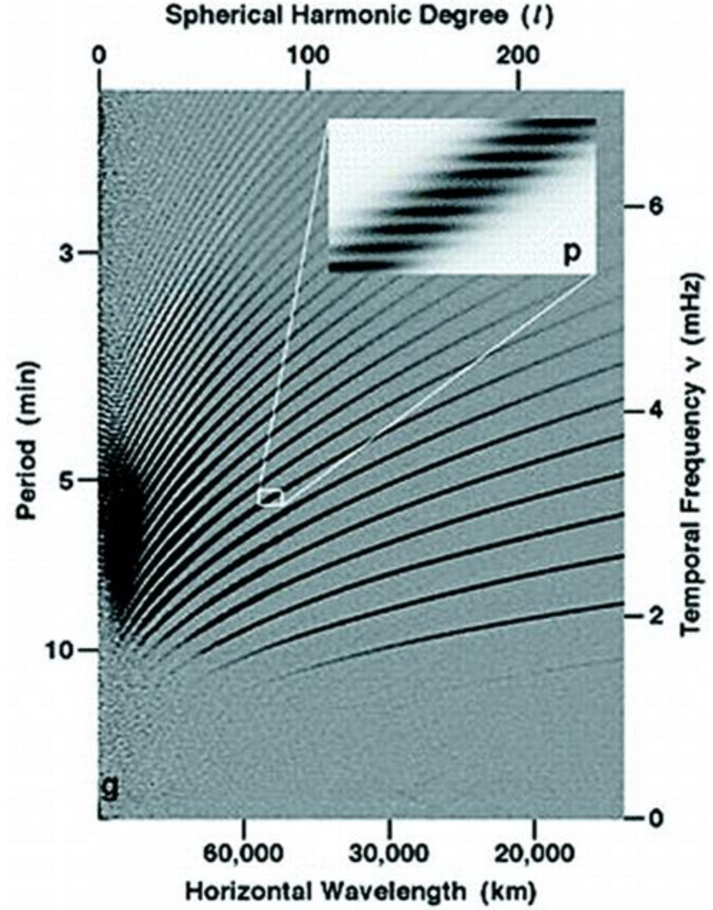
Example

$$\omega = \frac{2\pi}{300\mathrm{s}} = 0.02\mathrm{s}^{-1}$$

$$k_{\mathrm{hor}} = \frac{\ell}{R} = \frac{100}{700\,\mathrm{Mm}}$$

$$c_s = \frac{\omega}{k_{hor}} = 0.02 \times 7\,\frac{\mathrm{Mm}}{\mathrm{s}} = 140\,\frac{\mathrm{km}}{\mathrm{s}}$$

Which modes (k) needed to probe the core?



A. *kR*>100

B. *kR*<100

C. *kR*<10

D. *kR*<1 (i.e. impossible)

Number of nodes

$$n = \frac{L}{\lambda/2} = \frac{2k_z}{2\pi} L = k_z L / \pi$$

Continuous case

$$\pi n = \int_{z_{\mathrm{lower}}}^{z_{\mathrm{outer}}} k_z \, dz$$

$$k_z^2 = \frac{\omega^2}{c_s^2} - k_{\mathrm{hor}}^2$$

$$\pi n = \int_{z_{\mathrm{lower}}}^{z_{\mathrm{outer}}} \sqrt{\frac{\omega^2}{c_{\mathrm{s}}^2} - k_{\mathrm{hor}}^2} \, dz$$

Just a function of k/ω …

$$\pi n / \omega = \int_{z_{\mathrm{lower}}}^{z_{\mathrm{outer}}} \sqrt{\frac{1}{c_{\mathrm{s}}^2} - \frac{k_{\mathrm{hor}}^2}{\omega^2}} \, dz$$

Inversion: input/output

$$n\pi = \int_{r_0}^{R_0} k_r \,\mathrm{d}r$$

$$k_r = \sqrt{\frac{\omega^2}{c_s^2} - \frac{l(l+1)}{r^2}}$$

$$k_r = \frac{\omega}{r}\sqrt{\frac{r^2}{c_s^2} - \frac{l(l+1)}{\omega^2}}$$

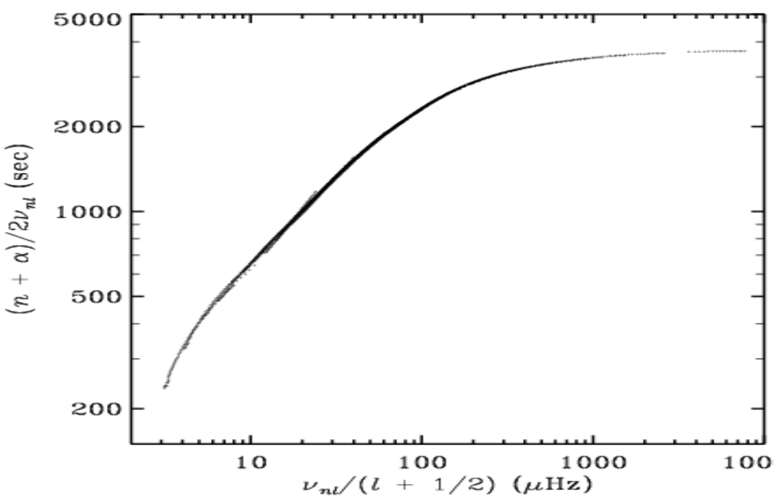
$$F(u) = \int_u^{\xi_0} \sqrt{\xi - u}\; G'(\xi)\,\mathrm{d}\xi$$

$$G(\xi) = \frac{2}{\pi}\int_{\xi}^{\xi_0} \frac{1}{\sqrt{\xi - u}}\, F'(u)\,\mathrm{d}u$$

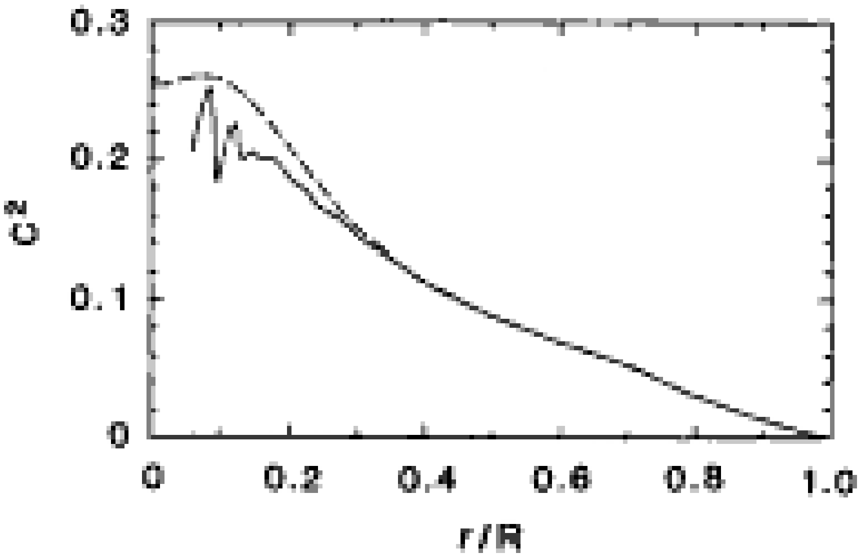
$$\xi \equiv \frac{r^2}{c_s^2}$$

$$u \equiv \frac{l(l+1)}{\omega^2}$$

$$G'(\xi) \equiv \frac{d\ln r}{d\xi}$$



Duvall law

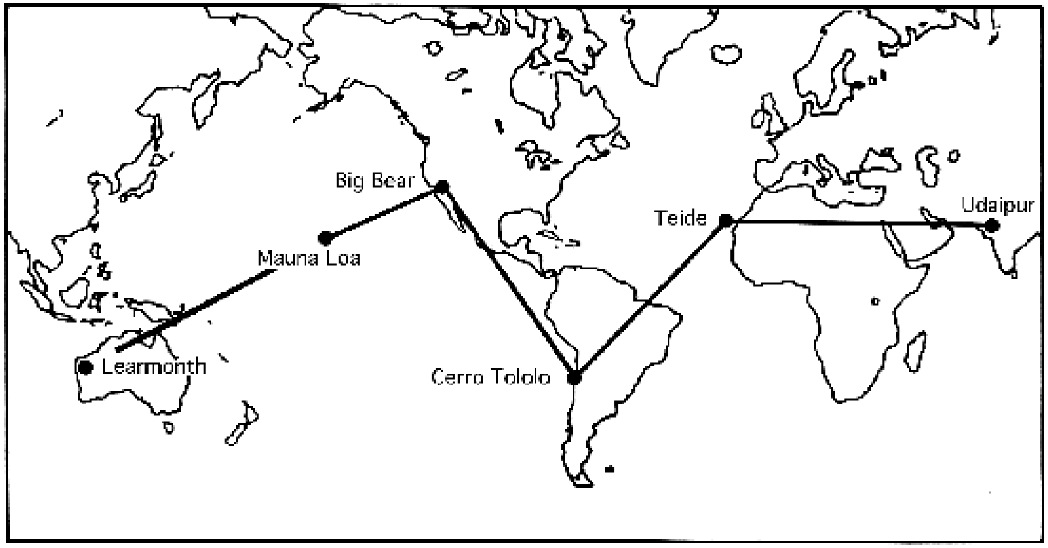


Sound speed

GONG

global oscillation network group

Since late 1980ties

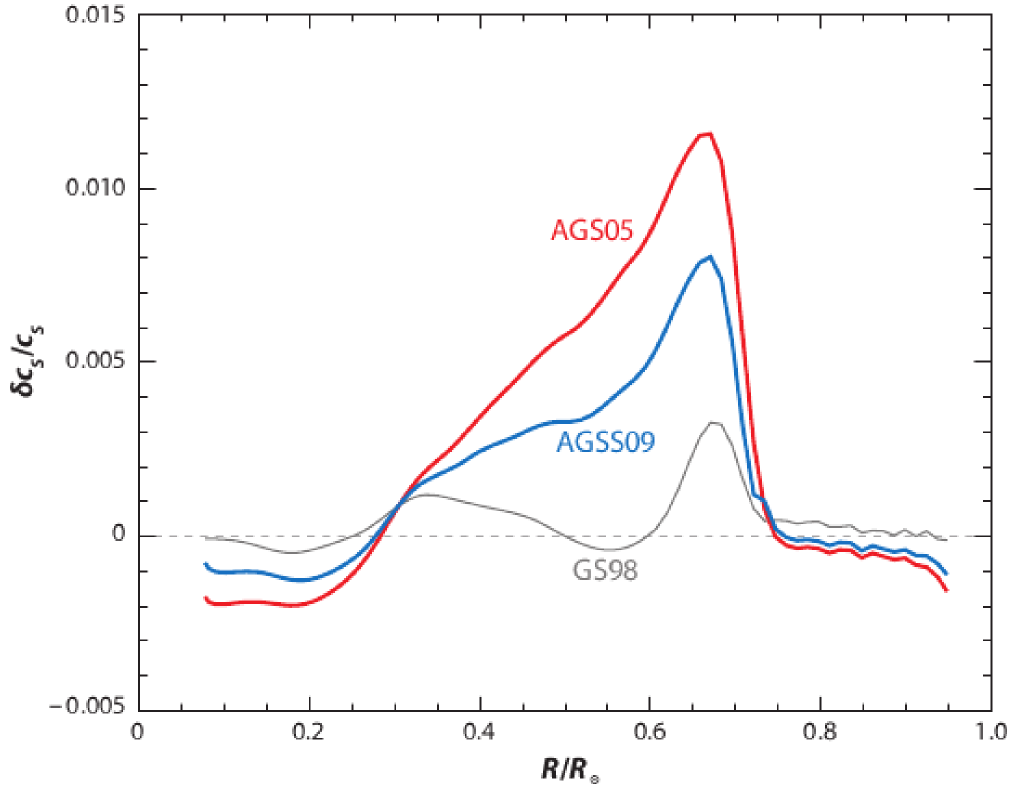


Open problems

Abundance of heavier elements (Z)

	X	Y	Z
GS98	.735	.249	.0231
AGS05	.739	.249	.0165

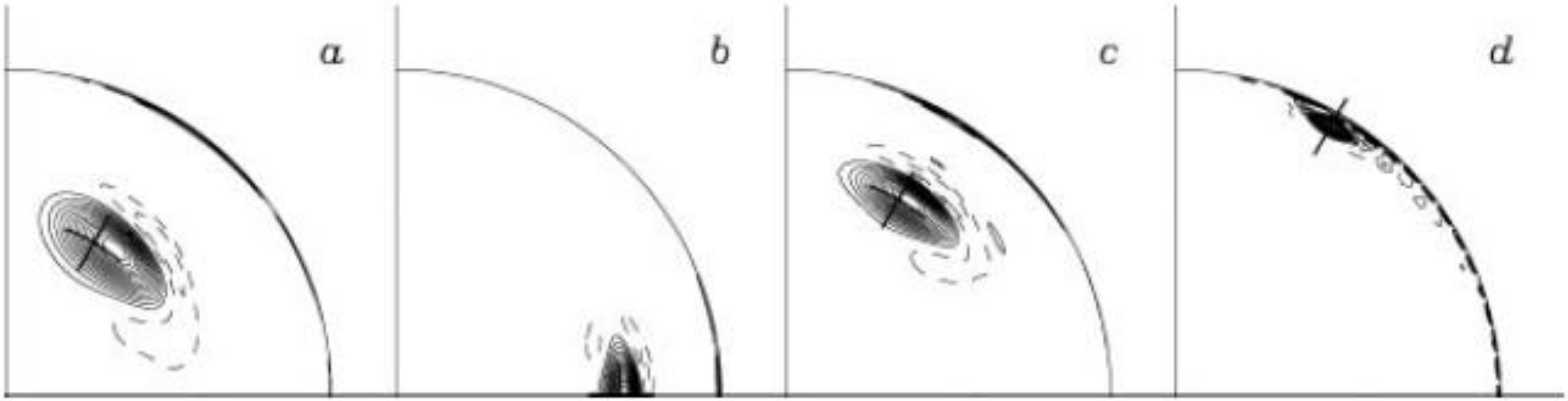
Opacity sensitive to Z
Theory of convection
Convective overshoot



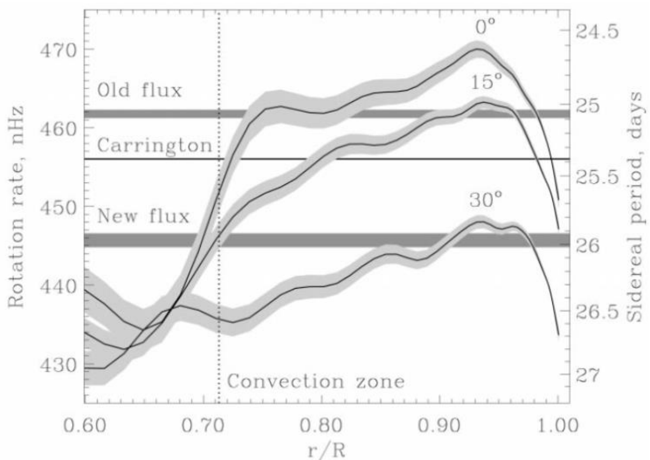
Internal angular velocity

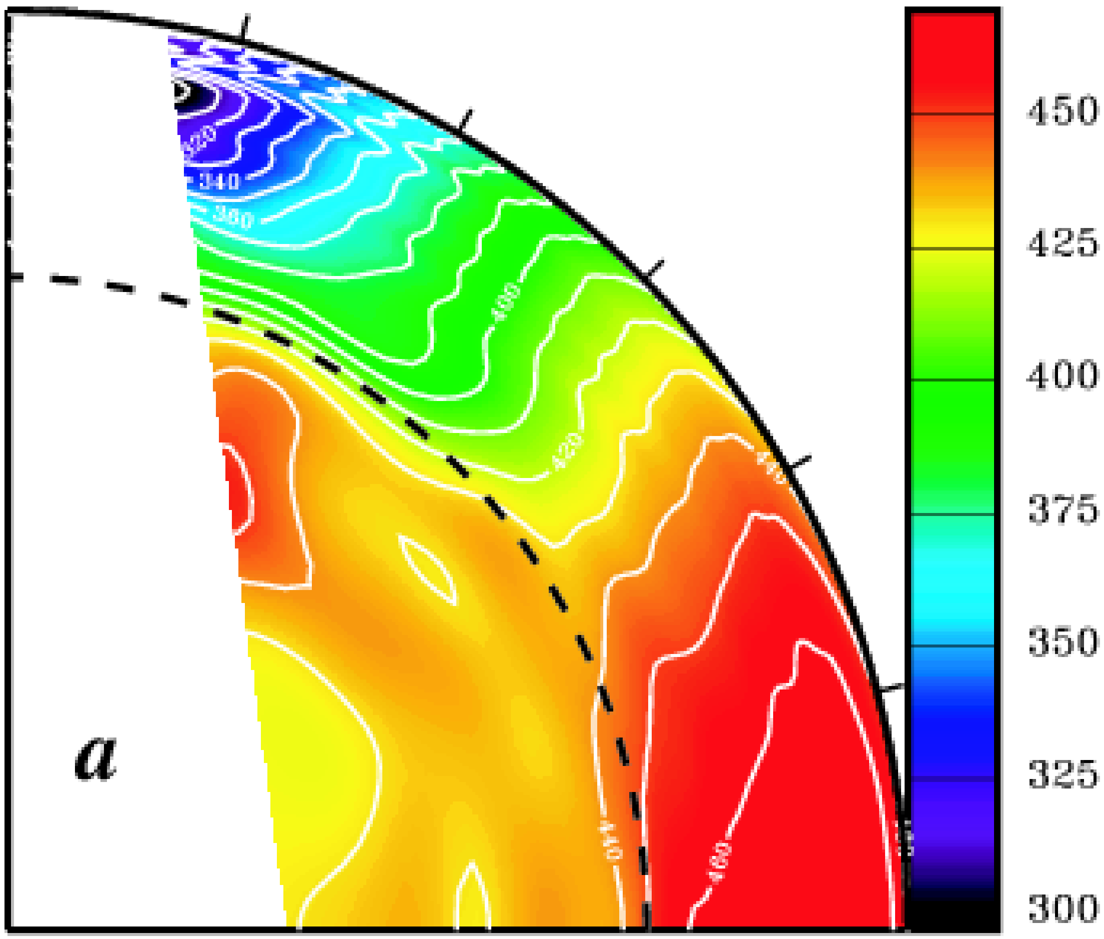
Rotational splittings

$$\omega_{nlm} - \omega_{nl0} = m \int_0^{\pi} \int_0^{R} K(r,\theta)\,\Omega(r,\theta)\, r\mathrm{d}r\, \mathrm{d}\theta$$



Internal angular velocity from helioseismology

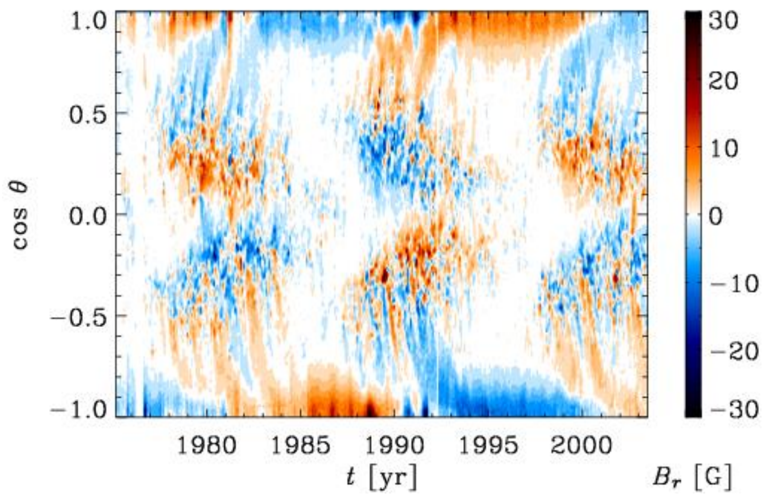


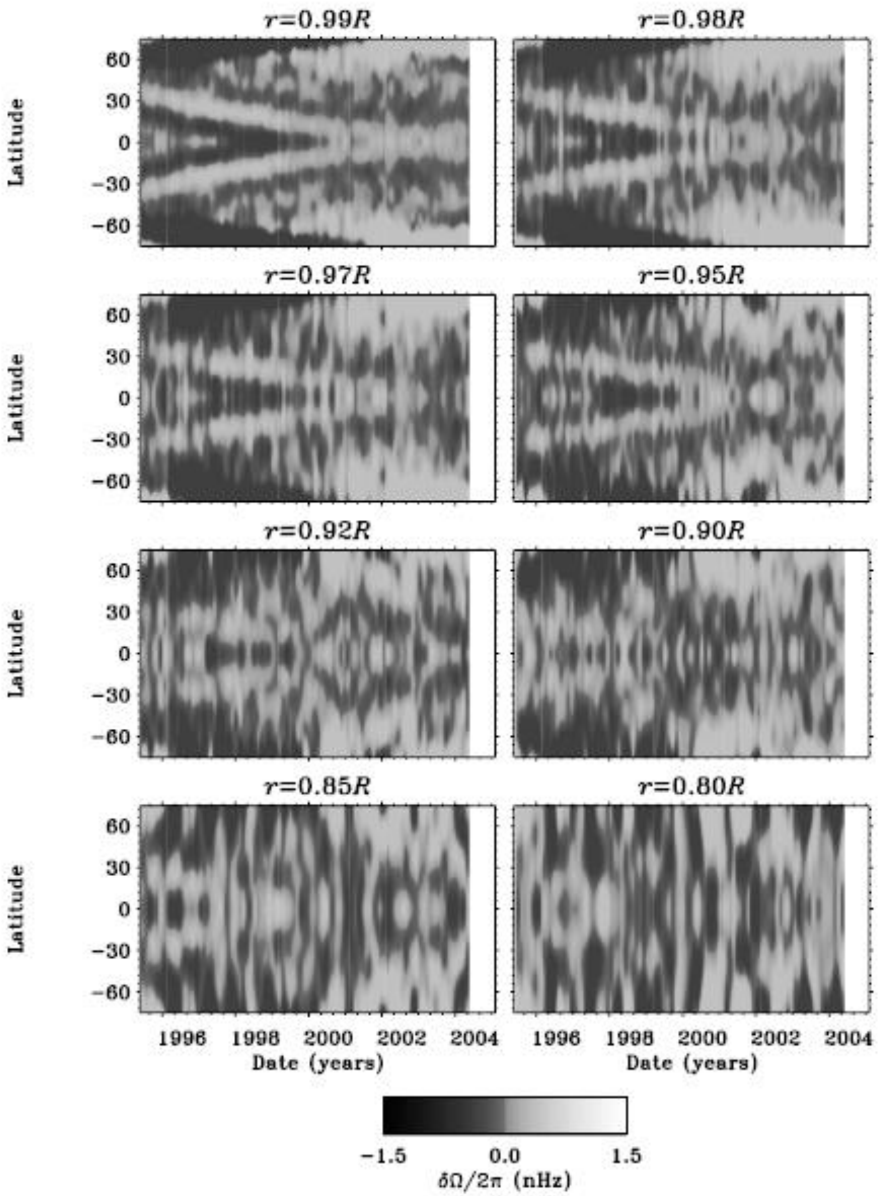


spoke-like at equ.
$d\Omega/dr>0$ at bottom

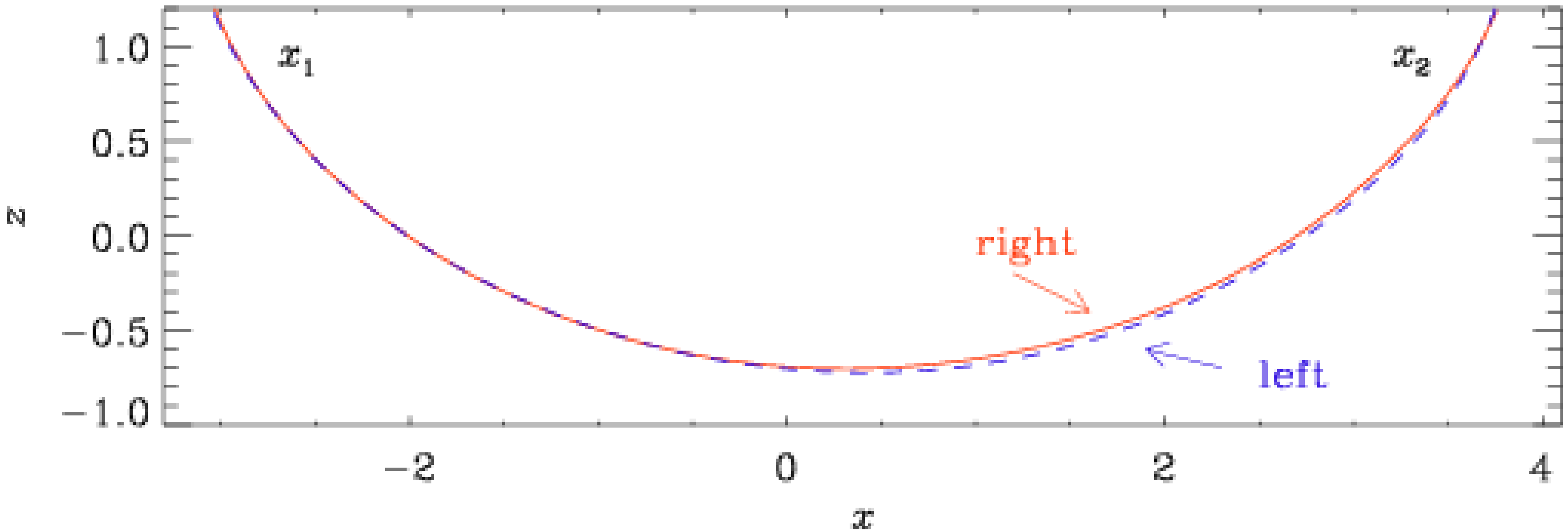
? $d\Omega/dr<0$ at top

Cycle dependence of $\Omega(r,\theta)$



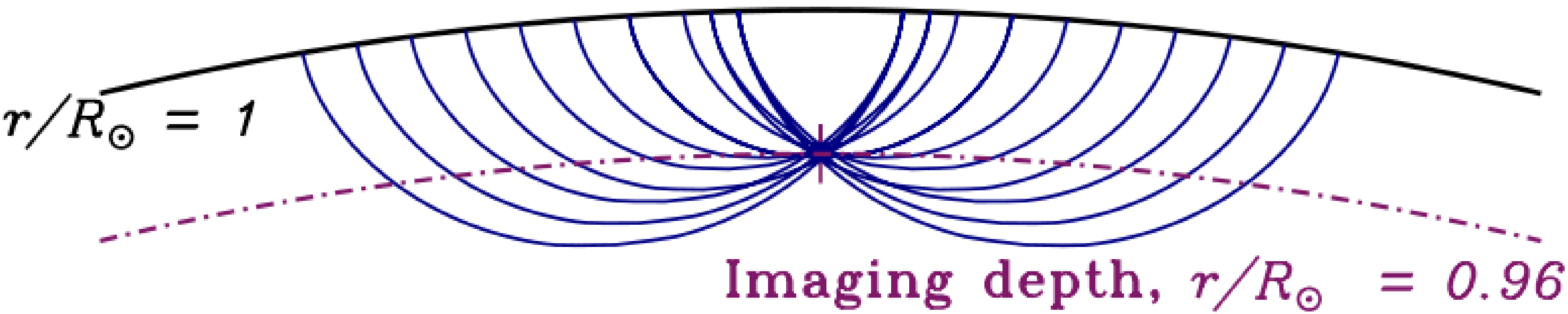


Travel time differences

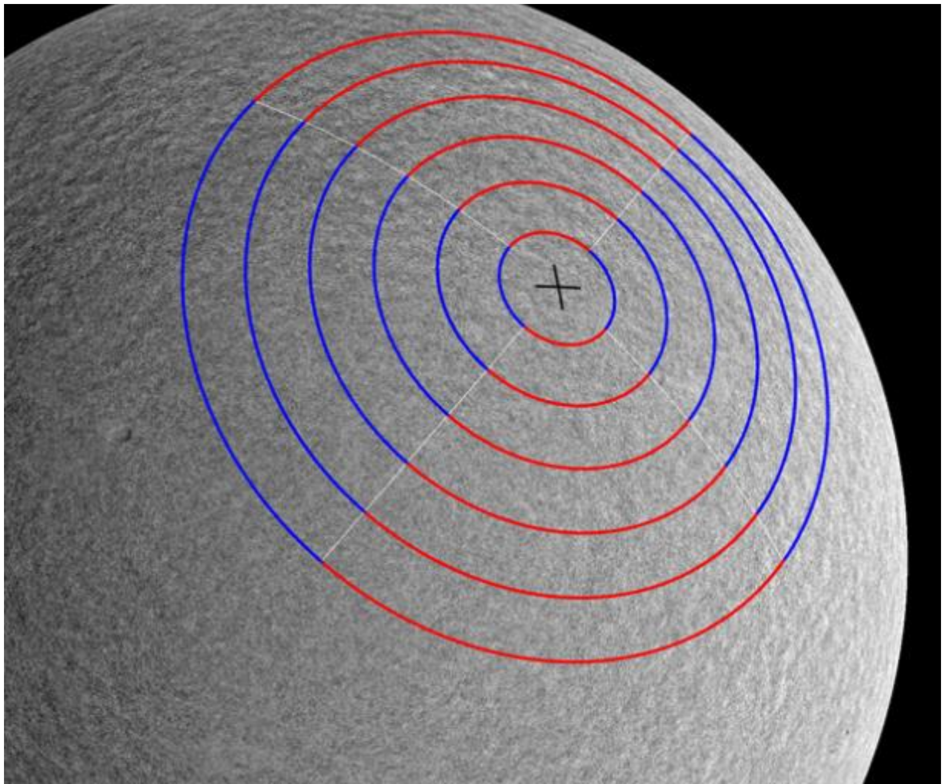


- Contrib. from whole path $$\tau_+ - \tau_- = 2\int \frac{\mathbf{u}\cdot\mathbf{ds}}{c_s^2}$$
- Esp. top layers (c_s small)
- → averaging over rays through same point

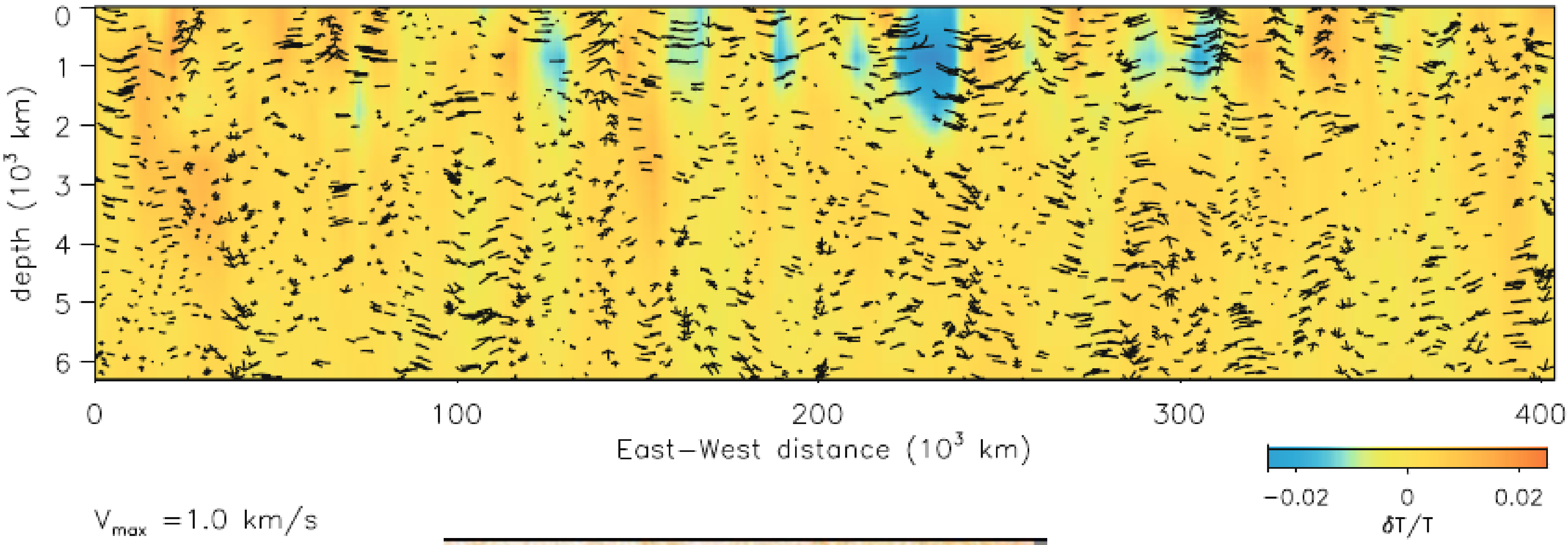
Deep-focusing geometry

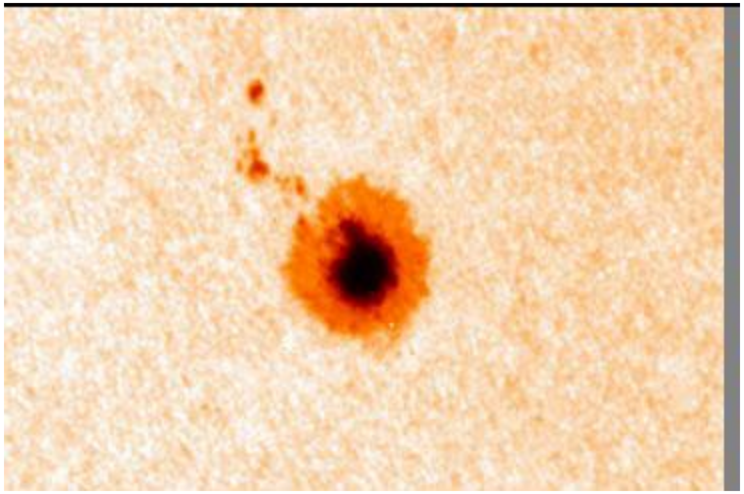


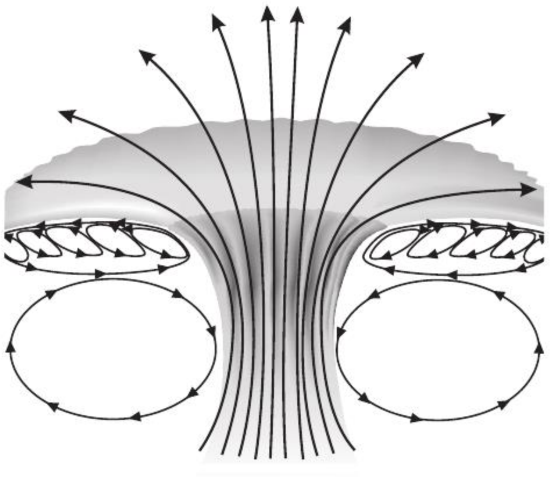
- Removes strong contributions from top layers
- Could they be right?



Turbulence imaging







Sunspot imaging
…but uncertain!

Summary

- Standing waves from 2 traveling ones
- granulation & oscillation different things
- Neutrino problem
- Solar abundance problem
- Internal angular velocity
- Interior convective flow measurements