ASTR/ATOC-5410: Fluid Instabilities, Waves, and Turbulence

Project description November 11, 2016, Axel Brandenburg

Project on solitons

Background

The f-mode is just like deep water waves. In that case it is well known that the presence of surface tension leads to additional modes known as capillary waves (Dias & Kharif, 1999). This leads to characteristic alterations of the f-mode, that tell us something about surface tension. This might also be astrophysically interesting, because magnetic fields leads to an extra tension-like force that could mimic the effects of surface tension.

Project details

1. Derive the linenar dispersion relation

$$\omega^2 = \left(g|\boldsymbol{k}| + T|\boldsymbol{k}|^3\right) \tanh|\boldsymbol{k}|h. \tag{1}$$

2. Following the review by Dias & Kharif (1999), explain the possibility of instabilities through resonance wave interactions in the form

$$n\boldsymbol{k} = \boldsymbol{k}_1 + \boldsymbol{k}_2, \tag{2}$$

$$n\omega(\boldsymbol{k}) = \omega(\boldsymbol{k}_1) + \omega(\boldsymbol{k}_2), \tag{3}$$

   where $n = 2$, 3, ...

3. Verify the effective wave equation given by Dias & Kharif (1999) in the form

$$2\mathrm{i}\frac{\partial A}{\partial t} + p\frac{\partial^2 A}{\partial x^2} + q\frac{\partial^2 A}{\partial y^2} + \gamma A|A|^2 = -\mathrm{i}\epsilon\left(+s\frac{\partial^3 A}{\partial x\partial y^2} + r\frac{\partial^3 A}{\partial x^3} + uA^2\frac{\partial A^*}{\partial x} - vA^2\frac{\partial A}{\partial x}\right) \tag{4}$$

4. Solve this equation numerically under simplifying assumptions.

5. Explain the concept of the bifurcation of waterwaves when phase and group velocities are nearly equal and derive

$$2\mathrm{i}\frac{\partial A}{\partial t} + p\frac{\partial^2 A}{\partial x^2} + q\frac{\partial^2 A}{\partial y^2} + \gamma A|A|^2 = 0 \tag{5}$$

   for this case.

References

Dias, F., & Kharif, C., "Nonlinear gravity and capillary-gravity waves," *Ann. Rev. Fluid Dyn.* **31**, 301-346 (1999).