## ASTR/ATOC-5410: Fluid Instabilities, Waves, and Turbulence CU Boulder Course Syllabus (Fall 2016)

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Course Times: Mon., Wed., Fri., 11:00–11:50 am

**Location:** Duane Physics, Room E-126

**Course web page:** http://lcd-www.colorado.edu/~axbr9098/teach/ASTR\_5410 **Office hours:** JILA A604: Wed. 2:00-3:30, by appointment, or if you see me

**SUMMARY** The hydrodynamic equations have many nonlinearities. Such equations have in general multiple solutions, some are stable and some are unstable. Most of the time one is only interested in the stable ones, because the unstable ones could be short-lived. But sometimes the opposite is true, for example in accretion disks where an instability leads to turbulence, which in turn leads to enhanced transport ("turbulent viscosity"), without which a disk would hardly accrete (molecular or microphysical viscosity is too small).

In this course we discuss many of the known instabilities. Here we focus on linear instabilities, which can be analyzed by linearizing the governing equations. Even then the equations are sometimes too complicated so that it is useful to learn how to solve them numerically. Waves are a natural counterpart to instabilities, because they are described by the same linearized equations. Those equations hold as long as the amplitudes are small enough. When this is no longer the case, things become interesting; finite amplitude waves (such as solitons) are possible, solutions can become chaotic or rather turbulent.

This course is an elective for the APS graduate major and minor. Pre-requisite courses include

### **COURSE GOALS** At the end of the course, you should be able to:

- understand many of the other instabilities that are out there.
- discover new instabilities
- employ numerical eigenvalue solvers to study dispersion relations
- characterize properties of important wave types in astro- and geophysics
- understand important aspects in turbulent, such as forward and inverse cascades
- characterize mixing properties of turbulence
- compute turbulent transport coefficients analytically and numerically

# **COURSE MATERIAL**

## **Primary textbooks:**

- Primary Text: Kundu, P. K., Cohen, I. M., & Dowling, D. R. 2016, Fluid Mechanics (6th ed., Academic Press) (KCD)
- Chandrasekhar, S. 1961, Hydrodynamic and Hydromagnetic Stability (Dover Publications, New York) (C61)
- Davidson, P. A. 2015, Turbulence: an introduction for scientists and engineers (2nd ed., Oxford University Press, Oxford) (D15)
- Haken, H. 1983, Synergetics An Introduction (3rd ed., Springer, Berlin) (H83), §§7.4 & 8.9–8.13

## **GRADING**

The final grade will be assembled from the following components:

5 Homework Sets	5/10
Midterm Exam	2/10
Final Project/Paper & Presentation	3/10

### SCHEDULE OF TOPICS

The dates listed here for each set of topics are approximate. There will be an actively maintained web page that stays up-to-date on the topics to be covered in each class session. Below, each sub-topic is listed along with relevant chapters from Kundu et al. It is highly recommended that students become familiar with these topics before their discussion in class.

### 1 Instabilities

# 1.1 Week 1: Local analyses

Thermal instability (Field 1965); Magnetorotational instability (MRI, Velikhov 1959, Balbus & Hawley 1991); Magnetothermal instability (MTI); Heat flux buoyancy instability (HBI);

#### 1.2 Week 2: Fluid between two boundaries

Rayleigh-Benard problem (KCD-11.4); Comments on the problem in spherical geometry (C61-VI); Comments on the weakly nonlinear problem (H83); Effect of rotation (C61-III); Effect of a magnetic field (C61-IV).

## 1.3 Week 3: Shear flows

Couette flow (KCD-11.6), effects of magnetic field in small gap width approximation; Inflection point instability

# 1.4 Week 4: Stability of superposed fluids

Rayleigh-Taylor instability (C61-X); Kelvin-Helmholtz instability (C61-XI); Application to Inertial Confinement Fusion; Jeans instability (C61-XIII); Baroclinic instability (KCD-13.15)

## 1.5 Week 5: Magnetic instabilities

Other instabilities (Bell instability, Dynamo instability, etc)

# 2 Waves

## 2.1 Week 6: Waves in a stratified layer

Acoustic waves in an isothermal layer (cf. KCD-15.2); Gravity waves in an isothermal layer (KCD-8.8); (& gravity wave with rotation).

# 2.2 Week 7: Geophysical & astrophysical applications of waves

WKB solution (KCD-13.12, & Lee wave); Helioseismology (Stix 2002).

## 2.3 Week 8: Waves in rivers, oceans, and the atmosphere

Solitons (Korteweg-de Vries equation, derivation from shallow water theory, KCD-8.6, p. 383); Capillary waves (KCD-8.3, p. 365), Rossby wave (KCD-13.13), (& Kelvin wave, KCD-13.10).

## 3 Turbulence

### 3.1 Week 9: Basics & basic tools

Dimensional arguments; Energy spectra, structure functions; Kolmogorov's 4/5th law (Dav15 §6.2); Intermittency (Dav15 §6.5).

## 3.2 Week 10: Laboratory, geophysical, & astrophysical applications

Geostrophic turbulence (KCD-13.16); Wind tunnel turbulence, turbulence in planetary boundary layer; Turbulence in solar wind and interstellar medium.

### 3.3 Week 11: Turbulence in a box

Periodic domains (Dav15 §7.2); Structure in chaos (Dav15 §7.3); Decaying turbulence (Dav15 §§6.1, 6.3); Forced turbulence; 2-D turbulence (§10.1), inverse cascade of energy.

## 3.4 Week 12: Turbulence with magnetic fields

MHD turbulence (Dav15 §§9.3, 9.4); Strong and weak turbulence, local anisotropy; Helical MHD turbulence, inverse cascade of magnetic helicity; Relation to conservation laws.

### 3.5 Week 13: Models & modeling results

Shell models; Coherent structures; Alignment properties with eigenvectors of rate of strain matrix; Compressible convection, compressible Boussinesq limit.

## 3.6 Week 14: Mean-field theory

Two-scale analysis, turbulent diffusion; Minimal  $\tau$  approximation,  $\alpha$  effect,  $\Lambda$  effect; Test-field method for determining transport coefficients from numerical (turbulence) simulations.

## 3.7 Week 15: Project presentations

#### HOMEWORK SETS

There will be approximately five homework assignments distributed throughout the semester. A detailed schedule of distribution and due dates will be given out in class and posted on the course web page. All but one of them will be mostly mathematical "problem sets." The remaining one will be a mini-project to find a popular news article on a topic relevant to this course, and critique it. Requirements for this written critique will be handed out later.

Hardcopy submissions are preferred, but email is fine, too. Students choosing the latter option are encouraged to write out solutions long-hand (neatly!) and scan them. This way you won't be tempted to leave out intermediate steps when typing in equations.

Homeworks are due at the beginning of class on the dates to be given. However, since it is our top priority that students have sufficient time to learn from the homework sets, we will grant one lateness exception per student: One homework set can be turned in up to three business days late with no penalty. (Though please let me know, in person or email, if/when you'll be taking this option.) Any other homework that is late will incur a penalty of a 5% lower grade per business day that it is late.

#### MIDTERM EXAM

There will be at least one full class period (maybe two) devoted to reviewing the relevant material prior to the exam. Details about its format will be forthcoming.

#### FINAL PROJECT OR PAPER

In lieu of a sit-down final exam, there will be a project or term paper that will enable you to explore a chosen topic in a bit more detail, and gain some extra experience with either scientific writing or computing. The project can involve either of the two following components (or, if you're ambitious, both):

- Some kind of mathematical or computational calculation that explores some topic relevant to the course. The types of things you could do include:
  - a. exploring a wider "parameter space" of a textbook model,
  - b. numerically solving an equation (that was presented in class) that has no analytic solution,
  - b. constructing your own model or simulation,
  - c. downloading and analyzing some publicly available data, or
  - d. testing (or debunking?) the claims made in a recent paper.

Feel free to use whatever tools you want (i.e., computing languages, software packages, output formats), but the whole thing—including source code and data—must be submitted.

Additional information, including lists of possible topic ideas and deadlines, will be distributed during the semester.

#### IN-CLASS ENGAGEMENT

Attendance is important, because frequently the class will separate either individually or into small groups. In these break-out sessions, you will work out some interesting implication, or draw some useful conclusion, from the lecture material. No written answers need to be submitted; just verbal discussion of the outcomes.

The grade for this component is essentially "try" or "not-try;" i.e., all you need to get your 1/10 (of the total grade) is to attend class regularly and show consistent engagement with the material. This can be demonstrated

through asking questions in class, answering questions that someone else has raised, and/or participating actively in the break-out sessions.

#### ACADEMIC INTEGRITY

All students at CU Boulder are responsible for knowing and adhering to the academic integrity policy of this institution. Violations of this policy may include: cheating, plagiarism, aid of academic dishonesty, fabrication, lying, bribery, and threatening behavior. All incidents of academic misconduct will be reported to the Honor Code Council (honor@colorado.edu; 303-735-2273). Students who are found to be in violation of the academic integrity policy will be subject to both academic sanctions from the faculty member and non-academic sanctions (including but not limited to university probation, suspension, or expulsion).

For this course, I encourage you to discuss the assignments and topics with your fellow students. However, everything that is written up and submitted must be your own independent work. If you do collaborate with other students, a good time to split off from the group is when you start to write up your answers. If someone were to ask you questions about your work, you should be able to explain everything about how & why you did it the way you did.

### STUDENTS WITH DISABILITIES

If you qualify for accommodations because of a disability, please submit to me a letter from Disability Services in a timely manner (for exam accommodations provide your letter at least one week prior to the exam) so that your needs can be addressed. Disability Services determines accommodations based on documented disabilities, and they can be contacted at 303-492-8671, by e-mail (dsinfo@colorado.edu), or on the web (disabilityservices.colorado.edu).

## **RELIGIOUS OBSERVANCES**

Campus policy regarding religious observances requires that faculty make every effort to deal reasonably and fairly with all students who, because of religious obligations, have conflicts with scheduled exams, assignments or required attendance. If you have religious obligations that result in schedule conflicts, please contact me in the first two weeks of class to make alternate arrangements.

## DISCRIMINATION AND HARASSMENT

CU Boulder is committed to maintaining a positive learning, working, and living environment. The University of Colorado does not discriminate on the basis of race, color, national origin, sex, age, disability, creed, religion, sexual orientation, or veteran status in admission and access to, and treatment and employment in, its educational programs and activities. CU Boulder will not tolerate acts of discrimination or harassment based upon protected classes or related retaliation against or by any employee or student. For purposes of this CU Boulder policy, "protected classes" refers to race, color, national origin, sex, pregnancy, age, disability, creed, religion, sexual orientation, gender identity, gender expression, or veteran status. Individuals who believe they have been discriminated against should contact the Office of Discrimination and Harassment (ODH) at 303-492-2127 or the Office of Student Conduct (OSC) at 303-492-5550.

#### **CLASSROOM BEHAVIOR**

Students and faculty each have responsibility for maintaining an appropriate learning environment. Those who fail to adhere to such behavioral standards may be subject to discipline. Professional courtesy and sensitivity are especially important with respect to individuals and topics dealing with differences of race, color, culture, religion, creed, politics, veteran status, sexual orientation, gender, gender identity and gender expression, age, disability, and nationalities. Class rosters are provided to the instructor with the student's legal name. I will

gladly honor your request to address you by an alternate name or gender pronoun. Please advise me of this preference early in the semester so that I may make appropriate changes to my records.