ASTR/ATOC-5410: Fluid Instabilities, Waves, and Turbulence

Project description November 14, 2016, Axel Brandenburg

Toroidal magnetic field instabilities

Taylor-Couette flow geometries have been used to model instabilities in astrophysical disks Stefani et al. (2006). They also provide an opportunity to study global aspects that are not captured by shearing boxes, but they are simpler than full spheres. An example is the Tayler instability (Tayler, 1973), which is now frequently applied to spherical geometries such as the Tayler–Spruit dynamo (Spruit, 2002).

Background

The eigenfunctions of the Tayler instability are helical. The total field remains non-helical if both contributions balance initially. A small imbalance can however be amplified, as was shown first by Chatterjee et al. (2011) in magnetic buoyancy instabilities and later for the Tayler instability Gellert et al. (2011).

Project details

We choose a basic state with zero velocity and zero axial component of the magnetic field (B_z). The azimuthal component of the magnetic field is

$$B_\varphi = B_0\,(s/s_0)\exp[-(s-s_0)^2/\sigma^2], \tag{1}$$

where B_0 is a normalization constant, $s_0 = 2$ and $\sigma = 0.2$. We choose B_0 and c_s in such a way that the sound speed is much larger than the Alfvén speed.

1. Verify that the unperturbed initial state remains in equilibrium for some time. You may use analytic or numerical techniques. A setup for the Pencil Code is listed at the bottom of the page `http://lcd-www.colorado.edu/~axbr9098/teach/PencilCode/MixedTopics.html`

2. Add a perturbation of the magnetic field with an infinitesimally small net helicity given by the following expression:
$$\boldsymbol{A} = \delta s \cos\left(z\frac{n_z\pi}{h}\right)\begin{pmatrix}\sin m\varphi\\ 0\\ \cos m\varphi\end{pmatrix}, \tag{2}$$
where δ is an arbitrary small amplitudes which we set to 10^{-7} for all the simulations and $k_z = q/s_\mathrm{in} = n_z\pi/h$ is the vertical wavenumber of the perturbation.

3. Check that the evolution equations for the two eigenmodes are described by
$$\frac{\partial\hat{\boldsymbol{L}}}{\partial t} = \gamma\hat{\boldsymbol{L}} - \left(\mu|\hat{\boldsymbol{L}}|^2 + \mu_*|\hat{\boldsymbol{R}}|^2\right)\hat{\boldsymbol{L}}, \tag{3}$$
$$\frac{\partial\hat{\boldsymbol{R}}}{\partial t} = \gamma\hat{\boldsymbol{R}} - \left(\mu|\hat{\boldsymbol{R}}|^2 + \mu_*|\hat{\boldsymbol{L}}|^2\right)\hat{\boldsymbol{R}}. \tag{4}$$

4. Discuss the connection with auto-catalysis of left- and right-handed bio-molecules, L and R, and assume that they are capable of polymerizing to form homochiral dimers,

$$\mathsf{L}+\mathsf{L}\xrightarrow{\mu}\mathsf{LL},\quad \mathsf{R}+\mathsf{R}\xrightarrow{\mu}\mathsf{RR}, \tag{5}$$

as well as heterochiral dimers,

$$\mathsf{L}+\mathsf{R}\xrightarrow{\mu_*}\mathsf{LR}. \tag{6}$$

5. Determine an empirical phase diagram that characterize the evolution of the handedness.

References

Bonanno, A., & Guarnieri, F., "On the possibility of helicity oscillations in the saturation of the Tayler instability," *Astron. Nachr.* submitted, arXiv:1610.00616 (2016).

Bonanno, A., Brandenburg, A., Del Sordo, F., & Mitra, D., "Breakdown of chiral symmetry during saturation of the Tayler instability," *Phys. Rev. E* **86**, 016313 (2012).

Chatterjee, P., Mitra, D., Brandenburg, A., & Rheinhardt, M., "Spontaneous chiral symmetry breaking by hydromagnetic buoyancy," *Phys. Rev. E* **84**, 025403R (2011).

Gellert, M., Rüdiger, G., & Hollerbach, R., "Helicity and alpha-effect by current-driven instabilities of helical magnetic fields," *Month. Not. Roy. Astron. Soc.* **414**, 2696-2701 (2011).,

Spruit, H. C., "Dynamo action by differential rotation in a stably stratified stellar interior," *Astron. Astrophys.* **381**, 923-932 (2002).

Stefani, F., Gundrum, T., Gerbeth, G., Rüdiger, G., Schultz, M., Szklarski, J., & Hollerbach, R., "Experimental evidence for magnetorotational instability in a Taylor-Couette flow under the influence of a helical magnetic field," *Phys. Rev. Lett.* **97**, 184502 (2006).

Tayler, R. J., "The adiabatic stability of stars containing magnetic fields-I.Toroidal fields," *Month. Not. Roy. Astron. Soc.* **161**, 365-380 (1973).